ISSN 2075-9827 e-ISSN 2313-0210
Carpathian Math. Publ. 2016, **8** (1), 127–133
doi:10.15330/cmp.8.1.127-133

http://www.journals.pu.if.ua/index.php/cmp
Карпатські матем. публ. 2016, Т.8, №1, С.127–133

MOZHYROVSKA Z.H.

HYPERCYCLIC OPERATORS ON ALGEBRA OF SYMMETRIC ANALYTIC FUNCTIONS ON ℓ_p

In the paper, it is proposed a method of construction of hypercyclic composition operators on $H(\mathbb{C}^n)$ using polynomial automorphisms of $\mathbb{C}^n$ and symmetric analytic functions on ℓ_p. In particular, we show that a "symmetric translation" operator is hypercyclic on a Fréchet algebra of symmetric entire functions on ℓ_p which are bounded on bounded subsets.

Key words and phrases: hypercyclic operators, functional spaces, symmetric functions.

Lviv University of Trade and Economics, 10 Tuhan-Baranovskyi str., 79005, Lviv, Ukraine
E-mail: zoryana.math@gmail.com

INTRODUCTION

The theory of hypercyclicity studies the long-term behavior of continuous operators on topological spaces. Let X be a Fréchet (linear complete metric) space.

Definition 1. *A continuous linear operator $T : X \to X$ is called hypercyclic if there is a vector $x_0 \in X$ for which the orbit under T, $\mathrm{Orb}(T, x_0) = \{x_0, Tx_0, T^2x_0, \ldots\}$ is dense in X. Every such vector x_0 is called a hypercyclic vector of T.*

The classical Birkhoff's theorem [6] asserts that any operator of composition with translation $x \mapsto x + a$, $T_a\colon f(x) \mapsto f(x + a)$ is hypercyclic on a space of entire functions $H(\mathbb{C})$ on a complex plane $\mathbb{C}$ if $a \neq 0$. The Birkhoff's translation T_a has also been regarded as a differentiation operator

$$T_a(f) = \sum_{n=0}^{\infty} \frac{a^n}{n!} D^n f.$$

A generalization of Birkhoff's theorem was proved by Godefroy and Shapiro in [9]. They showed that if $\varphi(z) = \sum_{|\alpha|\geq 0} c_\alpha z^\alpha$ is a non-constant entire function of exponential type on $\mathbb{C}^n$, then the operator

$$f \mapsto \sum_{|\alpha|\geq 0} c_\alpha D^\alpha f, \qquad f \in H(\mathbb{C}^n), \tag{1}$$

is hypercyclic. Moreover, in [9], it is proved that any continuous linear operator T on $H(\mathbb{C}^n)$, which commutes with translations and is not a scalar multiple of the identity, can be expressed by (1) and so is hypercyclic as well.

Let us recall that an operator C_Φ on $H(\mathbb{C}^n)$ is said to be a *composition operator* if $C_\Phi f(x) = f(\Phi(x))$ for some analytic map $\Phi\colon \mathbb{C}^n \to \mathbb{C}^n$. It is known that only translation operator T_a for

УДК 517.98
2010 *Mathematics Subject Classification:* 47A16, 46E10.

some $a \neq 0$ is a hypercyclic composition operator on $H(\mathbb{C})$ [5]. However, if $n > 1$, $H(\mathbb{C}^n)$ supports more hypercyclic composition operators. Bernal-González [4] established some necessary and sufficient conditions for a composition operator by an affine map to be hypercyclic.

In [14], it was proposed a method of construction of hypercyclic composition operators on $H(\mathbb{C}^n)$, which can not be described by formula (1), using symmetric analytic functions on ℓ_1. The purpose of this paper is a generalization of the method for the space ℓ_p, $1 < p < \infty$. Also similarly to [14], we show that a symmetric translation operator is hypercyclic on a Fréchet algebra $H^n_{bs}(\ell_p)$ of symmetric entire functions on ℓ_p which are bounded on bounded subsets. More about hypercyclic composition operators the reader can find in [13].

In Section 1, we discuss some relationship between polynomial automorphisms on $\mathbb{C}^n$ and an operation of changing of polynomial bases in an algebra of symmetric analytic functions on the Banach space of summing sequences, ℓ_p. In Section 2, we prove the hypercyclicity of a special operator on the algebra of symmetric analytic functions on ℓ_p which plays the role of translation in this algebra. We consider, in the third section, an algebra which is the completion of the space of symmetric polynomials on ℓ_p endowed with the uniform topology on bounded subsets and we prove hypercyclicity of our special operator on this algebra.

Let us recall a well known Kitai-Gethner-Shapiro's theorem which is also known as the Hypercyclicity Criterion.

Theorem 1 (Hypercyclicity Criterion)**.** *Let X be a separable complete linear metric space and $T\colon X \to X$ be a linear and continuous operator. Suppose there exist X_0, Y_0 dense subsets of X, a sequence (n_k) of positive integers and a sequence of mappings (possibly nonlinear, possibly not continuous) $S_n\colon Y_0 \to X$ so that*

1. *$T^{n_k}(x) \to 0$ for every $x \in X_0$ as $k \to \infty$,*
2. *$S_{n_k}(y) \to 0$ for every $y \in Y_0$ as $k \to \infty$,*
3. *$T^{n_k} \circ S_{n_k}(y) = y$ for every $y \in Y_0$.*

Then T is hypercyclic.

The operator T is called the operator that satisfy the *Hypercyclicity Criterion for full sequence* if we can chose $n_k = k$.

For details of the theory of analytic functions on Banach spaces we refer the reader to Dineen's book [8]. Note that an analogue of the Godefroy-Shapiro's theorem for a special class of entire functions on Banach space with separable dual was proved by Aron and Bés in [2]. Current state of theory of symmetric analytic functions on Banach spaces can be found in [1, 10]. A detailed survey of hypercyclic operators is given by Grosse-Erdmann in [3, 11, 12].

1 Algebra of symmetric functions

Let X be a Banach space with a symmetric basis $(e_i)_{i=1}^{\infty}$. A function g on X is called *symmetric* if for every $x = \sum\limits_{i=1}^{\infty} x_i e_i \in X$, $g(x) = g\Big(\sum\limits_{i=1}^{\infty} x_i e_i\Big) = g\Big(\sum\limits_{i=1}^{\infty} x_i e_{\sigma(i)}\Big)$ for an arbitrary permutation σ on the set $\{1, ..., m\}$ for any positive integer m. The sequence of homogeneous polynomials $(P_j)_{j=1}^{\infty}$, $\deg P_k = k$ is called a *homogeneous algebraic basis* in the algebra of symmetric polynomials, if for every symmetric polynomial P of degree n on X there exists a polynomial q on $\mathbb{C}^n$ such that $P(x) = q(P_1(x), \dots, P_n(x))$.

We denote by $\mathcal{P}_s(\ell_p)$ algebra symmetric continuous polynomials. Let $\lceil p \rceil$ be the smallest integer that is greater than or equal to p. In [10], it is proved that the polynomials

$$F_k\left(\sum_{i=1}^{\infty} a_i e_i\right) = \sum_{i=1}^{\infty} a_i^k \tag{2}$$

for $k = \lceil p \rceil, \lceil p \rceil + 1, \dots$ form an algebraic basis in $\mathcal{P}_s(\ell_p)$.

So, there are no symmetric polynomials of degree less than $\lceil p \rceil$ in $\mathcal{P}_s(\ell_p)$ and if $\lceil p_1 \rceil = \lceil p_2 \rceil$, then $\mathcal{P}_s(\ell_{p_1}) = \mathcal{P}_s(\ell_{p_2})$. Thus, without loss of generality we can consider $\mathcal{P}_s(\ell_p)$ only for integer values of p. Throughout, we will assume that p is an integer, $1 \leq p < \infty$.

Corollary 1 ([1]). *Given $(\xi_1, \dots, \xi_n) \in \mathbb{C}^n$, there is $x \in \ell_p^{n+p-1}$ such that*

$$F_p(x) = \xi_1, \dots, F_{n+p-1}(x) = \xi_n.$$

This result shows that any $P \in \mathcal{P}_s(\ell_p)$ has a unique representation in terms of $\{F_k\}$, in sense that if $q \in \mathcal{P}(\mathbb{C}^n)$ for some n is such that $P(x) = q(F_p(x), \dots, F_{n+p}(x))$, and if $q' \in \mathcal{P}(\mathbb{C}^m)$ for some m is such that $P(x) = q'(F_p(x), \dots, F_{m+p}(x))$, with, say, $n \leq m$, then $q'(\xi_1, \cdots, \xi_m) = q(\xi_1, \cdots, \xi_n)$.

Let us denote by $\mathcal{P}_s^n(\ell_p)$, $n \geq p$, the subalgebra of $\mathcal{P}_s(\ell_p)$ generated by $\{F_p, \dots, F_n\}$.

Denote by $H_{bs}^n(\ell_p)$ an algebra of entire symmetric functions on ℓ_p which is topologically generated by polynomials $F_p, \dots, F_n$. It means that $H_{bs}^n(\ell_p)$ is the completion of the algebraic span of $F_p, \dots, F_n$ in the uniform topology on bounded subsets. We say that polynomials $Q_p, \dots, Q_n$ (not necessary homogeneous) form an *algebraic basis* in $H_{bs}^n(\ell_p)$ if they topologically generate $H_{bs}^n(\ell_p)$. Evidently, if $(Q_j)_{j=1}^{\infty}$ is a homogeneous algebraic basis in $\mathcal{P}_s(\ell_p)$, then $(Q_p, \dots, Q_n)$ is an algebraic basis in $H_{bs}^n(\ell_p)$.

2 SYMMETRIC TRANSLATION

In this section, we construct a special operator on the algebra of symmetric analytic functions on ℓ_p. We start with an evident statement, which actually is a very special case of the Universal Comparison Principle (see [11, Proposition 4]).

Proposition 1. *Let T be a hypercyclic operator on X and A be an isomorphism of X. Then $A^{-1}TA$ is hypercyclic.*

We will say that $A^{-1}TA$ is a *similar* operator to T. If $T = C_\alpha$ is a composition operator on $H(\mathbb{C}^n)$ and $A = C_\Phi$ is a composition by an analytic automorphism Φ of $\mathbb{C}^n$, then $A^{-1}TA = C_{\Phi \circ \alpha \circ \Phi^{-1}}$ is a composition operator too. If A is a composition with a polynomial automorphism, we will say that $A^{-1}TA$ is *polynomially similar* to T. Now we consider operators which are similar to the translation composition $T_a\colon f(x) \mapsto f(x+a)$ on $H(\mathbb{C}^n)$.

Let us denote by $\mathcal{F}_p^n$ the mapping from ℓ_p to $\mathbb{C}^{n+1-p}$, $n \geq p$, given by

$$\mathcal{F}_p^n : x \mapsto (F_p(x), \dots, F_n(x)).$$

It is known (see [1]) that the map

$$C_{\mathcal{F}_p^n}\colon f(t_1, \dots, t_n) \mapsto f(F_p(x), \dots, F_n(x))$$

is a topological isomorphism from the algebra $H(\mathbb{C}^{n+1-p})$ to the algebra $H^n_{bs}(\ell_p)$.

Easy to see that for symmetric function $f(x)$ on ℓ_p the function $f(x+y)$ is not symmetric for some fixed $y \in \ell_p$. The space of symmetric function is not invariant respect to certain translation operator $f(x) \mapsto f(x+y)$. We propose another translation on ℓ_p, which keep the space of symmetric analytic functions.

Let $x, y \in \ell_p$, $x = (x_1, x_2, \ldots)$ and $y = (y_1, y_2, \ldots)$. We put

$$x \bullet y := (x_1, y_1, x_2, y_2, \ldots).$$

We note the basic properties of symmetric translation.

1. If $x = \sigma_1(u)$ i $y = \sigma_2(v)$ for some permutations σ_1, σ_2 then $x \bullet y = \sigma(u \bullet v)$ for some permutation σ on $\mathbb{N}$.
2. $\|x \bullet y\|^p = \|x\|^p + \|y\|^p$.
3. For any natural $n \geq p$
$$F_n(x \bullet y) = F_n(x) + F_n(y). \tag{3}$$

We define

$$\mathcal{T}_y(f)(x) := f(x \bullet y)$$

and will say that $x \mapsto x \bullet y$ is the *symmetric translation* and the operator $\mathcal{T}_y$ is the *symmetric translation operator*. It is clear that if f is a symmetric function, then $f(x \bullet y)$ is a symmetric function for any fixed y. In [7], it is proved that $\mathcal{T}_y$ is a topological isomorphism from the algebra of symmetric analytic functions to itself.

Let $g \in H^n_s(\ell_p)$ and $\alpha = (\alpha_1, \ldots, \alpha_n)$. Set for $f = (\mathcal{F}^{\mathbf{F}}_n)^{-1} g$

$$\mathcal{D}^\alpha g := \mathcal{F}^{\mathbf{F}}_n D^\alpha (\mathcal{F}^{\mathbf{F}}_n)^{-1} g = \left(\frac{\partial^{\alpha_1}}{\partial t_1^{\alpha_1}} \cdots \frac{\partial^{\alpha_n}}{\partial t_n^{\alpha_n}} f\right)(F_1(\cdot), \ldots, F_{p+n-1}(\cdot)).$$

Theorem 2. *Let $y \in \ell_p$ such that $(F_p(y), \ldots, F_{p+n-1}(y))$ is a nonzero vector in $\mathbb{C}^n$. Then the symmetric translation operator $\mathcal{T}_y$ is hypercyclic on $H^n_{bs}(\ell_p)$. Moreover, every operator $\mathcal{A}$ on $H^n_s(\ell_p)$ which commutes with $\mathcal{T}_y$ and is not a scalar multiple of the identity is hypercyclic and can be represented by*

$$\mathcal{A}(g) = \sum_{|\alpha| \geq 0} c_\alpha \mathcal{D}^\alpha g, \tag{4}$$

where c_α are coefficients of a non-constant entire function of exponential type on $\mathbb{C}^n$.

Proof. Let $a = (F_p(y), \ldots, F_{p+n-1}(y)) \in \mathbb{C}^n$. If $g \in H^n_{bs}(\ell_p)$, then

$$g(x) = C_{\mathcal{F}^n_p}(f)(x) = f(F_p(x), \ldots, F_{p+n-1}(x))$$

for some $f \in H^n_s(\ell_1)$ and property (3) implies that

$$\begin{aligned}
\mathcal{T}_y(g)(x) &= g(x \bullet y) = f(F_p(x \bullet y), \ldots, F_{p+n-1}(x \bullet y)) \\
&= f(F_p(x) + F_p(y), \ldots, F_{p+n-1}(x) + F_{p+n-1}(y)) \\
&= C_{\mathcal{F}^n_p}((f)(t+a)) = C_{\mathcal{F}^n_p}(T_a(f)(t)).
\end{aligned}$$

Since the set $(T^k_a(f))^\infty_{k=1}$ is dense in $H(\mathbb{C}^n)$, then set $(\mathcal{T}^k_y(g))^\infty_{k=1} = (C_{\mathcal{F}^n_p}(T^k_a(f)))^\infty_{k=1}$ is dense in $H^n_{bs}(\ell_p)$. So, the symmetric translation of operator $\mathcal{T}_y$ is hypercyclic on $H^n_{bs}(\ell_p)$. Since $\mathcal{T}_y(g)(x) = \mathcal{F}^{\mathbf{F}}_n T_a (\mathcal{F}^{\mathbf{F}}_n)^{-1}(g)(x)$, the proof of (4) follows from Proposition 1 and the Godefroy-Shapiro Theorem. ◻

A given algebraic basis $\mathbf{R}$ on $H_s^n(\ell_p)$ we set

$$T_{\mathbf{R},y} := (\mathcal{F}_n^{\mathbf{R}})^{-1}\mathcal{T}_y\mathcal{F}_n^{\mathbf{R}} \quad \textit{and} \quad D_{\mathbf{R}}^{\alpha} := (\mathcal{F}_n^{\mathbf{R}})^{-1}\mathcal{D}^{\alpha}\mathcal{F}_n^{\mathbf{R}}.$$

Corollary 2. *Let* $\mathbf{R}$ *be an algebraic basis on* $H_s^n(\ell_p)$ *and let* $y \in \ell_p$ *such that* $(F_p(y),\ldots,F_{p+n-1}(y)) \neq 0$. *Then the operator* $T_{\mathbf{R},y}$ *is hypercyclic on* $H(\mathbb{C}^n)$. *Moreover, every operator* A *on* $H(\mathbb{C}^n)$ *which commutes with* $T_{\mathbf{R},y}$ *and is not a scalar multiple of the identity is hypercyclic and can be represented by the form*

$$A(f) = \sum_{|\alpha|\geq 0} c_\alpha D_{\mathbf{R}}^{\alpha} f, \tag{5}$$

where c_α *as in (1).*

We need the next proposition.

Proposition 2 ([14]). *Let* $\Phi = (\Phi_1,\ldots,\Phi_n)$ *be a polynomial automorphism on* $\mathbb{C}^n$. *Then* $(\Phi_1(\mathbf{R}),\ldots,\Phi_n(\mathbf{R}))$ *is an algebraic basis in* $H_s^n(\ell_p)$ *for an arbitrary algebraic basis* $\mathbf{R} = (R_1,\ldots,R_n)$.

Conversely, if $(\Phi_1(\mathbf{R}),\ldots,\Phi_n(\mathbf{R}))$ *is an algebraic basis for some algebraic basis* $\mathbf{R} = (R_1,\ldots,R_n)$ *in* $H_s^n(\ell_p)$ *and a polynomial map* Φ *on* $\mathbb{C}^n$, *then* Φ *is a polynomial automorphism.*

Note that due to Proposition 2 the transformation $(\mathcal{F}_n^{\mathbf{R}})^{-1}\mathcal{T}_y\mathcal{F}_n^{\mathbf{R}}$ is nothing else than a composition with $\Phi\circ(I+a)\circ\Phi^{-1}$, where $\Phi(F_p,\ldots,F_{p+n-1}) = (R_p,\ldots,R_{p+n-1})$ and $a = (F_p(y),\ldots,F_{p+n-1}(y))$. Conversely, every polynomially similar operator to the translation can be represented by the form $(\mathcal{F}_n^{\mathbf{R}})^{-1}\mathcal{T}_y\mathcal{F}_n^{\mathbf{R}}$ for some algebraic basis of symmetric polynomials $\mathbf{R}$. This observation can be helpful in order to construct some examples of such operators.

The next algebraic bases of $\mathcal{P}_s(\ell_p)$ is useful for us: $(G_k^{(p)})_{k=1}^{\infty}$, where

$$G_k(x) = G_k^{(1)}(x) = \sum_{i_1<\cdots<i_k} x_{i_1}\cdots x_{i_k}$$

and $G_k^{(p)}(x)$ can be obtained from Newton's formula (see [16, §53]), putting $F_1(x) = F_2(x) = \cdots = F_{p-1}(x) = 0$. So, we get ([15])

$$\begin{aligned} nG_n^{(p)} &= (-1)^{p+1}F_p(x)G_{n-p}^{(p)}(x) + (-1)^{p+2}F_{p+1}(x)G_{n-p-1}^{(p)}(x) \\ &\quad + \cdots + (-1)^{n-p+1}F_{n-p}(x)G_p^{(p)}(x) + (-1)^{n+1}F_n(x), \end{aligned}$$

where $n > p$, $G_0^{(p)}(x) \equiv 1$, $F_0(x) \equiv 1$ and $G_1^{(p)}(x) = G_2^{(p)}(x) = \cdots = G_{p-1}^{(p)}(x) = 0$, $F_1(x) = F_2(x) = \cdots = F_{p-1}(x) = 0$. By another words, in (2) the terms $F_r(x)G_{q-r}^{(p)}(x) = 0$, if $r < p$ and $q - r < p$, where $p \leq r \leq n-p$, $p \leq q-r \leq n-p$.

Let us compute how looks the operator $T_{\mathbf{R},y}$ for $\mathbf{R} = \mathbf{G}$. We observe first that

$$G_m^{(p)}(x\bullet y) = \sum_{j+k=m} G_j^{(p)}(x)G_k^{(p)}(y), \qquad p \leq m \leq p+n-1,$$

where for the sake of convenience we take $G_0^{(p)} \equiv 1$. Thus

$$\begin{aligned}\mathcal{T}_y\mathcal{F}_n^{\mathbf{G}} f(t_1,\ldots,t_n) &= \mathcal{T}_y f(G_p^{(p)}(x),\ldots,G_{p+n}^{(p)}(x)) = f(G_p^{(p)}(x\bullet y),\ldots,G_{p+n}^{(p)}(x\bullet y))\\ &= f\Big(G_p^{(p)}(x)+G_p^{(p)}(y),\ldots,\sum_{i+k=m} G_i^{(p)}(x)G_k^{(p)}(y),\ldots,\sum_{i+k=p+n-1} G_i^{(p)}(x)G_k^{(p)}(y)\Big).\end{aligned}$$

Therefore,

$$T_{\mathbf{G},y} f(t_1,\ldots,t_n) = f\Big(t_p+b_p,\ldots,\sum_{j+k=m} t_j b_k,\ldots,\sum_{j+k=p+n-1} t_j b_k\Big), \tag{6}$$

where $t_1=0,\ldots,t_{p-1}=0$, $b_1=0,\ldots,b_{p-1}=0$, and $b_j=G_j^{(p)}(y)$ for $1\le j\le p+n-1$.

Godefroy and Shapiro proved that any continuous linear operator T on $H(\mathbb{C}^n)$, which commutes with translations and is not a scalar multiple of the identity, can be generated by (1). Composition with an affine map still does not commute with T_a. Indeed, by (6),

$$T_a\circ T_{\mathbf{G},y} f(t_1,\ldots,t_n) = f\Big(t_p+b_p+a_p,\ldots,\sum_{j=0}^{p+n-1} t_j b_{p+n-1-j}+a_{p+n-1}\Big);$$

$$T_{\mathbf{G},y}\circ T_a f(t_1,\ldots,t_n) = f\Big(t_p+b_p+a_p,\ldots,\sum_{j=0}^{p+n-1} (t_j+a_j) b_{p+n-1-j}\Big),$$

where $a_0=1$. Evidently, $T_a\circ T_{\mathbf{G},y} \neq T_{\mathbf{G},y}\circ T_a$ for some $a\neq 0$ whenever $b\neq(0,\ldots,0,b_{p+n-1})$.

3 The case of space $H_{bs}(\ell_p)$

Note that T_a satisfies the Hypercyclicity Criterion for full sequence [9] and so the symmetric shift $\mathcal{T}_y$ on $H_s^n(\ell_p)$ satisfies the Hypercyclicity Criterion for full sequence provided $(F_p(y),\ldots,F_{p+n-1}(y))\neq 0$.

We will establish our result about hypercyclic operators on the space of symmetric entire functions on ℓ_p. But before this, we need the following general auxiliary statement, which might be of some interest by itself.

Lemma 1 ([14])**.** *Let X be a Fréchet space and $X_1\subset X_2\subset\cdots\subset X_m\subset\cdots$ be a sequence of closed subspaces such that $\bigcup_{m=1}^{\infty} X_m$ is dense in X. Let T be an operator on X such that $T(X_m)\subset X_m$ for each m each restriction $T|_{X_m}$ satisfies the Hypercyclicity Criterion for full sequence on X_m. Then T satisfies the Hypercyclicity Criterion for full sequence on X.*

We denote by $H_{bs}(\ell_p)$ a Fréchet algebra of symmetric entire functions on ℓ_p which are bounded on bounded subsets. This algebra is the completion of the space of symmetric polynomials on ℓ_p endowed with the uniform topology on bounded subsets.

Theorem 3. *The symmetric translation operator $\mathcal{T}_y$ is hypercyclic on $H_{bs}(\ell_p)$ for every $y\neq 0$.*

Proof. Since $y\neq 0$, $F_{m_0}(y)\neq 0$ for some m_0 [1]. So, $\mathcal{T}_y$ is hypercyclic (and satisfies the Hypercyclicity Criterion for full sequence) on $H_s^m(\ell_p)$ whenever $m\ge m_0$. The set $\bigcup_{m=m_0}^{\infty} H_s^m(\ell_p)$ contains the space of all symmetric polynomials on ℓ_p and so it is dense in $H_{bs}(\ell_p)$. Also $H_s^m(\ell_p)\subset H_s^n(\ell_p)$, if $n>m$. Hence, by Lemma 1, $\mathcal{T}_y$ is hypercyclic. □

REFERENCES

[1] Alencar R., Aron R., Galindo P., Zagorodnyuk A. *Algebra of symmetric holomorphic functions on ℓ_p*. Bull. London Math. Soc. 2003, **35** (1), 55–64. doi:10.1112/S0024609302001431

[2] Aron R., Bès J. *Hypercyclic differentiation operators*. Contemp. Math. 1999, **232**, 39–46.

[3] Bayart F., Matheron E. Dynamics of linear operators. Cambridge University Press, New York, 2009.

[4] Bernal-González L. *Universal entire functions for affine endomorphisms of* $\mathbb{C}^N$. J. Math. Anal. Appl. 2005, **305** (2), 690–697. doi:10.1016/j.jmaa.2004.12.031

[5] Bès J. Three problems on hypercyclic operators. Ph. D. Thesis, Kent State University, 1998.

[6] Birkhoff G.D. *Démonstration d'un théorème élémentaire sur les fonctions entières*. C.R. Acad. Sci. Paris, 1929, **189**, 473–475.

[7] Chernega I., Galindo P., Zagorodnyuk A. *Some algebras of symmetric analytic functions and their spectra*. Proc. Edinb. Math. Soc. (2) 2011, **55** (1), 1–17. doi:10.1017/S0013091509001655

[8] Dineen S. Complex Analysis on Infinite Dimensional Spaces. In: Springer Monographs in Mathematics. Springer, London, 1999. doi:10.1007/978-1-4471-0869-6

[9] Godefroy G., Shapiro J.H. *Operators with dense, invariant, cyclic vector manifolds*. J. Funct. Anal. 1991, **98** (2), 229–269. doi:10.1016/0022-1236(91)90078-j

[10] Gonzaléz M., Gonzalo R., Jaramillo J. *Symmetric polynomials on rearrangement invariant function spaces*. J. London Math. Soc. 1999, **59** (2), 681–697. doi:10.1112/S0024610799007164

[11] Grosse-Erdmann K.-G. *Universal families and hypercyclic operators*. Bull. Amer. Math. Soc. 1999, **36**, 345–381.

[12] Grosse-Erdmann K. G., Peris Manguillot A. Linear chaos. In: Universitext. Springer-Verlag, London, 2011. doi:10.1007/978-1-4471-2170-1

[13] Mozhyrovska Z.H. *Hypercyclic composion operators*. Journal of Vasyl Stefanyk Precarpathian National University 2015, **2** (4), 75–92. doi:10.15330/jpnu.2.4.75-92

[14] Novosad Z., Zagorodnyuk A. *Polynomial automorphisms and hypercyclic operators on spaces of analytic functions*. Arch. Math. (Basel) 2007, **89** (2), 157–166. doi:10.1007/S00013-007-2043-4

[15] Chernega I. *Symmetric analytic functions on the space ℓ_p*. Appl. Probl. Mech. Math. 2012, **10**, 47–54. (in Ukrainian)

[16] Kurosh A.G. Course in higher algebra. Nauka, Moscow, 1968. (in Russian)

Received 02.04.2016

Revised 27.05.2016

Можировська З.Г. *Гіперциклічні оператори на алгебрі симетричних аналітичних функцій на ℓ_p*. // Карпатські матем. публ. — 2016. — Т.8, №1. — С. 127–133.

В статті запропоновано метод побудови гіперциклічних операторів композиції на просторі $H(\mathbb{C}^n)$ з використанням поліноміальних автоморфізмів на $\mathbb{C}^n$ і симетричних аналітичних функцій на ℓ_p. Зокрема, в роботі показано гіперциклічність оператора "симетричного зсуву" на алгебрі Фреше симетричних цілих функцій на ℓ_p, які є обмеженими на обмежених підмножинах.

Ключові слова і фрази: гіперциклічні оператори, функціональні простори.