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ON THE CONVERGENCE CRITERION FOR BRANCHED CONTINUED FRACTIONS WITH INDEPENDENT VARIABLES

In this paper, we consider the problem of convergence of an important type of multidimensional generalization of continued fractions, the branched continued fractions with independent variables. These fractions are an efficient apparatus for the approximation of multivariable functions, which are represented by multiple power series. We have established the effective criterion of absolute convergence of branched continued fractions of the special form in the case when the partial numerators are complex numbers and partial denominators are equal to one. This result is a multidimensional analog of the Worpitzky's criterion for continued fractions. We have investigated the polycircular domain of uniform convergence for multidimensional C-fractions with independent variables in the case of nonnegative coefficients of this fraction.

Key words and phrases: convergence, absolute convergence, uniform convergence, branched continued fraction with independent variables, multidimensional C-fraction with independent variables.

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INTRODUCTION

The problem of convergence of continued fractions whether their multidimensional generalizations, branched continued fractions, in particular, branched continued fractions with independent variables, is that on the basis of information about coefficients fraction to conclude its convergence or divergence. This class fractions was proposed by D.I. Bodnar [6], in the study of the convergence of branched continued fractions with positive elements for establishing a analog of the Seidel convergence criteria for continued fractions. In the thesis by Kh.Yo. Kuchminska [7] established the estimate of approximation of function by such fractions under the conditions of the type of Śleszyński-Pringsheim in the case of the two branches of branching. Further study of the convergence of branched continued fractions with independent variables, in particular, branched continued fraction of the special form

$$1+\sum_{i_1=1}^{N}\frac{c_{i(1)}}{1}+\sum_{i_2=1}^{i_1}\frac{c_{i(2)}}{1}+\sum_{i_3=1}^{i_2}\frac{c_{i(3)}}{1}+\cdots, \tag{1}$$

where N is fixed natural number, $c_{i(k)}$, $i(k)\in\mathcal{I}_k$, $k\geq 1$, are complex numbers,

$$\mathcal{I}_k=\left\{i(k):\ i(k)=(i_1,i_2,\ldots,i_k),\ 1\leq i_p\leq i_{p-1},\ 1\leq p\leq k,\ i_0=N\right\}$$

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denotes set of multiindices, $k \geq 1$, and branched continued fraction of the special form which is reciprocal to it

$$\frac{1}{1} {+} \sum_{i_1=1}^{N} \frac{c_{i(1)}}{1} {+} \sum_{i_2=1}^{i_1} \frac{c_{i(2)}}{1} {+} \sum_{i_3=1}^{i_2} \frac{c_{i(3)}}{1} {+} \cdots \tag{2}$$

received a continuation in the papers by O.E. Baran [3], where proved that (1) converges absolutely, if there exists the real numbers $0 \leq q_{i(k)} < 1$ or $0 < q_{i(k)} \leq 1$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, such that

$$|c_{i(k)}| \leq i_{k-1}^{-1} g_{i(k)}(1 - g_{i(k-1)}), \quad g_{i(0)} = 0, \; i(k) \in \mathcal{I}_k, \; k \geq 1, \tag{3}$$

and by O.E. Baran [2], where investigated a convergence of (2) for

$$|c_{i(k)}| \leq i_{k-1}^{-1} \rho(1 - \rho), \quad 0 < \rho \leq 2^{-1}, \quad i(k) \in \mathcal{I}_k, \; k \geq 1. \tag{4}$$

The next stage of the study of convergence of branched continued fractions with independent variables associated with the paper by T.M. Antonova and D.I. Bodnar [1], where proved that (1) converges absolutely for

$$|c_{i(k)}| \leq t_{i(k)} \left(1 - \sum_{i_{k+1}=1}^{i_k} t_{i(k+1)}\right), \quad t_{i(k)} \geq 0, \; \sum_{i_{k+1}=1}^{i_k} t_{i(k+1)} < 1, \quad i(k) \in \mathcal{I}_k, \; k \geq 1. \tag{5}$$

In addition, we note the paper by Kh.Yo. Kuchminska [8], where was proved a convergence of (2) with the elements that satisfy (4) in a slightly more general form than it was done in [2], and the paper by D.I. Bodnar and M.M. Bubnyak [5], where was investigated a convergence of one-periodic branched continued fractions of a special form with the elements that lie in disks whose radius form a geometric sequences with common ratio 4^{-1}.

We remark that the convergence criteria of branched continued fractions of the special form (1) and (2) established in the above mentioned works are multidimensional analogs of the Worpitzky's criterion for continued fractions [9].

Our research continues to establish the convergence criteria for the branched continued fractions with independent variables.

1 BRANCHED CONTINUED FRACTIONS OF THE SPECIAL FORM

Let

$$f_n = 1 + \sum_{i_1=1}^{N} \frac{c_{i(1)}}{1} {+} \sum_{i_2=1}^{i_1} \frac{c_{i(2)}}{1} {+} \cdots {+} \sum_{i_n=1}^{i_{n-1}} \frac{c_{i(n)}}{1}$$

be the nth approximant of (1), $n \geq 1$.

We shall prove the following result.

Theorem 1. *Let for the elements $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, of branched continued fraction of the special form (1) hold the following conditions*

$$|c_{i(k)}| \leq q_{i(k)}^{i_k} q_{i(k-1)}^{i_k - 1} (1 - q_{i(k-1)}), \quad i(k) \in \mathcal{I}_k, \; k \geq 1, \tag{6}$$

where $q_{i(0)}$ and $q_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, are constants which satisfy one or the other of the conditions

$$0 \leq q_{i(0)} < 1, \quad 0 \leq q_{i(k)} < 1, \quad i(k) \in \mathcal{I}_k, \; k \geq 1, \tag{7}$$

or

$$0 < q_{i(0)} \leq 1, \quad 0 < q_{i(k)} \leq 1, \quad i(k) \in \mathcal{I}_k,\ k \geq 1. \tag{8}$$

Then

(A) the branched continued fraction of special form (1) converges absolutely;

(B) the values of branched continued fraction of the special form (1) and of its approximants are in the disk

$$|z - 1| \leq 1 - q_{i(0)}^N; \tag{9}$$

(C) the disk (9) is the "best" set of values of branched continued fraction of the special form (1) and of its approximants for $q_{i(k)} = 2^{-1}$, $i(k) \in \mathcal{I}_k,\ k \geq 1$.

Proof. We show that branched continued fraction of the special form

$$1 - \sum_{i_1=1}^{N} \frac{q_{i(1)}^{i_1} q_{i(0)}^{i_1-1}(1 - q_{i(0)})}{1} - \sum_{i_2=1}^{i_1} \frac{q_{i(2)}^{i_2} q_{i(1)}^{i_2-1}(1 - q_{i(1)})}{1} - \sum_{i_3=1}^{i_2} \frac{q_{i(3)}^{i_3} q_{i(2)}^{i_3-1}(1 - q_{i(2)})}{1} - \cdots \tag{10}$$

is a majorant of (1).

For the tails of (1) we introduce the following notation:

$$Q_{i(s)}^{(s)} = 1, \quad i(s) \in \mathcal{I}_s,\ s \geq 1,$$

$$Q_{i(k)}^{(s)} = 1 + \sum_{i_{k+1}=1}^{i_k} \frac{c_{i(k+1)}}{1} + \sum_{i_{k+2}=1}^{i_{k+1}} \frac{c_{i(k+2)}}{1} + \cdots + \sum_{i_s=1}^{i_{s-1}} \frac{c_{i(s)}}{1}, \quad i(k) \in \mathcal{I}_k,\ 1 \leq k \leq s-1,\ s \geq 2.$$

It is clear that the following recurrence relations hold

$$Q_{i(k)}^{(s)} = 1 + \sum_{i_{k+1}=1}^{i_k} \frac{c_{i(k+1)}}{Q_{i(k+1)}^{(s)}}, \quad i(k) \in \mathcal{I}_k,\ 1 \leq k \leq s-1,\ s \geq 2. \tag{11}$$

Let s be arbitrary integer number, moreover $s \geq 0$. Using relations (11), by induction on k for arbitrary of multiindex $i(k) \in \mathcal{I}_k$ we show that the following inequalities are valid

$$|Q_{i(k)}^{(s)}| \geq \tilde{Q}_{i(k)}^{(s)}, \quad i(k) \in \mathcal{I}_k,\ 1 \leq k \leq s, \tag{12}$$

where $\tilde{Q}_{i(k)}^{(s)}$, $i(k) \in \mathcal{I}_k$, $1 \leq k \leq s$, denote the tails of (10), and

$$\tilde{Q}_{i(k)}^{(s)} > q_{i(k)}^{i_k}, \quad i(k) \in \mathcal{I}_k,\ 1 \leq k \leq s, \tag{13}$$

if the conditions (7) hold,

$$\tilde{Q}_{i(k)}^{(s)} \geq q_{i(k)}^{i_k}, \quad i(k) \in \mathcal{I}_k,\ 1 \leq k \leq s, \tag{14}$$

if the conditions (8) hold.

It is clear that for $k = s$, $i(s) \in \mathcal{I}_s$, relations (12)–(14) hold. By induction hypothesis that (12)–(14) hold for $k = p+1$, $p+1 \leq s$, $i(p+1) \in \mathcal{I}_{p+1}$, we prove (12)–(14) for $k = p$, $i(p) \in \mathcal{I}_p$.

Indeed, use of relations (11) for arbitrary of multiindex $i(p) \in \mathcal{I}_p$ lead to

$$|Q^{(s)}_{i(p)}| \geq 1 - \sum_{i_{p+1}=1}^{i_p} \frac{|c_{i(p+1)}|}{|Q^{(s)}_{i(p+1)}|} \geq 1 - \sum_{i_{p+1}=1}^{i_p} \frac{q^{i_{p+1}}_{i(p+1)} q^{i_{p+1}-1}_{i(p)} (1 - q_{i(p)})}{\tilde{Q}^{(s)}_{i(p+1)}} = \tilde{Q}^{(s)}_{i(p)}.$$

From (13) and (14) it follows that $\tilde{Q}^{(s)}_{i(p+1)} \neq 0$. Therefore, replacing $q^{i_{p+1}}_{i(p+1)}$ by $\tilde{Q}^{(s)}_{i(p+1)}$, inequalities (13) and (14) are obtained for $k = p$, $i(p) \in \mathcal{I}_p$.

Now, from (12)–(14) it follows that $Q^{(s)}_{i(k)} \neq 0$ and $\tilde{Q}^{(s)}_{i(k)} \neq 0$ for all indices. Applying the method suggested in [4, p. 28] and recurrence relations (11), for $m > n \geq 1$ we obtain

$$\begin{aligned}
|f_m - f_n| &\leq \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \frac{\prod\limits_{k=1}^{n+1} |c_{i(k)}|}{\prod\limits_{k=1}^{n+1} |Q^{(m)}_{i(k)}| \prod\limits_{k=1}^{n} |Q^{(n)}_{i(k)}|} \\
&\leq (-1)^{n+1} \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \frac{(-1)^{n+1} \prod\limits_{k=1}^{n+1} q^{i_k}_{i(k)} q^{i_k-1}_{i(k-1)} (1 - q_{i(k-1)})}{\prod\limits_{k=1}^{n+1} \tilde{Q}^{(m)}_{i(k)} \prod\limits_{k=1}^{n} \tilde{Q}^{(n)}_{i(k)}} = -(\tilde{f}_m - \tilde{f}_n),
\end{aligned}$$

where $\tilde{f}_k$, $k \geq 1$, denote the approximants of (10).

Hence,

$$|f_m - f_n| \leq \tilde{f}_n - \tilde{f}_m, \quad m > n \geq 1,$$

and

$$\sum_{r=1}^{k} |f_{r+1} - f_r| \leq \sum_{r=1}^{k} (\tilde{f}_r - \tilde{f}_{r+1}) = -\sum_{i_1=1}^{N} q^{i_1}_{i(1)} q^{i_1-1}_{i(0)} (1 - q_{i(0)}) - \tilde{f}_{k+1}, \quad k \geq 1. \tag{15}$$

From this it follows that the sequence $\{\tilde{f}_k\}$ is a monotonically decreases. Furthermore, from (13) and (14) for arbitrary $k \geq 1$ we have

$$\tilde{f}_k = 1 - \sum_{i_1=1}^{N} \frac{q^{i_1}_{i(1)} q^{i_1-1}_{i(0)} (1 - q_{i(0)})}{\tilde{Q}^{(k)}_{i(1)}} \geq q^N_{i(0)},$$

i.e. the sequence $\{\tilde{f}_k\}$ is bounded below. Therefore, the limit

$$\tilde{f} = \lim_{k \to \infty} \tilde{f}_k$$

exists and is a finite. Now, from (15) for $k \to \infty$ it follows that (1) converges absolutely. This proves part (A).

Next we prove part (B) and (C). Using (6), (13) and (14) for arbitrary $k \geq 1$ we have

$$|f_k - 1| \leq \sum_{i_1=1}^{N} \frac{|c_{i(1)}|}{|Q^{(k)}_{i(1)}|} \leq \sum_{i_1=1}^{N} \frac{q^{i_1}_{i(1)} q^{i_1-1}_{i(0)} (1 - q_{i(0)})}{\tilde{Q}^{(k)}_{i(1)}} \leq 1 - q^N_{i(0)}.$$

Therefore, the disk (9) includes the set of value of (1). We show that it coincides with (9) for $q_{i(k)} = 2^{-1}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$.

Let c be an arbitrary complex number such that $|c| < 1 - q^N_{i(0)}$. Then for the approximant f_1 of (1), where the $c_{i(1)} = c(1 - q^N_{i(0)})^{-1} q^{i_1-1}_{i(0)}(1 - q_{i(0)})$, $1 \leq i_1 \leq N$, and the $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 2$, are arbitrary complex numbers that satisfy (6), we obtain $f_1 = 1 + c$. If $|c| = 1 - q^N_{i(0)}$ then (1), where the $c_{i(1)} = 2^{-i_1} c(1 - q^N_{i(0)})^{-1} q^{i_1-1}_{i(0)}(1 - q_{i(0)})$, $1 \leq i_1 \leq N$, and the $c_{i(k)} = -4^{-i_k}$, $i(k) \in \mathcal{I}_k$, $k \geq 2$, satisfy (6) for $q_{i(k)} = 2^{-1}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, get value $1 + c$. We show it.

Indeed, in the above mentioned values of elements of (1) by equivalent transformation $\rho_{i(k)} = 2^{i_k-1}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, [4, pp. 29–33] we can write it in the form

$$1 + \sum_{i_1=1}^{N} \frac{2^{-1} c(1 - q^N_{i(0)})^{-1} q^{i_1-1}_{i(0)}(1 - q_{i(0)})}{2^{i_1-1}} \underset{-}{} \sum_{i_2=1}^{i_1} \frac{2^{i_1-i_2-2}}{2^{i_2-1}} \underset{-}{} \sum_{i_3=1}^{i_2} \frac{2^{i_2-i_3-2}}{2^{i_3-1}} \underset{-}{} \cdots . \tag{16}$$

To prove that the value of (16) is equal to $1 + c$ it is sufficient to prove the following relations

$$f^{(k)} = 2^{k-1} - \sum_{i_2=1}^{k} \frac{2^{k-i_2-2}}{2^{i_2-1}} \underset{-}{} \sum_{i_3=1}^{i_2} \frac{2^{i_2-i_3-2}}{2^{i_3-1}} \underset{-}{} \sum_{i_4=1}^{i_3} \frac{2^{i_3-i_4-2}}{2^{i_4-1}} \underset{-}{} \cdots = 2^{-1}, \quad 1 \leq k \leq N. \tag{17}$$

By induction on k we show that the relations (17) are valid.

It is easy to shown that for $k = 1$ relation (17) holds. By induction hypothesis that (17) hold for $k = n - 1$, $n \geq 2$, we prove (17) for $k = n$. We have

$$f^{(n)} = 2^{n-1} - \frac{2^{n-3}}{f^{(1)}} - \frac{2^{n-4}}{f^{(2)}} - \ldots - \frac{1}{f^{(n-2)}} - \frac{2^{-1}}{f^{(n-1)}} - \frac{2^{-2}}{f^{(n)}}. \tag{18}$$

Since $f^{(k)} = 2^{-1}$, $1 \leq k \leq n - 1$, $n \geq 2$, and

$$2^{n-1} - 2^{n-2} - 2^{n-3} - \ldots - 2^0 = 2^{n-1} - 2^{n-2} \frac{2^{1-n} - 1}{2^{-1} - 1} = 1,$$

than from (18) we obtain $f^{(n)} = 2^{-1}$. From this it follows that the value of (16) is equal to $1 + c$. Finally, it follows from concept of equivalent transformation [4, pp. 29–33] that the value of (1) is also equal to $1 + c$. □

It is now a simple matter to prove the following theorem.

Theorem 2. *Let for the elements $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, of branched continued fraction of the special form (2) hold the conditions (6), where $q_{i(0)}$ and $q_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, are constants which satisfy one or the other of the conditions*

$$0 < q_{i(0)} \leq 1, \quad 0 \leq q_{i(k)} < 1, \quad i(k) \in \mathcal{I}_k, \ k \geq 1, \tag{19}$$

or

$$0 < q_{i(0)} \leq 1, \quad 0 < q_{i(k)} \leq 1, \quad i(k) \in \mathcal{I}_k, \ k \geq 1. \tag{20}$$

Then

(A) the branched continued fraction of special form (2) converges absolutely;

(B) the values of the branched continued fraction of the special form (2) and of its approximants are in the disk

$$\left| z - \frac{1}{q_{i(0)}^N(2-q_{i(0)}^N)} \right| \leq \frac{1-q_{i(0)}^N}{q_{i(0)}^N(2-q_{i(0)}^N)}; \tag{21}$$

(C) the disk (21) is the "best" set of values of branched continued fraction of the special form (2) and of its approximants for $q_{i(k)} = 2^{-1}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$.

Proof. By analogous considerations as in the proof of Theorem 1, it is easy to shown that a majorant of (2) is the following branched continued fraction of the special form

$$\frac{1}{1} - \sum_{i_1=1}^{N} \frac{q_{i(1)}^{i_1} q_{i(0)}^{i_1-1}(1-q_{i(0)})}{1} - \sum_{i_2=1}^{i_1} \frac{q_{i(2)}^{i_2} q_{i(1)}^{i_2-1}(1-q_{i(1)})}{1} - \sum_{i_3=1}^{i_2} \frac{q_{i(3)}^{i_3} q_{i(2)}^{i_3-1}(1-q_{i(2)})}{1} - \cdots. \tag{22}$$

From the fact that the approximants of (22) form the sequence, which is a monotonically increasing and bounded above, it follows that (2) converges absolutely.

We write the kth approximant of (2) in the form

$$z = \left(1 + \sum_{i_1=1}^{N} \frac{c_{i(1)}}{Q_{i(1)}^{(k-1)}}\right)^{-1} = \frac{1}{1+w}.$$

Using relations (19), (20) and conditions (6), we have

$$|w| \leq \sum_{i_1=1}^{N} \frac{|c_{i(1)}|}{|Q_{i(1)}^{(k-1)}|} \leq \sum_{i_1=1}^{N} \frac{q_{i(1)}^{i_1} q_{i(0)}^{i_1-1}(1-q_{i(0)})}{\tilde{Q}_{i(1)}^{(k-1)}} \leq 1 - q_{i(0)}^N.$$

Therefore,

$$\left|\frac{1-z}{z}\right| = |w| \leq 1 - q_{i(0)}^N,$$

from where we obtain (21).

Since $0 < q_{i(0)} \leq 1$ then (21) contains the point 1. In view of proof part (C) of Theorem 1, to show that (21) is the "best" set, it suffices to note that values of the particular branched continued fraction of special form

$$z = \frac{1}{1} + \sum_{i_1=1}^{N} \frac{2^{-1}c(1-q_{i(0)}^N)^{-1} q_{i(0)}^{i_1-1}(1-q_{i(0)})}{2^{i_1-1}} - \sum_{i_2=1}^{i_1} \frac{2^{i_1-i_2-2}}{2^{i_2-1}} - \sum_{i_3=1}^{i_2} \frac{2^{i_2-i_3-2}}{2^{i_3-1}} - \cdots = \frac{1}{1+c}.$$

fill the disk (21) as c ranges over the set $|c| \leq 1 - q_{i(0)}^N$. □

2 MULTIDIMENSIONAL C-FRACTIONS WITH INDEPENDENT VARIABLES

In this section we have two convergence criteria for the multidimensional C-fractions with independent variables. Their proof is a simple application of Theorems 1 and 2 respectively.

Corollary 2.1. *Let $a_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, be nonnegative numbers such that*

$$a_{i(k)} \leq q_{i(k)}^{i_k} q_{i(k-1)}^{i_k-1}(1 - q_{i(k-1)}), \quad i(k) \in \mathcal{I}_k, \; k \geq 1, \tag{23}$$

where $q_{i(0)}$ and $q_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, are constants which satisfy one or the other of the conditions (7) or (8). Then the multidimensional C-fraction with independent variables

$$1 + \sum_{i_1=1}^{N} \frac{a_{i(1)} z_{i_1}}{1} + \sum_{i_2=1}^{i_1} \frac{a_{i(2)} z_{i_2}}{1} + \sum_{i_3=1}^{i_2} \frac{a_{i(3)} z_{i_3}}{1} + \cdots$$

converges uniformly in the domain

$$G = \left\{ \mathbf{z} = (z_1, z_2, \ldots, z_N) \in \mathbb{C}^N : \; |z_k| < 1, \; 1 \leq k \leq N \right\}. \tag{24}$$

Corollary 2.2. *Let $a_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, be nonnegative numbers such that satisfy the inequalities (23), where $q_{i(0)}$ and $q_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, are constants which satisfy one or the other of the conditions (19) or (20). Then the multidimensional C-fraction with independent variables*

$$\frac{1}{1} + \sum_{i_1=1}^{N} \frac{a_{i(1)} z_{i_1}}{1} + \sum_{i_2=1}^{i_1} \frac{a_{i(2)} z_{i_2}}{1} + \sum_{i_3=1}^{i_2} \frac{a_{i(3)} z_{i_3}}{1} + \cdots$$

converges uniformly in the domain (24).

CONCLUSION

The convergence criteria (6), as well as (3) and (5), is an effective criterion for investigating the convergence of branched continued fractions with independent variables.

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Досліджується питання збіжності багатовимірних узагальнень неперервних дробів — гіллястих ланцюгових дробів з нерівнозначними змінними. Ці дроби є ефективним апаратом при наближенні функцій, заданих кратними степеневими рядами. Встановлено ефективні умови абсолютної збіжності гіллястих ланцюгових дробів з нерівнозначними змінними у випадку коли частинні чисельники комплексні числа, а частинні знаменники дорівнюють одиниці. Отриманий результат є багатовимірним аналогом критерію Ворпітського для неперервних дробів. Досліджено полікругову область рівномірної збіжності для багатовимірних C-дробів з нерівнозначними змінними у випадку невід'ємних коефіцієнтів дробу.

Ключові слова і фрази: збіжність, абсолютна збіжність, рівномірна збіжність, гіллястий ланцюговий дріб з нерівнозначними змінними, багатовимірний C-дріб з нерівнозначними змінними.