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FIRST REFORMULATED ZAGREB INDICES OF SOME CLASSES OF GRAPHS

A topological index of a graph is a parameter related to the graph; it does not depend on labeling or pictorial representation of the graph. Graph operations plays a vital role to analyze the structure and properties of a large graph which is derived from the smaller graphs. The Zagreb indices are the important topological indices found to have the applications in Quantitative Structure Property Relationship (QSPR) and Quantitative Structure Activity Relationship (QSAR) studies as well. There are various studies of different versions of Zagreb indices. One of the most important Zagreb indices is the reformulated Zagreb index which is used in QSPR study.

In this paper, we obtain the first reformulated Zagreb indices of some derived graphs such as double graph, extended double graph, thorn graph, subdivision vertex corona graph, subdivision graph and triangle parallel graph. In addition, we compute the first reformulated Zagreb indices of two important transformation graphs such as the generalized transformation graph and generalized Mycielskian graph.

Key words and phrases: Zagreb index, reformulated Zagreb index, derived graphs.

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INTRODUCTION

All the graphs considered in this paper are connected and simple. For vertex $u \in V(G)$, the degree of the vertex u in G, denoted by $d_G(u)$, is the number of edges incident to u in G. A *topological index* of a graph is a parameter related to the graph; it does not depend on labeling or pictorial representation of the graph. In theoretical chemistry, molecular structure descriptors (also called topological indices) are used for modeling physicochemical, pharmacologic, toxicologic, biological and other properties of chemical compounds. Several types of such indices exist, especially those based on vertex and edge distances. One of the most intensively studied topological indices is the Wiener index. Two of these topological indices are known under various names, the most commonly used one are the first and second Zagreb indices.

The Zagreb indices have been introduced more than thirty years ago by Gutman I. and Trinajstić N. [6]. They are defined as

$$M_1(G) = \sum_{u\in V(G)} d_G(u)^2, \qquad M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

Note that the first Zagreb index may also be written as

$$M_1(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v)).$$

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The Zagreb indices are found to have appilications in QSPR and QSAR studies as well. For the survey on theory and application of Zagreb indices see [7]. Feng L. et al. [5] have given the sharp bounds for the Zagreb indices of graphs with a given matching number. Khalifeh M.H. et al. [12] have obtained the Zagreb indices of the Cartesian product, composition, join, disjunction and symmetric difference of graphs. The extremal values of Zagreb coindices over some special class of graphs determined by Ashrafi A.R. et al. [1].

Milićević A. et al. [15] in 2004 reformulated the Zagreb indices in terms of edge-degrees instead of vertex-degrees $EM_1(G) = \sum_{e\in E(G)} d(e)^2$, where $d(e)$ denotes the degree of the edge e in G, which is defined by $d(e) = d(u) + d(v) - 2$ with $e = uv$. The use of these descriptors in QSPR study was also discussed in their report [15]. Reformulated Zagreb index, particularly its upper/lower bounds has attracted recently theat tention of many mathematicians and computer scientists, see [3, 4, 10, 11, 15, 17, 20]. The aim of this paper is to obtain, the first reformulated Zagreb indices of some derived graphs such as double, extended double, thorn graph, subdivision vertex corona of graphs, subdivision graph and triangle parallel graph. In addition, we compute the first reformulated Zagreb indices of two important transformation graphs such as the generalized transformations graphs and generalized Mycielskian graphs.

1 MAIN RESULTS

The hyper Zagreb index and its coindex are defined as

$$HM(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v))^2 \qquad \text{and} \qquad \overline{HM}(G) = \sum_{uv\notin E(G)} (d_G(u) + d_G(v))^2.$$

The *F-index* of a graph G is defined as $F = F(G) = \sum_{u\in V(G)} d_G^3(u) = \sum_{uv\in E(G)} (d_G^2(u) + d_G^2(v))$.

1.1 Double graph and extended double cover

Let us denote the double graph of a graph G by G^*, which is constructed from two copies of G in the following manner [9, 2]. Let the vertex set of G be $V(G) = \{v_1, v_2, \ldots, v_n\}$, and the vertices of G^* are given by the two sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. Thus for each vertex $v_i \in V(G)$, there are two vertices x_i and y_i in $V(G^*)$. The *double graph* G^* includes the initial edge set of each copies of G, and for any edge $v_iv_j \in E(G)$, two more edges x_iy_j and x_jy_i are added, see Figure 1. Now we compute the first reformulated Zagreb index of the double of a given graph.

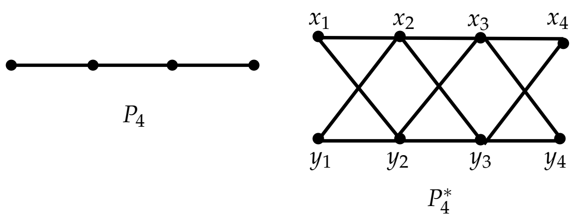

Figure 1: The double graph of P_4.

Theorem 1. *Let G be a connected graph with m edges. If G^* is a double graph of G, then $EM_1(G^*) = 16HM(G) - 32M_1(G) + 16m$.*

Proof. From the definition of a double graph it is clear that $d_{G^*}(x_i) = d_{G^*}(y_i) = 2d_G(v_i)$, where $v_i \in V(G)$ and $x_i, y_i \in V(G^*)$ are corresponding clone vertices of v_i. By the definition of EM_1,

$$\begin{aligned}
EM_1(G^*) &= \sum_{uv \in E(G^*)} (d_{G^*}(u) + d_{G^*}(v) - 2)^2 \\
&= \sum_{x_i x_j \in E(G^*)} (d_{G^*}(x_i) + d_{G^*}(x_j) - 2)^2 + \sum_{y_i y_j \in E(G^*)} (d_{G^*}(y_i) + d_{G^*}(y_j) - 2)^2 \\
&+ \sum_{x_i y_j \in E(G^*)} (d_{G^*}(x_i) + d_{G^*}(y_j) - 2)^2 + \sum_{x_j y_i \in E(G^*)} (d_{G^*}(x_j) + d_{G^*}(y_i) - 2)^2 \\
&= 4 \sum_{v_i v_j \in E(G)} (2d_G(v_i) + 2d_G(v_j) - 2)^2 = 16 \sum_{v_i v_j \in E(G)} (d_G(v_i) + d_G(v_j) - 1)^2 \\
&= 16 \sum_{v_i v_j \in E(G)} \left[(d_G(v_i) + d_G(v_j))^2 - 2(d_G(v_i) + d_G(v_j)) + 1 \right] \\
&= 16HM(G) - 32M_1(G) + 16m.
\end{aligned}$$

□

Let G be a simple connected graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$. The *extended double cover* of G, denoted by G^{**} is the bipartite graph with bipartition (X, Y) where $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ in which x_i and y_j are adjacent if and only if either v_i and v_j are adjacent in G or $i = j$, see Figure 2. This construction of the extended double cover was introduced by Alon N. [2] in 1986. Here we obtain the first reformulated Zagreb index of extended double cover of a given graph.

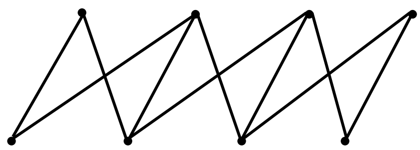

Figure 2: Extended double cover of P_4.

Theorem 2. *Let G be a graph and G^{**} its extended double cover. Then $EM_1(G^{**}) = 2HM(G)$.*

Proof. Let G be a graph with n vertices and m edges. The definition of the extended double cover implies that G^{**} consists of $2n$ vertices and $n + 2m$ edges. Moreover, $d_{G^{**}}(x_i) = d_{G^{**}}(y_i) = d_G(v_i) + 1$, for $i = \{1, 2, \ldots, n\}$. Here, $v_i \in V(G)$ and $x_i, y_i \in V(G^{**})$ are corresponding clone vertices of v_i. Hence

$$\begin{aligned}
EM_1(G^{**}) &= \sum_{uv \in E(G^{**})} (d_{G^{**}}(u) + d_{G^{**}}(v) - 2)^2 \\
&= \sum_{x_i y_j \in E(G^{**})} (d_{G^{**}}(x_i) + d_{G^{**}}(y_j) - 2)^2 + \sum_{x_j y_i \in E(G^{**})} (d_{G^{**}}(x_j) + d_{G^{**}}(y_i) - 2)^2 \\
&+ \sum_{i=1}^{n} (d_{G^{**}}(x_i) + d_{G^{**}}(y_i) - 2)^2 = 2 \sum_{v_i v_j \in E(G)} (d_G(v_i) + 1 + d_G(v_j) + 1 - 2)^2 \\
&= 2 \sum_{v_i v_j \in E(G)} (d_G(v_i) + d_G(v_j))^2 = 2HM(G).
\end{aligned}$$

□

1.2 Thorn Graph

An edge $e = uv$ of a graph G is called a *thorn* if either $d_G(u) = 1$ or $d_G(v) = 1$. The concept of thorn graph was introduced by Gutman I. [8] by joining a number of thorn to each vertex of any given graph G. Some of the topological indices of thorn graphs are studied in [13, 18, 19].

Let $V(G)$ and $V(G^T)$ be the vertex sets of G and its thorn graph G^T respectively. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $V^T(G) = V(G) \cup V_1 \cup V_2 \cup \ldots \cup V_n$, where V_i are the set of degree one vertices attached to the vertices v_i in G^T and $Vi \cup V_j = \varphi$, $i \neq j$. Let the vertices of the set V_i are denoted by v_{ij} for $j = 1, 2, \ldots, p_i$ and $i = 1, 2, \ldots, n$. Thus $|V(G^T)| = n + z$ where, $z = \sum_{i=1}^{n} p_i$. Then the degree of the vertices v_i in G^T are given by $d_{G^T}(v_i) = d_G(v_i) + p_i$, for $i = 1, 2, \ldots, n$. Now we compute the first reformulated Zagreb index of thorn of a given graph.

Theorem 3. *Let G be a graph. Then*

$$EM_1(G^T) = HM(G) + \sum_{v_iv_j \in E(G)} (p_i + p_j - 2)^2 + 2 \sum_{v_iv_j \in E(G)} (d_G(v_i) + d_G(v_j))(p_i + p_j - 2)$$
$$+ \sum_{i=1}^{n} p_i \Big[d_G^2(v_i) + (p_i - 1)^2 + 2d_G(v_i)(p_i - 1)\Big].$$

Proof. From the definition of reformulated first Zagreb index,

$$\begin{aligned}
EM_1(G^T) &= \sum_{v_iv_j \in E(G^T)} (d_{G^T}(v_i) + d_{G^T}(v_j) - 2)^2 \\
&= \sum_{v_iv_j \in E(G^T)} (d_{G^T}(v_i) + d_{G^T}(v_j) - 2)^2 + \sum_{i=1}^{n} \sum_{j=1}^{p_i} (d_{G^T}(v_i) + d_{G^T}(v_j) - 2)^2 \\
&= \sum_{v_iv_j \in E(G)} (d_G(v_i) + p_i + d_G(v_j) + p_j - 2)^2 + \sum_{i=1}^{n} \sum_{j=1}^{p_i} (d_G(v_i) + p_i + 1 - 2)^2 \\
&= \sum_{v_iv_j \in E(G)} \Big[(d_G(v_i) + d_G(v_j))^2 + (p_i + p_j - 2)^2 \\
&+ 2(d_G(v_i) + d_G(v_j))(p_i + p_j - 2)\Big] + \sum_{i=1}^{n} p_i (d_G(v_i) + p_i - 1)^2 \\
&= HM(G) + \sum_{v_iv_j \in E(G)} (p_i + p_j - 2)^2 + 2 \sum_{v_iv_j \in E(G)} (d_G(v_i) + d_G(v_j))(p_i + p_j - 2) \\
&+ \sum_{i=1}^{n} p_i \Big[d_G^2(v_i) + (p_i - 1)^2 + 2d_G(v_i)(p_i - 1)\Big].
\end{aligned}$$

□

1.3 Subdivision Vertex Corona of Graphs

Let G_1 and G_2 be any two simple connected graphs with n_1 and n_2 number of vertices and m_1 and m_2 number of edges respectively. The *subdivision vertex corona* of G_1 and G_2 is denoted by $G_1 \circ G_2$ and was introduced by Lu P. and Miao Y. [14]. The graph $G_1 \circ G_2$ is obtained from the subdivision graph $S(G_1)$ and n_1 copies of G_2, by joining the i-th vertex of $V(G_1)$ to every vertex in the i-th copy of G_2. Let $V(G_1) = \{v_1, v_2, \ldots, v_{n_1}\}$, $I(G_1) = \{v_1^e, v_2^e, \ldots, v_{m_1}^e\}$

and $V(G_2) = \{u_1, u_2, \ldots, u_{n_2}\}$, so that $V(S(G)) = V(G) \cup I(G)$. Let $u_1^i, u_2^i, \ldots, u_{n_2}^i$ denote the vertices of the i-th copy of $G_{2,i}$, $i = 1, 2, \ldots, n_1$, so that

$$V(G_1 \circ G_2) = V(G_1) \cup I(G_1) \cup [V(G_{2,1}) \cup V(G_{2,2}) \cup \ldots \cup V(G_{2,n_1})].$$

Here we compute the first reformulated Zagreb index of Subdivision vertex corona of graphs.

Theorem 4. *Let G_1 and G_2 be two graphs with n_1, n_2 and m_1, m_2 edges, respectively. Then*

$$EM_1(G_1 \circ G_2) = n_1 HM(G_2) + F(G_1) + 3n_2 M_1(G_1) + n_1 M_1(G_2) + n_2\Big[2m_1 n_2 + n_1(n_2-1)^2\Big] + 8m_1m_2 + 4\Big[m_1(n_2-1) + m_2(n_1-1)\Big].$$

Proof. The degree of the vertices of $G_1 \circ G_2$ is given by $d_{G_1 \circ G_2}(v_i) = d_{G_1}(v_i) + n_2$ for $i = 1, 2, \ldots, n_1$, $d_{G_1 \circ G_2}(e_i) = 2$ for $i = 1, 2, \ldots, m_1$, $d_{G_1 \circ G_2}(u_j^i) = d_{G_2}(u_j) + 1$ for $i = 1, 2, \ldots, n_1$ and $j = 1, 2, \ldots, n_2$. Let the vertex set of $G_1 \circ G_2$ can be partitioned into three subsets $E_1 = \{xy \in E(G_1 \circ G_2) | x, y \in V(G_{2,i}), i = 1, 2, \ldots, n_1\}$, $E_2 = \{xy \in E(G_1 \circ G_2) | x \in V(G_1), y \in I(G_1)\}$, and $E_3 = \{xy \in E(G_1 \circ G_2) | x \in V(G_1), y \in V(G_{2,i}),\ i = 1, 2, \ldots, n_1\}$. The contribution of the edges in E_1 to the first reformulated Zagreb index of $G_1 \odot G_2$ is given by

$$\begin{aligned} EM_1(G_1 \odot G_2) &= \sum_{xy \in E_1} (d_{G_1 \odot G_2}(x) + d_{G_1 \odot G_2}(y) - 2)^2 \\ &= \sum_{i=1}^{n_1} \sum_{u_iu_j \in E(G_2)} (d_{G_2}(u_i) + 1 + d_{G_2}(u_j) + 1 - 2)^2 \\ &= \sum_{i=1}^{n_1} \sum_{u_iu_j \in E(G_2)} (d_{G_2}(u_i) + d_{G_2}(u_j))^2 = n_1 HM(G_2). \end{aligned}$$

Similarly, the contribution of the edges in E_2 to the first reformulated Zagreb index of $G_1 \odot G_2$ is given by

$$\begin{aligned} EM_1(G_1 \odot G_2) &= \sum_{xy \in E_2} (d_{G_1 \odot G_2}(x) + d_{G_1 \odot G_2}(y) - 2)^2 = \sum_{i=1}^{n} (d_{G_1}(v_i) + n_2 + 2 - 2)^2 d_{G_1}(v_i) \\ &= \sum_{i=1}^{n} \Big[d_{G_1}^2(v_i) + n_2^2 + 2d_{G_1}(v_i) n_2\Big] d_{G_1}(v_i) \\ &= F(G_1) + 2n_2 M_1(G_1) + 2m_1 n_2^2. \end{aligned}$$

The contribution of the edges in E_3 to the first reformulated Zagreb index of $G_1 \odot G_2$ is given by

$$\begin{aligned} EM_1(G_1 \odot G_2) &= \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} (d_{G_1}(v_i) + n_2 + d_{G_2}(u_j) + 1 - 2)^2 \\ &= \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} (d_{G_1}(v_i) + d_{G_2}(u_j) + (n_2 - 1))^2 \\ &= \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \Big[d_{G_1}^2(v_i) + d_{G_2}^2(u_j) + (n_2-1)^2 + 2d_{G_1}(v_i) d_{G_2}(u_j) \\ &\quad + 2d_{G_2}(u_j)(n_2-1) + 2d_{G_1}(v_i)(n_2-1)\Big] = n_2 M_1(G_1) + n_1 M_1(G_2) \\ &\quad + n_1 n_2 (n_2-1)^2 + 8m_1m_2 + 4m_2(n_2-1) + 4m_1(n_2-1). \end{aligned}$$

The desired expression for the first reformulated Zagreb index of $G_1 \odot G_2$ is obtained by summing the above three expressions. □

1.4 Some derived graphs

The *subdivision graph*$S(G)$ is the graph obtained from G by replacing each edge of G by a path of length two. The *triangle parallel graph* of a graph G is denoted by $R(G)$ and is obtained from G by replacing each edge of G by a triangle. Now we compute the first reformulated Zagreb index of $S(G)$ and $R(G)$ for a given graph G.

Theorem 5. *Let G be a graph. Then* $EM_1(S(G)) = F(G)$.

Proof. Observe that $V(S(G)) = \Big(V(S(G)) \cap V(G)\Big) \cup \Big(V(S(G)) \setminus V(G)\Big)$, that is $|V(S(G))| = p + q$ and $|E(S(G))| = 2q$. Note that for $x \in V(S(G)) \cap V(G)$, $d_{S(G)}(x) = d_G(x)$ and for $x \in V(S(G)) \setminus V(G)$, $d_{S(G)}(x) = 2$. The first reformulated Zagreb index is given by

$$\begin{aligned} EM_1(S(G)) &= \sum_{uv \in E(S(G))} (d_{S(G)}(u) + d_{S(G)}(v) - 2)^2 = \sum_{u \in V(S(G))} (d_{S(G)}(u) + 2 - 2)^2 \\ &= \sum_{u \in V(G)} d_G(u)(d_G(u))^2 = F(G). \end{aligned}$$

□

Theorem 6. *Let G be a graph on m edges. Then*

$$EM_1(R(G)) = 4HM(G) - 8M_1(G) + 4F(G) + 4m.$$

Proof. From the definition of $R(G)$, we have

$$\begin{aligned} RM_1(R(G)) &= \sum_{uv \in E(R(G))} (d_{R(G)}(u) + d_{R(G)}(v) - 2)^2 \\ &= \sum_{u,v \in V(G),\, uv \in E(R(G))} (d_{R(G)}(u) + d_{R(G)}(v) - 2)^2 \\ &+ \sum_{x \in V(G),\, y \in V(R(G))/V(G),\, xy \in E(R(G))} (d_{R(G)}(x) + d_{R(G)}(y) - 2)^2 \\ &= \sum_{uv \in E(G)} (2d_G(u) + 2d_G(v) - 2)^2 + \sum_{p \in V(G)} (2d_G(p) + 2 - 2)^2 d_G(p) \\ &= \sum_{uv \in E(G)} [(2d_G(u) + 2d_G(v))^2 + 4 - 4(2d_G(u) + 2d_G(v))] + \sum_{p \in V(G)} 4d_G^3(p) \\ &= 4HM(G) - 8M_1(G) + 4F(G) + 4m. \end{aligned}$$

□

1.5 Generalized transformation graphs

Sampathkumar E. and Chikkodimath S.B. [16] defined the *semitotal-point graph* of given graph. Based on this definition, Gutman introduced some new graphical transformations. These generalize the concept of a semitotal-point graph.

Let $G = (V, E)$ be a graph, and let α, β be two elements of $V(G) \cup E(G)$. We say that the associativity of α and β is $+$ if they are adjacent or incident in G, otherwise is $-$. Let ab be a $2-$permutation of the set $\{+, -\}$. We say that α and β correspond to the first term a of ab if both α and β are in $V(G)$, whereas α and β correspond to the second term b of ab if one of α and β is in $V(G)$ and the other is in $E(G)$. The generalized transformation graph G^{ab} of G is defined on the vertex set $V(G) \cup E(G)$. Two vertices α and β of G^{ab} are joined by an edge if and only if their associativity in G is consistent with the corresponding term of ab.

In view of above, one can obtain four graphical transformations of graphs, since there are four distinct $2-$permutations of $\{+-\}$. Note that G^{++} is just the semitotal-point graph $T_2(G)$ of G, whereas the other generalized transformation graphs are G^{+-}, G^{-+} and G^{--}. In other words, the generalized transformation graph G^{ab} is a graph whose vertex set is $V(G) \cup E(G)$, and $\alpha, \beta \in V(G^{ab})$. α and β are adjacent in G^{ab} if and only if either (i) and (ii) holds:

(i) for any $\alpha, \beta \in V(G), \alpha, \beta$ are adjacent in G if $a = +$ and; α, β are not adjacent in G if $a = -$;

(ii) for any $\alpha \in V(G)$ and $\beta \in E(G), \alpha, \beta$ are incident in G if $b = +$ and; α, β are not incident in G if $b = -$.

The vertex v_i of G^{ab} corresponding to a vertex v_i of G is referred to as a point vertex. The vertex e_i of G^{ab} corresponding to an edge e_i of G is referred to as a line vertex.

Theorem 7. *Let G be a connected graph on n vertices and m edges. Then* $EM_1(G^{++}) = 4HM(G) - 8M_1(G) + 4F(G) + 4m$.

Proof. One can observe that the number of vertices and edges of G^{++} are $n + m$ and $2m$, respectively. $d_{G^{++}}(v_i) = 2d_G(v_i)$ and $d_{G^{++}}(e_i) = 2$.

$$\begin{aligned} EM_1(G^{++}) &= \sum_{uv \in E(G^{++})} (d_{G^{++}}(u) + d_{G^{++}}(v) - 2)^2 \\ &= \sum_{uv \in E(G^{++}) \cap E(G)} (d_{G^{++}}(u) + d_{G^{++}}(v) - 2)^2 \\ &+ \sum_{uv \in E(G^{++}) - E(G)} (d_{G^{++}}(u) + d_{G^{++}}(v) - 2)^2 \\ &= \sum_{uv \in E(G)} (2d_G(u) + 2d_G(v) - 2)^2 + \sum_{uv \in E(G^{++}) - E(G)} (2 + 2d_G(v) - 2)^2 \\ &= 4HM(G) - 8M_1(G) + 4 \sum_{v \in V(G)} d_G^3(v) + 4m \\ &= 4HM(G) - 8M_1(G) + 4F(G) + 4m. \end{aligned}$$

□

Theorem 8. *Let G be a connected graph on n vertices and m edges. Then*

$$EM_1(G^{+-}) = 4m(m-1)^2 + (nm - 2m)(n + m - 4)^2.$$

Proof. Note that $|V(G^{+-})| = n + m$ and $|E(G^{+-})| = m(n-1)$. Moreover, $d_{G^{++}}(v_i) = m$ and $d_{G^{++}}(e_i) = n - 2$.

$$\begin{aligned} EM_1(G^{+-}) &= \sum_{uv \in E(G^{+-})} (d_{G^{+-}}(u) + d_{G^{+-}}(v) - 2)^2 \\ &= \sum_{uv \in E(G^{+-}) \cap E(G)} (d_{G^{+-}}(u) + d_{G^{+-}}(v) - 2)^2 \\ &+ \sum_{uv \in E(G^{+-}) - E(G)} (d_{G^{+-}}(u) + d_{G^{+-}}(v) - 2)^2 \\ &= \sum_{uv \in E(G)} (2m - 2)^2 + \sum_{uv \in E(G^{+-}) - E(G)} (m + (n-2) - 2)^2 \\ &= m(2m-2)^2 + (m(n-1) - m)(n + m - 4)^2. \end{aligned}$$

□

Theorem 9. *Let G be a connected graph on n vertices and m edges. Then*

$$EM_1(G^{-+}) = 2n^2(n(n-1)-2m) + 2m(n-1)^2.$$

Proof. Note that $|V(G^{-+})| = n+m$ and $|E(G^{-+})| = m + \frac{n(n-1)}{2}$. Moreover, $d_{G^{-+}}(v_i) = n-1$ and $d_{G^{++}}(e_i) = 2$.

$$\begin{aligned}
EM_1(G^{-+}) &= \sum_{uv\in E(G^{-+})} (d_{G^{-+}}(u) + d_{G^{-+}}(v) - 2)^2 \\
&= \sum_{uv\in E(G^{-+})\cap E(\overline{G})} (d_{G^{-+}}(u) + d_{G^{-+}}(v) - 2)^2 \\
&\quad + \sum_{uv\in E(G^{-+})-E(\overline{G})} (d_{G^{-+}}(u) + d_{G^{-+}}(v) - 2)^2 \\
&= [\frac{n(n-1)}{2} - m](2n)^2 + (n-1)^2(\frac{n(n-1)}{2} + m - \frac{n(n-1)}{2} + m) \\
&= 4n^2(\frac{n(n-1)}{2} - m) + 2m(n-1)^2.
\end{aligned}$$

□

Theorem 10. *Let G be a connected graph on n vertices and m edges. Then*

$$\begin{aligned}
EM_1(G^{--}) &= 4\overline{HM}(G) - 8(n+m-2)\overline{M}_1(G) + 2(n+m-2)^2(n^2-n-m) \\
&\quad + (2n+m-5)^2 m(n-2) + 4\sum_{uv\in E(G^{--})-E(\overline{G})} \Big(d_G^2(v) - (2n+m-5)d_G(v)\Big).
\end{aligned}$$

Proof. Note that $|V(G^{--})| = p+q$ and $|E(G^{--})| = \frac{p(p-1)}{2} + q(p-3)$. Moreover, $d_{G^{--}}(v_i) = p+q-1-2d_G(v_i)$ and $d_{G^{--}}(e_i) = p-2$.

$$\begin{aligned}
EM_1(G^{--}) &= \sum_{uv\in E(G^{--})} (d_{G^{--}}(u) + d_{G^{--}}(v) - 2)^2 = \sum_{uv\in E(G^{--})\cap E(\overline{G})} (d_{G^{--}}(u) + d_{G^{--}}(v) - 2)^2 \\
&+ \sum_{uv\in E(G^{--})-E(\overline{G})} (d_{G^{--}}(u) + d_{G^{--}}(v) - 2)^2 \\
&= \sum_{uv\in E(\overline{G})} ((n+m-1) - 2d_G(u) + (n+m-1) - 2d_G(v) - 2)^2 \\
&+ \sum_{uv\in E(G^{--})-E(\overline{G})} (n-2+n+m-1-2d_G(v)-2)^2 \\
&= \sum_{uv\in E(\overline{G})} (2(n+m-1) - 2(d_G(u)+d_G(v)) - 2)^2 \\
&+ \sum_{uv\in E(G^{--})-E(\overline{G})} (2n+m-5-2d_G(v))^2 \\
&= \sum_{uv\in E(\overline{G})} ((2(n+m-1)-2)^2 + 4(d_G(u)+d_G(v))^2 \\
&- 4(2(n+m-1)-2)(d_G(u)+d_G(v))) \\
&+ \sum_{uv\in E(G^{--})-E(\overline{G})} ((2n+m-5)^2 + 4d_G^2(v) - 4(2n+m-5)d_G(v)) \\
&= 4\overline{HM}(G) - 8(n+m-2)\overline{M}_1(G) + 2(n+m-2)^2(n^2-n-m) \\
&+ (2n+m-5)^2 m(n-2) + 4\sum_{uv\in E(G^{--})-E(\overline{G})} \Big(d_G^2(v) - (2n+m-5)d_G(v)\Big).
\end{aligned}$$

□

1.6 Generalized Mycielskian graphs

Let G be a simple connected graph with n vertices and m edges, $V(G) = \{v_1, v_2, \ldots, v_n\}$. For a graph $G = (V, E)$, the *Mycielskian* of G is the graph $\mu(G)$ with the vertex set consisting of the disjoint union $V \cup V' \cup \{u\}$, where $V' = \{x' | x \in V\}$ and edge set $E \cup \{x'y, xy' | xy \in E\} \cup \{x'u | x' \in V'\}$.

For a graph $G = (V, E)$, the *generalized Mycielskian*, denoted by $\mu_k(G)$, of G is the graph whose vertex set is the disjoint union $V \cup (\bigcup_{i=1}^{k} V^i) \cup \{u\}$, where $V^i = \{x^i | x \in V\}$ is an independent set, $1 \leq i \leq k$, and edge set $E(\mu_k(G)) = E \cup \{\bigcup_{i=1}^{k} \{y^{i-1}x^i; x^{i-1}y^i | xy \in E\}\} \bigcup \{x^k u | x^k \in V^k\}$, where $x^0 = x$ and $y^0 = y$.

The proof of the following lemma easily follows from the definition of the generalized Mycielskian of G.

Lemma 1. *Let G be a connected graph. Then*

(i) $|V(\mu_k(G))| = (k+1)n + 1$;

(ii) $|E(\mu_k(G))| = (2k+1)m + n$;

(iii) *If* $u^0v^0 \in E(G)$, *then* $u^0v^0, u^iv^{i+1}, u^{i+1}v^i \in E(\mu_k(G))$ *for* $0 \leq i \leq k-1$;

(iv) $d_{\mu_k(G)}(v^i) = 2d_G(v), 0 \leq i \leq k-1$;

(v) $d_{\mu_k(G)}(v^k) = d_G(v) + 1$ *for all* $v \in V(G)$;

(vi) $d_{\mu_k(G)}(u) = n$.

Here we obtain the first reformulated Zagreb index of $\mu_k(G)$.

Theorem 11. *Let G be a connected graph with n vertices and m edges. Then*

$$\begin{aligned} EM_1(\mu_k(G)) &= 2(4k-1)HM(G) + 6F(G) - (16k-1)M_1(G) + 4M_2(G) \\ &\quad + n(n-1)^2 + 2m(4k+2n-3). \end{aligned}$$

Proof. By the definition of EM_1, we have

$$EM_1(\mu_k(G)) = \sum_{uv \in E(\mu_k(G))} \Big(d_{\mu_k(G)}(u) + d_{\mu_k(G)}(v) - 2\Big)^2.$$

By Lemma 1, we get

$$\begin{aligned} EM_1(\mu_k(G)) = &\sum_{uv \in E(G)} \Big(2d_G(u) + 2d_G(v) - 2\Big)^2 + 2(k-1) \sum_{uv \in E(G)} \Big(2d_G(u) + 2d_G(v) - 2\Big)^2 \\ &+ \sum_{uv \in E(G)} \Big(2d_G(u) + (d_G(v)+1) - 2\Big)^2 + \sum_{uv \in E(G)} \Big(2d_G(v) + (d_G(u)+1) - 2\Big)^2 \\ &+ \sum_{v \in V(G)} \Big((d_G(v)+1) + n - 2\Big)^2 \\ &= (2k-1) \sum_{uv \in E(G)} \Big(2d_G(u) + 2d_G(v) - 2\Big)^2 \\ &+ \sum_{uv \in E(G)} \Big(2d_G(u) + d_G(v) - 1\Big)^2 + \sum_{uv \in E(G)} \Big(2d_G(v) + d_G(u) - 1\Big)^2 \\ &+ \sum_{v \in V(G)} \Big(d_G(v) + n - 1\Big)^2 \\ &= S_1 + S_2 + S_3 + S_4, \end{aligned}$$

where

$$\begin{aligned} S_1 &= (2k-1) \sum_{uv\in E(G)} \Big(2d_G(u)+2d_G(v)-2\Big)^2 \\ &= 4(2k-1) \sum_{uv\in E(G)} \Big((d_G(u)+d_G(v))^2 - 2(d_G(u)+d_G(v)) + 1\Big) \\ &= (2k-1)\Big(4HM(G) - 8M_1(G) + 4m\Big), \end{aligned}$$

$$\begin{aligned} S_2 &= \sum_{uv\in E(G)} \Big(2d_G(u)+d_G(v)-1\Big)^2 \\ &= \sum_{uv\in E(G)} \Big((d_G(u)+d_G(v))^2 + (d_G(u))^2 + 2(d_G(u)+d_G(v))d_G(u) \\ &\quad -2(d_G(u)+d_G(v)) - 2d_G(u) + 1\Big) \\ &= HM(G) + 3\sum_{v\in V(G)} d_G(v)(d_G(v))^2 + 2M_2(G) - 2M_1(G) - 2\sum_{v\in V(G)} (d_G(v))^2 + m \\ &= HM(G) + 3F(G) + 2M_2(G) - 4M_1(G) + m. \end{aligned}$$

Similarly,

$$S_3 = \sum_{uv\in E(G)} \Big(2d_G(v)+d_G(u)-1\Big)^2 = HM(G) + 3F(G) + 2M_2(G) - 4M_1(G) + m,$$

$$\begin{aligned} S_4 = \sum_{v\in V(G)} \Big(d_G(v)+n-1\Big)^2 &= \sum_{v\in V(G)} \Big((d_G(v))^2 + 2(n_1)d_G(v) + (n-1)^2\Big)^2 \\ &= M_1(G) + n(n-1)^2 + 4(n-1)m. \end{aligned}$$

The desired expression for the first reformulated Zagreb index of $\mu_k(G)$ is obtained by summing S_1 to S_4. □

REFERENCES

[1] Ashrafi A.R., Doslić T., Hamzeha A. *Extremal graphs with respect to the Zagreb coindices*. MATCH Commun. Math.Comput. Chem. 2011, **65**, 85–92.

[2] Alon N. *Eigenvalues and expanders*. Combinatorica 1986, **6** (2), 83–96. doi:10.1007/BF02579166

[3] De N. *Some bounds of reformulated Zagreb indices*. Appl. Math. Sci. 2012, **6** (101), 5005–5012.

[4] Furtula B., Gutman I. *A forgotten topological index*. J. Math. Chem. 2015, **53** (4), 1184–1190. doi:10.1007/s10910-015-0480-z

[5] Feng L., Ilić A. *Zagreb, Harary and hyper-Wiener indices of graphs with a given matching number*. Appl. Math. Lett. 2010, **23** (8), 943–948. doi:10.1016/j.aml.2010.04.017

[6] Gutman I., Trinajstić N. *Graph theory and molecular orbitals. Total φ-electron energy of alternant hydrocarbons*. Chemical Physics Letters 1972, **17** (4), 535–538. doi:10.1016/0009-2614(72)85099-1

[7] Gutman I., Das K.C. *The first Zagerb index 30 years after*. MATCH Commun. Math. Comput. Chem. 2004, **50**, 83–92.

[8] Gutman I. *Distance in thorny graph*. Publ. Inst. Math. 1998, **63**, 31–36.

[9] Hua H., Zhang S., Xu K. *Further results on the eccentric distance sum*. Discrete Appl. Math. 2012, **160** (1-2), 170–180. doi:10.1016/j.dam.2011.10.002

[10] Ilić A., Stevanović D. *On comparing Zagreb indices.* MATCH Commun. Math. Comput. Chem. 2009, **62**, 681–687.

[11] Ilić A., Zhou B. *On reformulated Zagreb indices.* Discrete Appl. Math. 2012, **160** (3), 204–209. doi: 10.1016/j.dam.2011.09.021

[12] Khalifeh M.H., Yousefi-Azari H., Ashrafi A.R. *The first and second Zagreb indices of some graph operations.* Discrete Appl. Math. 2009, **157** (4), 804–811. doi:10.1016/j.dam.2008.06.015

[13] Kathiresan K.M., Parameswaran C. *Certain generalized thorn graphs and their Wiener indices.* J. Appl. Math. Inform. 2012, **30** (5-6), 793–807.

[14] Lu P., Miao Y. *Spectra of the subdivision-vertex and subdivision-edge coronae.* arXiv:1302.0457v2

[15] Milićević A., Nikolić S., Trinajstić N. *On reformulated Zagreb indices.* Mol. Divers. 2004, **8** (4), 93–399.

[16] Sampathkumar E., Chikkodimath S.B. *Semitotal graphs of a graph-I.* J. Karnatak Univ. Sci. 1973, **18**, 274–280.

[17] Su G., Xiong L., Xu L., Ma B. *On the maximum and minimum first reformulated Zagreb index of graphs with connectivity at most k.* Filomat 2011, **25** (4) 75–83. doi:10.2298/FIL1104075S

[18] Walikar H.B., Ramane H.S., Sindagi L., Shirakol S.S., Gutman I. *Hosoya polynomial of thorn trees, rods, rings, and stars.* Kragujevac J. Sci. 2006, **28**, 47–56.

[19] Zhou B. *On modified Wiener indices of thorn trees*. Kragujevac J. Math. 2005, **27**, 5–9.

[20] Zhou B., Trinajstić N. *Some properties of the reformulated Zagreb indices.* J. Math. Chem. 2010, **48** (3), 714–719. doi:10.1007/s10910-010-9704-4

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Каладеві В., Муруґешан Р., Паттабіраман К. *Перші перевизначені індекси Загреба для деяких класів графів* // Карпатські матем. публ. — 2017. — Т.9, №2. — С. 134–144.

Топологічний індекс графа — це параметр, пов'язаний з графом; він не залежить від маркування або наочного зображення графа. Операції з графами відіграють важливу роль для аналізу структури і властивостей великого графа, що породжений від менших графів. Індекси Загреба є важливими топологічними показниками, які знайшли застосування в вивченні кількісної структури відносин властності (QSPR) та кількісної структури відносин активності (QSAR). Є різні дослідження окремих видів індексів Загреба. Один з найважливіших індексів Загреба — це переформульований індекс Загреба, який використовується в дослідженні QSPR.

У статті ми отримуємо значення перших переформульованих індексів Загреба деяких похідних графів, таких як подвійний граф, подовжений подвійний граф, шиповий граф, напівподілений вершиний коронний граф, напівподілений граф та паралельний трикутний граф. Крім того обчислено перші переформульовані індекси Загреба для двох важливих перетворень графів таких як граф узагальненого перетворення та узагальнений граф Міцельскіяна.

Ключові слова і фрази: індекс Загреба, перевизначений індекс Загреба, похідні графи.