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PROPERTIES OF INTEGRALS WHICH HAVE THE TYPE OF DERIVATIVES OF VOLUME POTENTIALS FOR ONE KOLMOGOROV TYPE ULTRAPARABOLIC ARBITRARY ORDER EQUATION

In weighted Hölder spaces it is studied the smoothness of integrals, which have the structure and properties of derivatives of volume potentials which generated by fundamental solutions of the Cauchy problem for one ultraparabolic arbitrary order equation of the Kolmogorov type. The coefficients in this equation depend only on the time variable. Special distances and norms are used for constructing of the weighted Hölder spaces.

The results of the paper can be used for establishing of the correct solvability of the Cauchy problem and estimates of solutions of the given non-homogeneous equation in corresponding weighted Hölder spaces.

Key words and phrases: ultraparabolic Kolmogorov type arbitrary order equation, an integral which have the type of derivatives of the volume potential, weight Hölder norm, Hölder space of increasing functions.

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INTRODUCTION

Properties of the corresponding volume potentials are very important when the fundamental solution is being constructed and investigated, correct solvability of the Cauchy problem is being established and estimates of solutions for parabolic equations are being obtained. Such properties have been established for parabolic equations in the sense of Petrovsky and for $\overrightarrow{2b}$-parabolic equations in the sense of Eidelman without any degenerations in works [5, 6, 8] and for equations with degenerations on the initial hyperplane in works [6, 7, 10, 12, 13]. Volume potentials for the degenerated arbitrary order parabolic equations of the Kolmogorov type (ultraparabolic equations of the Kolmogorov type) were studied in [1–4, 6] and properties of volume potentials with density from Hölder spaces of bounded functions which are increasing as $|x| \to \infty$ were established only for the second order equations.

It is convenient to obtain such properties if the statements about properties of integrals which have the type of derivatives of volume potentials are proved first at all. These properties are described by belonging such integrals to corresponding functional spaces according to the type of spaces which density and kernel of the integral belong to. Statesments of such type are

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proved in works [6,8,9,11] for parabolic equations in the sense of Petrovsky and for parabolic equations in the sense of Eidelman. By the way they have their own value.

In this paper there is an attempt to prove the corresponding statements in case of the Kolmogorov type parabolic equations. The major part of these equations are parabolic in the sense of Petrovsky with respect to basic indepedent variables.

1 NOTATIONS AND ASSUMPTIONS

Let b, n_1, n_2, n_3 be given positive integer numbers such that $1 \le n_3 \le n_2 \le n_1$, $n := n_1 + n_2 + n_3$; $x := (x_1, x_2, x_3) \in \mathbb{R}^n$, $x_l := (x_{l1}, \dots, x_{ln_j}) \in \mathbb{R}^{n_l}$, $l \in L := \{1, 2, 3\}$; T is a positive number; if $k_1 := (k_{11}, \dots, k_{1n_1}) \in \mathbb{Z}_+^{n_1}$ is a n_1-dimensional index, then $|k_1| := k_{11} + \ldots + k_{1n_1}$, $\partial_{x_1}^{k_1} := \partial_{x_{11}}^{k_{11}} \cdot \ldots \cdot \partial_{x_{1n_1}}^{k_{1n_1}}$.

The paper is concerned with the study of properties of integrals of the type

$$u(t,x) := \int\limits_0^t d\tau \int\limits_{\mathbb{R}^n} M(t,x;\tau,\xi) f(\tau,\xi) d\xi, \quad (t,x) \in \Pi_{(0,T]} := (0,T] \times \mathbb{R}^n. \tag{1}$$

The kernel M is a complex-valued function which has properties of the derivatives of the fundamental solution G of the Cauchy problem for the equation

$$(\partial_t - \sum_{j=1}^{n_2} x_{1j}\partial_{x_{2j}} - \sum_{j=1}^{n_3} x_{2j}\partial_{x_{3j}} - \sum_{|k_1| \le 2b} a_{k_1}(t)\partial_{x_1}^{k_1}) u(t,x) = f(t,x), \quad (t,x) \in \Pi_{(0,T]}. \tag{2}$$

In the equation (2) $\partial_t - \sum\limits_{|k_1| \le 2b} a_{k_1}(t)\partial_{x_1}^{k_1}$ is parabolic by Petrovsky differential expression, and coefficients a_{k_1} are continuous on $[0,T]$ functions.

The equation (2) belongs to a class of ultraparabolic equations arbitrary order $2b$ and it generalize known equation of A.N.Kolmogorov of diffusion with inertia. In [6] it was established a structure and properties of the function G and its derivatives.

Let us describe properties of the kernel M of integral (1). For the purpose we denote: $q := 2b/(2b-1)$, $N := (n_1 + (2b+1)n_2 + (4b+1)n_3)/(2b)$, $\Delta_x^{x'} f(t,x) := f(t,x) - f(t,x')$, $\rho(t,x,\xi) := t^{1-q} \sum\limits_{j=1}^{n_1} |x_{1j} - \xi_{1j}|^q + t^{1-2q} \sum\limits_{j=1}^{n_2} |x_{2j} + t x_{1j} - \xi_{2j}|^q + t^{1-3q} \sum\limits_{j=1}^{n_3} |x_{3j} + t x_{2j} + 2^{-1} t^2 x_{1j} - \xi_{3j}|^q$, $d(x;x') := \sum\limits_{l=1}^{3} |x_l - x_l'|^{1/(2b(l-1)+1)}$, $d_1(x;x';\lambda) := |x_1 - x_1'|^\lambda + \sum\limits_{l=2}^{3} |x_l - x_l'|^{(\lambda+1)/(2b(l-1)+1)}$, $d_2(x;x';\lambda) := |x_1 - x_1'|^\lambda + |x_2 - x_2'|^{(\lambda+1)/(2b+1)} + |x_3 - x_3'|^{(\lambda+2b+1)/(4b+1)}$, if $t \in (0,T]$, $\{x, x', \xi\} \subset \mathbb{R}^n$, $\lambda \in (0,1]$.

Note, that if $d(x;x') < 1$, then

$$d_2(x;x';\lambda) \le d_1(x;x';\lambda) \le 4^{1-\lambda} d(x;x')^\lambda, \quad \{x,x'\} \subset \mathbb{R}^n, \lambda \in (0,1].$$

As the kernel of the integral (1), let us take the function M, which can be represented in the form

$$M(t,x;\tau,\xi) := (t-\tau)^{-\nu-N} \Omega(t,x;\tau,\xi), \quad 0 \le \tau < t \le T, \ \{x,\xi\} \subset \mathbb{R}^n, \tag{3}$$

where $\nu \in (0, 2b + 1/(2b)]$, and the function Ω, with the values in $\mathbb{C}$, is continuous and it satisfies the conditions below with some numbers $c > 0$ and $\gamma \in (0,1]$:

A_1. $\forall\{t,\tau\}\subset(0,T],\ \tau<t,\ \forall x\in\mathbb{R}^n$:

$$\int\limits_{\mathbb{R}^n}\Omega(t,x;\tau,\xi)d\xi=0\ \text{ for } \nu\in(1-1/(2b),1],$$

$$\int\limits_{\mathbb{R}^{n_2+n_3}}\Omega(t,x;\tau,\xi)d\xi_2d\xi_3=0\ \text{ for } \nu\in(1,1+1/(2b)], \tag{4}$$

$$\int\limits_{\mathbb{R}^{n_3}}\Omega(t,x;\tau,\xi)d\xi_3=0\ \text{ for } \nu\in(1+1/(2b),2b+1/(2b)];$$

A_2. $\exists C>0\ \forall\{t,\tau\}\subset(0,T],\ \tau<t,\ \forall\{x,\xi\}\subset\mathbb{R}^n$:

$$|\Omega(t,x;\tau,\xi)|\le C\exp\{-c\rho(t-\tau,x,\xi\}; \tag{5}$$

A_3. $\exists C>0\ \forall\{t,\tau\}\subset(0,T],\ \tau<t,\ \forall\{x,x',\xi\}\subset\mathbb{R}^n,\ d(x;x')<(t-\tau)^{1/(2b)}$:

$$|\Delta_x^{x'}\Omega(t,x;\tau,\xi)|\le C(d(x;x'))^\gamma(t-\tau)^{-\gamma/(2b)}\exp\{-c\rho(t-\tau,x,\xi)\}. \tag{6}$$

The definition of the function M contains the number ν, c, and γ, which assume are considered to be given. By $\mathcal{M}(\nu,c,\gamma)$ we denote a set of all functions M determined by formula (3), in which the function Ω satisfies conditions A_1 – A_3 with given $\gamma\in(0,1]$, $\nu\in(0,2b+1/(2b)]$, $c\in\mathbb{R}_+$.

It should be noted that for $\nu\in[1,2b+1/(2b)]$ integral (1) with the function $M\in\mathcal{M}(\nu,c,\gamma)$ is treated as the limit

$$\lim_{h\to0}\int\limits_0^{t-h}d\tau\int\limits_{\mathbb{R}^n}M(t,x;\tau,\xi)f(\tau,\xi)d\xi,$$

which exists for suitable f, because of condition A_1.

Let us define spaces to which the functions f and u belong. They are the spaces of functions which are continuous or satisfy Hölder condition and which have certain restrictions as $|x|\to\infty$. Their behavior as $|x|\to\infty$ will be described by the functions

$$\varphi(t,x):=\exp\sum_{l=1}^{3}k_l(t,a_l)|x_l|^q$$

or

$$\psi(t,x):=\exp\sum_{l=1}^{3}s_l(t)|x_l|^q,\ t\in[0,T],\ x\in\mathbb{R}^n.$$

Here for a fixed number c_0 from the interval $(0,c)$, where c is the constant from conditions A_2 and A_3, and for a set $a:=(a_1,a_2,a_3)$ of non-negative numbers a_l, $l\in L$, such that $T<\min\limits_{l\in L}(c_0/a_l)^{(2b-1)/(2b(l-1)+1)}$:

$$k_l(t,a_l):=c_0a_l(c_0^{2b-1}-a_l^{2b-1}t^{2b(l-1)+1})^{1-q},\ l\in L;$$

$$s_1(t):=k_1(t,a_1)+2^{q-1}t^qk_2(t,a_2)+2^{q-2}t^{2q}k_3(t,a_3),$$

$$s_2(t):=2^{q-1}k_2(t,a_2)+4^{q-1}t^qk_3(t,a_3),\ \ s_3(t):=4^{q-1}k_3(t,a_3),\ t\in[0,T].$$

The functions $k(t) := (k_1(t,a_1), k_2(t,a_2), k_3(t,a_3))$ and $s(t) := (s_1(t), s_2(t), s_3(t))$, $t \in [0,T]$, have the following properties [6]:

$$k(0) = a,\ a_l \le k_l(\tau, a_l) < k_l(t, a_l) < s_l(t),\ 0 \le \tau < t \le T,\ l \in L; \tag{7}$$

$$k_l(t-\tau, k_l(\tau, a_l)) \le k_l(t, a_l),\ 0 \le \tau < t \le T,\ l \in L; \tag{8}$$

$$-c_0\rho(t-\tau, x, \xi) + \sum_{l=1}^{3} a_l |\xi_l|^q \le \sum_{l=1}^{3} k_l(t, a_l) |\bar{x}_l(t)|^q \le \sum_{l=1}^{3} s_l(t) |x_l|^q,$$
$$0 \le \tau < t \le T, \{x, \xi\} \subset \mathbb{R}^n, \tag{9}$$

where $\bar{x}_l(t) := (\bar{x}_{l1}(t), \bar{x}_{l2}(t), \ldots, \bar{x}_{ln_l}(t))$, $l \in L$; $\bar{x}_{1j}(t) := x_{1j}$, $j \in \{1, \ldots, n_1\}$; $\bar{x}_{2j}(t) := x_{2j} + tx_{1j}$, $j \in \{1, \ldots, n_2\}$; $\bar{x}_{3j}(t) := x_{3j} + tx_{2j} + 2^{-1}t^2 x_{1j}$, $j \in \{1, \ldots, n_3\}$.

From these properties it is follows that

$$\varphi(\tau, X_1(t-\tau)) \le \varphi(t, X_1(t)) \le \psi(t, x),$$

$$\exp\{-c_0\rho(t-\tau, x, \xi)\}\varphi(\tau, \xi) \le \psi(t, x),\ \ 0 \le \tau < t \le T,\ \ \{x, \xi\} \subset \mathbb{R}^n, \tag{10}$$

where $X_1(t) := (\bar{x}_1(t), \bar{x}_2(t), \bar{x}_3(t))$.

For a given number $\lambda \in (0,1]$ we denote by C^0, C^λ_φ, $C^\lambda_{1,\varphi}$ and $C^\lambda_{2,\varphi}$ spaces of continuous functions $u : \Pi_{[0,T]} \to \mathbb{C}$, for which the corresponding norms $||u||^0_\varphi$, $||u||^\lambda_\varphi := ||u||^0_\varphi + [u]^\lambda_\varphi$, $||u||^\lambda_{1,\varphi} := ||u||^0_\varphi + [u]^\lambda_{1,\varphi}$ and $||u||^\lambda_{2,\varphi} := ||u||^0_\varphi + [u]^\lambda_{2,\varphi}$, where

$$||u||^0_\varphi := \sup_{(t,x)\in\Pi_{[0,T]}} \frac{|u(t,x)|}{\varphi(t,x)},$$

$$[u]^\lambda_\varphi := \sup_{\substack{\{(t,x),(t,x')\}\subset\Pi_{[0,T]} \\ (t,x)\neq(t,x')}} \frac{|\Delta^{x'}_x u(t,x)|}{(d(x;x'))^\lambda(\varphi(t,x)+\varphi(t,x'))},$$

$$[u]^\lambda_{1,\varphi} := \sup_{\substack{\{(t,x),(t,x')\}\subset\Pi_{[0,T]} \\ (t,x)\neq(t,x')}} \frac{|\Delta^{x'}_x u(t,x)|}{d_1(x;x';\lambda)(\varphi(t,x)+\varphi(t,x'))},$$

$$[u]^\lambda_{2,\varphi} := \sup_{\substack{\{(t,x),(t,x')\}\subset\Pi_{[0,T]} \\ (t,x)\neq(t,x')}} \frac{|\Delta^{x'}_x u(t,x)|}{d_2(x;x';\lambda)(\varphi(t,x)+\varphi(t,x'))}$$

are finite.

Except these spaces we will use the space C^λ_ψ. The definition of this space is obtained if in the definition of the space C^λ_φ the function φ replace by the function ψ.

2 MAIN THEOREM

Let us formulate the main results of this paper.

Theorem. *Let $M \in \mathcal{M}(\nu, c, \gamma)$ and function u is determined by formula* (1). *Then the following statements are valid:*

a) *if $\nu \leq 1 - 1/(2b)$ and $f \in C^0$, then $u \in C^{\gamma}_{\psi}$ and*

$$||u||^{\gamma}_{\psi} \leq C||f||^{0}_{\varphi}; \tag{11}$$

b) *if $\nu \in (1 - 1/(2b), 1]$ and $f \in C^{\lambda}_{\varphi}$, $\lambda \in (0, 1]$, then with $\nu + (\gamma - \lambda)/(2b) < 1$ we have $u \in C^{\gamma}_{\psi}$ and*

$$||u||^{\gamma}_{\psi} \leq C||f||^{\lambda}_{\varphi}, \tag{12}$$

and with $\nu + (\gamma - \lambda)/(2b) > 1$ we have $u \in C^{\lambda}_{\psi}$ and

$$||u||^{\lambda}_{\psi} \leq C||f||^{\lambda}_{\varphi}; \tag{13}$$

c) *if $\nu \in (1, 1 + 1/(2b)]$ and $f \in C^{\lambda}_{1,\varphi}$, $\lambda \in (0, 1]$, then with $\nu + (\gamma - 1 - \lambda)/(2b) < 1$ we have $u \in C^{\gamma}_{\psi}$ and*

$$||u||^{\gamma}_{\psi} \leq C||f||^{\lambda}_{1,\varphi}, \tag{14}$$

and with $\nu + (\gamma - 1 - \lambda)/(2b) > 1$ we have $u \in C^{\lambda}_{\psi}$ and

$$||u||^{\lambda}_{\psi} \leq C||f||^{\lambda}_{1,\varphi}; \tag{15}$$

d) *if $\nu \in (1 + 1/(2b), 2b + 1/(2b)]$ and $f \in C^{\lambda}_{2,\varphi}$, $\lambda \in (0, 1]$, then with $\nu + 1 - 2b + (\gamma - 1 - \lambda)/(2b) < 1$ we have $u \in C^{\gamma}_{\psi}$ and*

$$||u||^{\gamma}_{\psi} \leq C||f||^{\lambda}_{2,\varphi}, \tag{16}$$

and with $\nu + 1 - 2b + (\gamma - 1 - \lambda)/(2b) > 1$ we have $u \in C^{\lambda}_{\psi}$ and

$$||u||^{\lambda}_{\psi} \leq C||f||^{\lambda}_{2,\varphi}. \tag{17}$$

The constants C in inequalities (11)–(17) *depend only on the constant C from conditions A_2 and A_3, and also they depend on the numbers $n_1, n_2, n_3, b, \nu, c, \gamma$ and λ.*

Proof. Below various constants we will denote by same letters if we have no interest in constant's values.

a) Using the equality [6]

$$\int\limits_{\mathbb{R}^n} (t-\tau)^{-N} \exp\{-c'\rho(t-\tau, x, \xi)\} d\xi = C, \ \ 0 < \tau < t \leq T, \ x \in \mathbb{R}^n, \ c' > 0, \tag{18}$$

with the help of (3), (5), (10) and of the definition of the norm $||f||^{0}_{\varphi}$ we have

$$|u(t,x)| \leq C \int\limits_0^t (t-\tau)^{-\nu-N} d\tau \int\limits_{\mathbb{R}^n} \exp\{-c\rho(t-\tau,x,\xi)\}|f(\tau,\xi)|d\xi = C \int\limits_0^t (t-\tau)^{-\nu-N} d\tau$$
$$\int\limits_{\mathbb{R}^n} \exp\{-c_0\rho(t-\tau,x,\xi)\}\varphi(\tau,\xi)\frac{|f(\tau,\xi)|}{\varphi(\tau,\xi)} \exp\{-(c-c_0)\rho(t-\tau,x,\xi)\}d\xi \tag{19}$$
$$\leq C\psi(t,x) \int\limits_0^t (t-\tau)^{-\nu} d\tau ||f||^{0}_{\varphi} = C\psi(t,x)t^{1-\nu}||f||^{0}_{\varphi}, (t,x) \in \Pi_{(0,T]}.$$

Let x and x' be arbitrary fixed points from $\mathbb{R}^n$ and $d := d(x; x')$. Let us estimate the difference $\Delta_x^{x'} u$.

When $d^{2b} > t$, with the help of estimate (19) we obtain

$$\begin{aligned} &|\Delta_x^{x'} u(t,x)| \le |u(t,x)| + |u(t,x')| \le C(\psi(t,x) + \psi(t,x'))t^{1-\nu}||f||_{\varphi}^0 \\ &\le C(\psi(t,x) + \psi(t,x'))(d(x;x'))^{\gamma} t^{1-\nu-\gamma/(2b)}||f||_{\varphi}^0, \quad t \in (0,T], \ \{x,x'\} \subset \mathbb{R}^n, \ \gamma \in (0,1]. \end{aligned} \tag{20}$$

Let us consider the case $d^{2b} < t$. We have

$$|\Delta_x^{x'} u(t,x)| \le \int_0^t d\tau \int_{\mathbb{R}^n} |\Delta_x^{x'} M(t,x;\tau,\xi)| \, |f(\tau,\xi)| d\xi, \quad t \in (0,T], \ \{x,x'\} \subset \mathbb{R}^n. \tag{21}$$

Let us prove for the difference $\Delta M := \Delta_x^{x'} M(t,x;\tau,\xi)$ the inequality

$$|\Delta M| \le C d^{\gamma} (t-\tau)^{-\gamma/(2b)-\nu-N} \exp\{-c\rho(t-\tau,x,\xi)\}. \tag{22}$$

We shall distinguish the following cases: 1) $d^{2b} \ge t-\tau$, 2) $d^{2b} < t-\tau$.

In the first case, we obtain estimate (22) immediately from (3), (5) and from the inequality $|\Delta M| \le |M(t,x;\tau,\xi)| + |M(t,x';\tau,\xi)|$. In case 2) note that

$$\Delta M = (t-\tau)^{-\nu-N} \Delta_x^{x'} \Omega(t,x;\tau,\xi).$$

Because of (6) we have estimate (22) in case 2).

With the help of (10), (18), (21) and (22) we get

$$\begin{aligned} &|\Delta_x^{x'} u(t,x)| \le C(\psi(t,x) + \psi(t,x')) d^{\gamma} t^{1-\nu-\gamma/(2b)} ||f||_{\varphi}^0, \\ &t \in (0,T], \ \{x,x'\} \subset \mathbb{R}^n, \gamma \in (0, 1-1/(2b)]. \end{aligned} \tag{23}$$

From (20) and (23) the estimate

$$[u]_{\psi}^{\gamma} \le C||f||_{\varphi}^0$$

follows and by this result and (19) the estimate (11) holds.

b) Let $\nu \in (1-1/(2b), 1]$. Because of the first condition from (4) we represent integral (1) in the form

$$u(t,x) = \int_0^t d\tau \int_{\mathbb{R}^n} M(t,x;\tau,\xi) \Delta_{\xi}^{X_1(t-\tau)} f(\tau,\xi) d\xi, \quad (t,x) \in \Pi_{(0,T]}, \tag{24}$$

where $X_1(t) := (\bar{x}_1(t), \bar{x}_2(t), \bar{x}_3(t))$ as in (10).

With the help of (3), (5) and (7)–(10) we get

$$\begin{aligned} |u(t,x)| &\le C \int_0^t (t-\tau)^{-\nu-N} d\tau \int_{\mathbb{R}^n} \exp\{-(c-c_0)\rho(t-\tau,x,\xi)\} \exp\{-c_0\rho(t-\tau,x,\xi)\} \\ &\times (\varphi(\tau,\xi) + \varphi(\tau, X_1(t-\tau))) \frac{|\Delta_{\xi}^{X_1(t-\tau)} f(\tau,\xi)|}{\varphi(\tau,\xi) + \varphi(\tau,X_1(t-\tau))} d\xi \le C \int_0^t (t-\tau)^{-\nu-N} d\tau \\ &\times \int_{\mathbb{R}^n} \exp\{-(c-c_0)\rho(t-\tau,x,\xi)\} (d(\xi, X_1(t-\tau)))^{\lambda} d\xi \psi(t,x) [f]_{\varphi}^{\lambda}. \end{aligned}$$

Now let us use the inequality [6]

$$(d(\xi, X_1(t-\tau)))^\lambda \exp\{-\bar{c}\rho(t-\tau,x,\xi)\} \le C(t-\tau)^{\lambda/(2b)} \exp\{-\bar{c}_1\rho(t-\tau,x,\xi)\},$$
$$0 \le \tau < t \le T,\ \{x,\xi\} \subset \mathbb{R}^n,\ 0 < \bar{c}_1 < \bar{c},\ \lambda \in (0,1]. \tag{25}$$

For $\bar{c} = c - c_0$ with the help of (18) we have

$$|u(t,x)| \le C \int_0^t (t-\tau)^{-\nu-N+\lambda/(2b)} d\tau \int_{\mathbb{R}^n} \exp\{-\bar{c}_1\rho(t-\tau,x,\xi)\} d\xi \psi(t,x)[f]^\lambda_\varphi$$
$$= C\psi(t,x)[f]^\lambda_\varphi \int_0^t (t-\tau)^{-\nu+\lambda/(2b)} d\tau = C\psi(t,x)[f]^\lambda_\varphi t^{1-\nu+\lambda/(2b)},\ \ (t,x) \in \Pi_{(0,T]}. \tag{26}$$

Then

$$||u||^0_\psi \le C[f]^\lambda_\varphi. \tag{27}$$

Let us estimate the difference $\Delta^{x'}_x u$. If $d^{2b} \ge t$, where $d := d(x;x')$, then under condition (26) we have the estimate

$$|\Delta^{x'}_x u(t,x)| \le C(\psi(t,x) + \psi(t,x'))[f]^\lambda_\varphi t^{1-\nu+\lambda/(2b)},\ \ t \in (0,T],\ \{x,\xi\} \subset \mathbb{R}^n.$$

We obtain

$$|\Delta^{x'}_x u(t,x)| \le C(\psi(t,x) + \psi(t,x'))[f]^\lambda_\varphi d^\lambda t^{1-\nu}$$
$$\le C(\psi(t,x) + \psi(t,x'))d^\lambda [f]^\lambda_\varphi,\ t \in (0,T],\ \{x,\xi\} \subset \mathbb{R}^n; \tag{28}$$

and with $\nu + (\gamma-\lambda)/(2b) < 1$ we receive

$$|\Delta^{x'}_x u(t,x)| \le C(\psi(t,x) + \psi(t,x'))[f]^\lambda_\varphi t^{1-\nu-(\gamma-\lambda)/(2b)} t^{\gamma/(2b)}$$
$$\le C(\psi(t,x) + \psi(t,x'))[f]^\lambda_\varphi t^{1-\nu-(\gamma-\lambda)/(2b)} d^\gamma$$
$$\le C(\psi(t,x) + \psi(t,x'))d^\gamma [f]^\lambda_\varphi,\ t \in (0,T],\ \{x,\xi\} \subset \mathbb{R}^n. \tag{29}$$

It is sufficient to consider the case, where $d^{2b} < t$. By the first condition from (4) like (24) we write

$$\Delta^{x'}_x u(t,x) = \int_0^{t-d^{2b}} d\tau \int_{\mathbb{R}^n} \Delta^{x'}_x M(t,x;\tau,\xi) \Delta^{X_1(t-\tau)}_\xi f(\tau,\xi) d\xi$$
$$+ \int_{t-d^{2b}}^t d\tau \int_{\mathbb{R}^n} M(t,x;\tau,\xi) \Delta^{X_1(t-\tau)}_\xi f(\tau,\xi) d\xi$$
$$- \int_{t-d^{2b}}^t d\tau \int_{\mathbb{R}^n} M(t,x';\tau,\xi) \Delta^{X'_1(t-\tau)}_\xi f(\tau,\xi) d\xi =: \sum_{l=1}^3 K_l,$$

where $X'_1(t) := X_1(t)|_{x=x'}$.

Using (3), (6), the second inequality from (9), (10), we get

$$|K_1| \le C \int_0^{t-d^{2b}} (t-\tau)^{-\nu-N} d\tau \int_{\mathbb{R}^n} (d(x;x'))^\gamma (t-\tau)^{-\gamma/(2b)} \exp\{-c\rho(t-\tau,x,\xi)\}$$

$$\times(\varphi(\tau,\xi)+\varphi(\tau,X_1(t-\tau)))\frac{|\Delta_\xi^{X_1(t-\tau)}f(\tau,\xi)|}{\varphi(\tau,\xi)+\varphi(\tau,X_1(t-\tau))}d\xi \le C\int\limits_0^{t-d^{2b}}(t-\tau)^{-\nu-N-\gamma/(2b)}d\tau$$

$$\times\int\limits_{\mathbb{R}^n}\psi(t,x)\exp\{-(c-c_0)\rho(t-\tau,x,\xi)\}(d(\xi;X_1(t-\tau)))^\lambda d\xi d^\gamma[f]^\lambda_\varphi.$$

Now let us use the inequality (25) and equality (18). We get

$$|K_1|\le Cd^\gamma\int\limits_0^{t-d^{2b}}(t-\tau)^{-\nu-(\gamma-\lambda)/(2b)}d\tau\psi(t,x)[f]^\lambda_\varphi. \tag{30}$$

If $\nu+(\gamma-\lambda)/(2b)<1$, then from (30) we obtain

$$|K_1|\le Cd^\gamma\psi(t,x)[f]^\lambda_\varphi(t-\tau)^{1-\nu-(\gamma-\lambda)/(2b)}|^0_{\tau=t-d^{2b}}$$

$$=Cd^\gamma\psi(t,x)[f]^\lambda_\varphi(t^{1-\nu-(\gamma-\lambda)/(2b)}-d^{2b(1-\nu)-\gamma+\lambda})\le Cd^\gamma\psi(t,x)[f]^\lambda_\varphi.$$

If $\nu+(\gamma-\lambda)/(2b)>1$, then from (30) we obtain

$$|K_1|\le Cd^\gamma\psi(t,x)[f]^\lambda_\varphi(t-\tau)^{1-\nu-(\gamma-\lambda)/(2b)}|^{t-d^{2b}}_{\tau=0}=Cd^\gamma\psi(t,x)[f]^\lambda_\varphi(d^{2b(1-\nu)-\gamma+\lambda}$$

$$-t^{1-\nu-(\gamma-\lambda)/(2b)})\le Cd^{2b(1-\nu)+\lambda}\psi(t,x)[f]^\lambda_\varphi=Cd^\lambda\psi(t,x)[f]^\lambda_\varphi.$$

Let us estimate K_2. With the help of (3), (9), (10) and (25) we obtain

$$|K_2|\le C\int\limits_{t-d^{2b}}^t(t-\tau)^{-\nu-N}d\tau\int\limits_{\mathbb{R}^n}(d(\xi;X_1(t-\tau)))^\lambda\exp\{-c\rho(t-\tau,x,\xi)\}$$

$$\times(\varphi(\tau,\xi)+\varphi(\tau,X_1(t-\tau)))d\xi[f]^\lambda_\varphi\le C\int\limits_{t-d^{2b}}^t(t-\tau)^{-\nu-N}d\tau$$

$$\times\int\limits_{\mathbb{R}^n}(d(\xi;X_1(t-\tau)))^\lambda\exp\{-(c-c_0)\rho(t-\tau,x,\xi)\}\psi(t,x)d\xi[f]^\lambda_\varphi$$

$$\le C\int\limits_{t-d^{2b}}^t(t-\tau)^{-\nu-N+\lambda/(2b)}d\tau\int\limits_{\mathbb{R}^n}\exp\{-\bar{c}_1\rho(t-\tau,x,\xi)\}\psi(t,x)d\xi[f]^\lambda_\varphi.$$

Using (18) with $c'=\bar{c}_1$, we have

$$|K_2|\le C\int\limits_{t-d^{2b}}^t(t-\tau)^{-\nu+\lambda/(2b)}d\tau\psi(t,x)[f]^\lambda_\varphi.$$

Since $-\nu+\lambda/(2b)>-1$, we obtain

$$|K_2|\le C(t-\tau)^{1-\nu+\lambda/(2b)}|^{t-d^{2b}}_{\tau=t}\psi(t,x)[f]^\lambda_\varphi=Cd^{2b(1-\nu)+\lambda}\psi(t,x)[f]^\lambda_\varphi \tag{31}$$

and thus, we have

$$|K_2|\le Cd^\lambda d^{2b(1-\nu)}\psi(t,x)[f]^\lambda_\varphi\le Cd^\lambda\psi(t,x)[f]^\lambda_\varphi,$$

if $\nu + (\gamma - \lambda)/(2b) > 1$. In case, where $\nu + (\gamma - \lambda)/(2b) < 1$, we receive from (31) the following inequality

$$|K_2| \leq Cd^{\gamma} d^{2b(1-\nu)+\lambda-\gamma} \psi(t,x)[f]^{\lambda}_{\varphi} \leq Cd^{\gamma}\psi(t,x)[f]^{\lambda}_{\varphi}.$$

By the similar way we obtain

$$|K_3| \leq Cd^{\lambda}\psi(t,x')[f]^{\lambda}_{\varphi}$$

in case, where $\nu \in (1 - 1/(2b), 1]$, and

$$|K_3| \leq Cd^{\gamma}\psi(t,x')[f]^{\lambda}_{\varphi}$$

in case, where $\nu \in (1 - 1/(2b), 1]$ and $\nu - (\gamma - \lambda)/(2b) < 1$.

From (27), (28), (29) and from the estimates for K_l, $l \in L$, the estimates (12) and (13) follow with $\nu \in (1 - 1/(2b), 1]$.

c) Let $\nu \in (1, 1 + 1/(2b)]$. Because of the second condition from (4) we represent integral (1) in the form

$$u(t,x) = \int\limits_0^t d\tau \int\limits_{\mathbb{R}^{n_1}} \Big(\int\limits_{\mathbb{R}^{n_2+n_3}} (t-\tau)^{-\nu-N} \Omega(t,x;\tau,\xi) \Delta_{\xi}^{X_2(t-\tau)} f(\tau,\xi) d\xi_2 d\xi_3 \Big) d\xi_1, \tag{32}$$
$$(t,x) \in \Pi_{(0,T]},$$

where $X_2(t) := (\xi_1, \bar{x}_2(t), \bar{x}_3(t))$, with $\bar{x}_l(t)$, $l \in \{2,3\}$, which were determined in (9).

With the help of (3), (5) and (7)–(10) we get

$$|u(t,x)| \leq C \int\limits_0^t (t-\tau)^{-\nu-N} d\tau \int\limits_{\mathbb{R}^n} \exp\{-(c-c_0)\rho(t-\tau,x,\xi)\}$$

$$\times \exp\{-c_0\rho(t-\tau,x,\xi)\}(\varphi(\tau,\xi) + \varphi(\tau,X_2(t-\tau))) \frac{|\Delta_{\xi}^{X_2(t-\tau)} f(\tau,\xi)|}{\varphi(\tau,\xi) + \varphi(\tau,X_2(t-\tau))} d\xi$$

$$\leq C \int\limits_0^t (t-\tau)^{-\nu-N} d\tau \int\limits_{\mathbb{R}^n} \exp\{-(c-c_0)\rho(t-\tau,x,\xi)\} d_1(\xi; X_2(t-\tau);\lambda) d\xi \psi(t,x)[f]^{\lambda}_{1,\varphi}.$$

The inequality below follows from definitions of d, d_1 and X_2.

$$\begin{aligned} d_1(\xi; X_2(t-\tau);\lambda) &= \sum_{l=2}^{3} |\xi_l - \bar{x}_l(t-\tau)|^{(\lambda+1)/(2b(l-1)+1)} \\ &\leq C \left(\sum_{l=2}^{3} |\xi_l - \bar{x}_l(t-\tau)|^{1/(2b(l-1)+1} \right)^{\lambda+1} = C\left(d(\xi; X_2(t-\tau))\right)^{\lambda+1}, \\ &\qquad 0 \leq \tau < t \leq T, \quad \{x,\xi\} \subset \mathbb{R}^n, \quad \lambda \in (0,1]. \end{aligned}$$

Here $C > 0$ is some constant. Then taking into account inequality (25) we have

$$\begin{aligned} d_1(\xi; X_2(t-\tau);\lambda) \exp\{-\bar{c}\rho(t-\tau,x,\xi)\} &\leq C(d(\xi; X_2(t-\tau)))^{1+\lambda} \exp\{-\bar{c}\rho(t-\tau,x,\xi)\} \\ &\leq C(t-\tau)^{(1+\lambda)/(2b)} \exp\{-\bar{c}_1\rho(t-\tau,x,\xi)\}, \\ &0 \leq \tau < t \leq T,\ \{x,\xi\} \subset \mathbb{R}^n,\ 0 < \bar{c}_1 < \bar{c},\ \lambda \in (0,1]. \end{aligned} \tag{33}$$

For $\bar{c} = c - c_0$ with the help of (18) we have

$$\begin{aligned} |u(t,x)| &\le C \int_0^t (t-\tau)^{-\nu-N+(1+\lambda)/(2b)} d\tau \int_{\mathbb{R}^n} \exp\{-\bar{c}_1 \rho(t-\tau,x,\xi)\} d\xi \psi(t,x) [f]^{\lambda}_{1,\varphi} \\ &= C\psi(t,x)[f]^{\lambda}_{1,\varphi} \int_0^t (t-\tau)^{-\nu+(1+\lambda)/(2b)} d\tau \\ &= C\psi(t,x)[f]^{\lambda}_{1,\varphi} t^{1-\nu+(1+\lambda)/(2b)}, \ \ (t,x) \in \Pi_{(0,T]}. \end{aligned} \tag{34}$$

Then

$$||u||^0_{\psi} \le C[f]^{\lambda}_{1,\varphi}. \tag{35}$$

Let us estimate the difference $\Delta^{x'}_x u$. If $d^{2b} \ge t$, where $d := d(x;x')$, then under estimate (34) we have the inequality

$$\begin{aligned} |\Delta^{x'}_x u(t,x)| &\le C(\psi(t,x) + \psi(t,x'))[f]^{\lambda}_{1,\varphi} d^{\lambda} t^{1-\nu+1/(2b)} \\ &\le C(\psi(t,x) + \psi(t,x')) d^{\lambda} [f]^{\lambda}_{1,\varphi}, \ t \in (0,T], \ \{x,\xi\} \subset \mathbb{R}^n, \end{aligned}$$

and with $\nu + (\gamma - 1 - \lambda)/(2b) < 1$ we receive

$$\begin{aligned} |\Delta^{x'}_x u(t,x)| &\le C(\psi(t,x) + \psi(t,x'))[f]^{\lambda}_{1,\varphi} t^{1-\nu-(\gamma-1-\lambda)/(2b)} t^{\gamma/(2b)} \\ &\le C(\psi(t,x) + \psi(t,x'))[f]^{\lambda}_{1,\varphi} t^{1-\nu-(\gamma-1-\lambda)/(2b)} d^{\gamma} \\ &\le C(\psi(t,x) + \psi(t,x')) d^{\gamma} [f]^{\lambda}_{1,\varphi}, \ t \in (0,T], \ \{x,\xi\} \subset \mathbb{R}^n. \end{aligned} \tag{36}$$

It is sufficient to consider the case, where $d^{2b} < t$. By the second condition from (4) like (32) we write

$$\begin{aligned} \Delta^{x'}_x u(t,x) = & \int_0^{t-d^{2b}} d\tau \int_{\mathbb{R}^{n_1}} \Big(\int_{\mathbb{R}^{n_2+n_3}} \Delta^{x'}_x M(t,x;\tau,\xi) \Delta^{X_2(t-\tau)}_{\xi} f(\tau,\xi) d\xi_2 d\xi_3 \Big) d\xi_1 \\ & + \int_{t-d^{2b}}^{t} d\tau \int_{\mathbb{R}^{n_1}} \Big(\int_{\mathbb{R}^{n_2+n_3}} M(t,x;\tau,\xi) \Delta^{X_2(t-\tau)}_{\xi} f(\tau,\xi) d\xi_2 d\xi_3 \Big) d\xi_1 \\ & - \int_{t-d^{2b}}^{t} d\tau \int_{\mathbb{R}^{n_1}} \Big(\int_{\mathbb{R}^{n_2+n_3}} M(t,x';\tau,\xi) \Delta^{X_2'(t-\tau)}_{\xi} f(\tau,\xi) d\xi_2 d\xi_3 \Big) d\xi_1 =: \sum_{l=1}^{3} K_l', \end{aligned} \tag{37}$$

where $X_2'(t) := X_2(t)|_{x=x'}$.

Using (3), (6), the second inequality from (9), (10), we get

$$\begin{aligned} |K_1'| \le C & \int_0^{t-d^{2b}} (t-\tau)^{-\nu-N} d\tau \int_{\mathbb{R}^n} (d(x;x'))^{\gamma} (t-\tau)^{-\gamma/(2b)} \exp\{-c\rho(t-\tau,x,\xi)\} \\ & \times (\varphi(\tau,\xi) + \varphi(\tau, X_2(t-\tau))) \frac{|\Delta^{X_2(t-\tau)}_{\xi} f(\tau,\xi)|}{\varphi(\tau,\xi) + \varphi(\tau,X_2(t-\tau))} d\xi \le C \int_0^{t-d^{2b}} (t-\tau)^{-\nu-N-\gamma/(2b)} d\tau \\ & \times \int_{\mathbb{R}^n} \psi(\tau,x) \exp\{-(c-c_0)\rho(t-\tau,x,\xi)\} d_1(\xi; X_2(t-\tau);\lambda) d\xi d^{\gamma} [f]^{\lambda}_{1,\varphi}. \end{aligned}$$

Now let us use the inequalities (33) and equality (18). We get

$$|K_1'| \le Cd^\gamma \int_0^{t-d^{2b}} (t-\tau)^{-\nu-N-\gamma/(2b)+(1+\lambda)/(2b)} d\tau \int_{\mathbb{R}^n} \psi(\tau,x)\exp\{-\bar{c}_1\rho(t-\tau,x,\xi)\}d\xi$$

$$\times d^\gamma [f]^\lambda_{1,\varphi} = Cd^\gamma \int_0^{t-d^{2b}} (t-\tau)^{-\nu-(\gamma-1-\lambda)/(2b)} d\tau \psi(t,x)[f]^\lambda_{1,\varphi}.$$

If $\nu + (\gamma-1-\lambda)/(2b) > 1$, then

$$|K_1'| \le Cd^\gamma \psi(t,x)[f]^\lambda_{1,\varphi}(t-\tau)^{1-\nu-(\gamma-1-\lambda)/(2b)}|^{t-d^{2b}}_{\tau=0} = Cd^\gamma\psi(t,x)[f]^\lambda_{1,\varphi}(d^{2b(1-\nu)-\gamma+1+\lambda}$$

$$-t^{1-\nu-(\gamma-1-\lambda)/(2b)}) \le Cd^{2b(1-\nu)+1+\lambda}\psi(t,x)[f]^\lambda_{1,\varphi} \le Cd^\lambda\psi(t,x)[f]^\lambda_{1,\varphi}.$$

If $\nu + (\gamma-1-\lambda)/(2b) < 1$, then

$$|K_1'| \le Cd^\gamma\psi(t,x)[f]^\lambda_{1,\varphi}(t-\tau)^{1-\nu-(\gamma-1-\lambda)/(2b)}|^0_{\tau=t-d^{2b}} = Cd^\gamma\psi(t,x)[f]^\lambda_{1,\varphi}$$

$$\times(t^{1-\nu-(\gamma-1-\lambda)/(2b)} - d^{2b(1-\nu)-\gamma+1+\lambda}) \le Cd^\gamma\psi(t,x)[f]^\lambda_{1,\varphi}.$$

Let us estimate K_2'. With the help of (3), (9), (10) and (33) we obtain

$$|K_2'| \le C\int_{t-d^{2b}}^t (t-\tau)^{-\nu-N}d\tau \int_{\mathbb{R}^n} d_1(\xi;X_2(t-\tau);\lambda)\exp\{-c\rho(t-\tau,x,\xi)\}$$

$$\times(\varphi(\tau,\xi)+\varphi(\tau,X_2(t-\tau)))d\xi[f]^\lambda_{1,\varphi} \le C\int_{t-d^{2b}}^t (t-\tau)^{-\nu-N}d\tau$$

$$\times\int_{\mathbb{R}^n} d_1(\xi;X_2(t-\tau);\lambda)\exp\{-(c-c_0)\rho(t-\tau,x,\xi)\}\psi(t,x)d\xi[f]^\lambda_{1,\varphi}$$

$$\le C\int_{t-d^{2b}}^t (t-\tau)^{-\nu-N+(1+\lambda)/(2b)}d\tau\int_{\mathbb{R}^n}\exp\{-\bar{c}_1\rho(t-\tau,x,\xi)\}\psi(t,x)d\xi[f]^\lambda_{1,\varphi}.$$

Using (18) with $c' = \bar{c}_1$, we have

$$|K_2'| \le C\int_{t-d^{2b}}^t (t-\tau)^{-\nu+(1+\lambda)/(2b)}d\tau\psi(t,x)[f]^\lambda_{1,\varphi}.$$

Since $\nu - (1+\lambda)/(2b) < 1$, we obtain

$$|K_2'| \le C(t-\tau)^{1-\nu+(1+\lambda)/(2b)}|^{t-d^{2b}}_{\tau=t}\psi(t,x)[f]^\lambda_{1,\varphi} = Cd^{2b(1-\nu)+1+\lambda}\psi(t,x)[f]^\lambda_{1,\varphi}. \tag{38}$$

The estimate

$$|K_2'| \le Cd^\gamma d^{2b(1-\nu)+1+\lambda-\gamma}\psi(t,x)[f]^\lambda_{1,\varphi} \le Cd^\gamma\psi(t,x)[f]^\lambda_{1,\varphi}$$

follow from (38) if $\nu + (\gamma - 1 - \lambda)/(2b) < 1$, and the estimate

$$|K_2'| \leq C d^\lambda d^{2b(1-\nu)+1} \psi(t,x)[f]^\lambda_{1,\varphi} \leq C d^\lambda \psi(t,x)[f]^\lambda_{1,\varphi}$$

if $\nu + (\gamma - 1 - \lambda)/(2b) > 1$.

By the similar way we obtain

$$|K_3'| \leq C d^\gamma \psi(t,x')[f]^\lambda_{1,\varphi}$$

in case, where $\nu + (\gamma - 1 - \lambda)/(2b) < 1$, and

$$|K_3'| \leq C d^\lambda \psi(t,x')[f]^\lambda_{1,\varphi}$$

in the case, where $\nu + (\gamma - 1 - \lambda)/(2b) > 1$.

From (35), (36), (37) and from estimates for K_l', $l \in L$, the estimates (14) and (15) follow.

d) This case can be proved by the similar way as the case **c)**. We must use the third equality from (4); representation of the integral (1) in the form

$$u(t,x) = \int\limits_0^t d\tau \int\limits_{\mathbb{R}^{n_1+n_2}} \Big(\int\limits_{\mathbb{R}^{n_3}} (t-\tau)^{-\nu-N} \Omega(t,x;\tau,\xi) \Delta_\xi^{X_3(t-\tau)} f(\tau,\xi) d\xi_3 \Big) d\xi_1 d\xi_2, \ (t,x) \in \Pi_{(0,T]},$$

where $X_3(t) := (\xi_1, \xi_2, \bar{x}_3(t))$, with $\bar{x}_3(t)$, which was determined in (9); and estimates

$$\begin{aligned} d_2(\xi; X_3(t-\tau); \lambda) \exp\{-\bar{c}\rho(t-\tau,x,\xi)\} &\leq C(d(\xi; X_3(t-\tau)))^{1+2b+\lambda} \exp\{-\bar{c}\rho(t-\tau,x,\xi)\} \\ &\leq C(t-\tau)^{(1+2b+\lambda)/(2b)} \exp\{-\bar{c}_1\rho(t-\tau,x,\xi)\}, \\ &0 \leq \tau < t \leq T, \qquad \{x,\xi\} \subset \mathbb{R}^n, \\ &0 < \bar{c}_1 < \bar{c}, \qquad \lambda \in (0,1]. \end{aligned}$$

These estimates are obtained in the same way as estimates (33). □

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Дронь В.С., Івасишен С.Д., Мединський І.П. *Властивості інтегралів типу похідних від об'ємного потенціалу для одного ультрапараболічного рівняння типу Колмогорова довільного порядку* // Карпатські матем. публ. — 2019. — Т.11, №2. — C. 268–280.

Розглядаються інтеграли, які мають структуру та властивості, подібні до похідних від об'ємних потенціалів, породжених фундаментальним розв'язком задачі Коші для ультрапараболічного рівняння типу Колмогорова довільного порядку. Коефіцієнти цього рівняння залежать тільки від часової змінної. Встановлюється належність цих інтегралів до відповідних вагових просторів Гельдера, залежно від того, до яких просторів належить густина та ядро інтеграла.

Для побудови просторів Гельдера використовуються спеціальні відстані та вагові норми. Відстані враховують анізотропність за просторовими змінними рівняння, яке породжує інтеграли, що розглядаються. Ваговими функціями є експоненти, які необмежено зростають при $|x| \to \infty$ і тип їх зростання спеціальним способом залежить від змінної t.

Результати роботи можуть бути використані для встановлення коректної розв'язності задачі Коші та оцінок розв'язків даного неоднорідного рівняння у відповідних вагових просторах Гельдера.

Ключові слова і фрази: ультрапараболічне рівняння типу Колмогорова довільного порядку, інтеграл типу похідних від об'ємного потенціалу, вагова гельдерова норма, простір Гельдера зростаючих функцій.