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ANALOGUES OF THE NEWTON FORMULAS FOR THE BLOCK-SYMMETRIC POLYNOMIALS ON $\ell_p(\mathbb{C}^s)$

The classical Newton formulas gives recurrent relations between algebraic bases of symmetric polynomials. They are true, of course, for symmetric polynomials on infinite-dimensional Banach sequence spaces.

In this paper, we consider block-symmetric polynomials (or MacMahon symmetric polynomials) on Banach spaces $\ell_p(\mathbb{C}^s)$, $1 \le p \le \infty$. We prove an analogue of the Newton formula for the block-symmetric polynomials for the case $p = 1$. In the case $1 < p$ we have no classical elementary block-symmetric polynomials. However, we extend the obtained Newton type formula for $\ell_1(\mathbb{C}^s)$ to the case of $\ell_p(\mathbb{C}^s)$, $1 < p \le \infty$, and in this way we found a natural definition of elementary block-symmetric polynomials on $\ell_p(\mathbb{C}^s)$.

Key words and phrases: symmetric polynomials, block-symmetric polynomials, algebraic basis, Newton's formula.

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1 Introduction

Let X be a Banach space, and let $\mathcal{P}(X)$ be the algebra of all continuous polynomials defined on X. Let $\mathcal{P}_0(X)$ be a subalgebra of $\mathcal{P}(X)$. A sequence $(Q_i)_i$ of polynomials is called an algebraic basis of $\mathcal{P}_0(X)$ if for every $P \in \mathcal{P}_0(X)$ there is a unique polynomial $q \in \mathcal{P}(\mathbb{C}^n)$ for some n such that $P(x) = q(Q_1(x), \ldots, Q_n(x))$. In other words, if Q is mapping $x \in X \rightsquigarrow (Q_1(x), \ldots, Q_n(x)) \in \mathbb{C}^n$, then $P = q \circ Q$ and this representation is unique. Subalgebras of polynomials with countable algebraic bases were considered by many authors (see e. g. [4, 8, 9, 11, 12]). Typical examples of such kind of algebras are algebras of polynomials which are invariant with respect to a (semi)group $\mathcal{S}$ of operators on X. If X has an unconditional basis (e_n), we can consider the group $\mathcal{S} = S_\infty$ of all permutations of natural numbers $\mathbb{N}$ acting on X by

$$\sigma \colon x = \sum_{n=1}^{\infty} x_n e_n \rightsquigarrow \sum_{n=1}^{\infty} x_{\sigma(n)} e_n.$$

S_∞-invariant polynomials on X are called *symmetric*. Symmetric polynomials and analytic functions on ℓ_p were investigated in [1–3, 5, 6, 8]. Linear bases of symmetric polynomials on ℓ_1 were considered in [7].

Let $\mathcal{P}_s(\ell_p)$ be the algebra of all symmetric polynomials on ℓ_1. In [10], it is proved that polynomials

$$F_k = \sum_{i=1}^{\infty} x_i^k,$$

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$k \geq \lceil p \rceil$ form an algebraic basis in $\mathcal{P}_s(\ell_p)$, where $\lceil p \rceil$ is the smallest integer, greater than p. Polynomials F_k are called *power* symmetric polynomials. In the case $p = 1$ we can consider another natural algebraic basis in $\mathcal{P}_s(\ell_1)$, which is called the *basis of elementary symmetric polynomials*, $\{G_k\}_{k=1}^{\infty}$,

$$G_k = \sum_{i_1 < i_2 < \ldots < i_k}^{\infty} x_{i_1} x_{i_2} \ldots x_{i_k}, \tag{1}$$

The relation between power symmetric polynomials and elementary symmetric polynomials can be given by the well-known Newton formulas (see, e.g., [17]):

$$nG_n = F_1 G_{n-1} - F_2 G_{n-2} + F_3 G_{n-3} - \ldots + (-1)^{n-2} F_{n-1} G_1 + (-1)^{n-1} F_n, \quad n \in \mathbb{N}.$$

In the case $p > 1$ we have no elementary symmetric polynomials, because the series (1) does not converge for any k. But putting in the Newton formulas $F_k = 0$ for $k < p$, we can define elementary symmetric polynomials on ℓ_p by

$$G_n^{(p)} = \sum_{k=\lceil p \rceil}^{n-\lceil p \rceil} (-1)^{k-1} F_k G_{n-k}.$$

It is easy to check that the sequence $\{G_n^{(p)}\}_{n>p}$ forms an algebraic basis in $\mathcal{P}_s(\ell_p)$.

There are other natural representations of S_∞ in Banach spaces with bases. For example, if $\mathcal{X}$ is a directs sum of infinite many of "blocks" which are copies of a Banach space X, then S_∞ acts permutating the "blocks". For this case we can consider the algebra of block-symmetric analytic functions consisting of invariants of this group. Note that this algebra is much more complicated and in the finitely-dimensional case has no algebraic basis (see, e.g., [15, 19]).

A generalization of the Newton formula for block-symmetric polynomials on $\ell_1(\mathbb{C}^2)$ was proved in [13]. In this paper we propose a generalization of this formula for block-symmetric polynomials on $\ell_p(\mathbb{C}^s)$.

2 MAIN RESULT

Let us denote by $\ell_p(\mathbb{C}^s)$, $1 \leq p < \infty$, the vector space of all sequences

$$x = (x_1, x_2, \ldots, x_m, \ldots),$$

where $x_j = (x_j^{(1)}, \ldots, x_j^{(s)}) \in \mathbb{C}^s$ for $j \in \mathbb{N}$, such that the series $\sum\limits_{j=1}^{\infty} \sum\limits_{r=1}^{s} \left| x_j^{(r)} \right|^p$ is convergent. The space $\ell_p(\mathbb{C}^s)$ with norm

$$\|x\| = \left(\sum_{j=1}^{\infty} \sum_{r=1}^{s} \left| x_j^{(r)} \right|^p \right)^{1/p}$$

is a Banach space. A polynomial P on the space $\ell_p(\mathbb{C}^s)$ is called block-symmetric (or vector-symmetric) if

$$P(x_1, x_2, \ldots, x_m, \ldots) = P(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(m)}, \ldots)$$

for every permutation $\sigma \in S_\infty$, where $x_j \in \mathbb{C}^s$ for all $j \in \mathbb{N}$. Let us denote by $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$ the algebra of all block-symmetric polynomials on $\ell_p(\mathbb{C}^s)$.

The algebra $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$ was considered in [14, 16]. Note that in Combinatorics, block-symmetric polynomials on finite-dimension spaces are called *MacMahon symmetric polynomials* (see [18]).

For a multi-index $\mathbf{k} = (k_1, k_2, \ldots, k_s) \in \mathbb{Z}_+^s$ let $m = |\mathbf{k}| = k_1 + k_2 + \ldots + k_s$.

In [14] it was proved that polynomials

$$H_m^{\mathbf{k}}(x) = H_m^{k_1,k_2,\ldots,k_s}(x) = \sum_{j=1}^{\infty} \prod_{\substack{r=1 \\ |\mathbf{k}|\geq\lceil p\rceil}}^{s} (x_j^{(r)})^{k_r} \tag{2}$$

form an algebraic basis in $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$, $1 \leq p < \infty$, where $x = (x_1, \ldots, x_m, \ldots) \in \ell_p(\mathbb{C}^s)$, $x_j = (x_j^{(1)}, \ldots, x_j^{(s)}) \in \mathbb{C}^s$.

In the case of the space $\ell_1(\mathbb{C}^s)$ there are elementary block-symmetric polynomials

$$R_m^{\mathbf{k}}(x) = R_m^{k_1,k_2,\ldots,k_s}(x) = \sum_{\substack{i_1<\ldots<i_{k_1} \\ j_1<\ldots<j_{k_2} \\ \cdots \\ l_1<\ldots<l_{k_s} \\ i_{k_p}\neq j_{k_q}\neq\ldots\neq l_{k_r}}}^{\infty} x_{i_1}^{(1)} \ldots x_{i_{k_1}}^{(1)} x_{j_1}^{(2)} \ldots x_{j_{k_2}}^{(2)} \ldots x_{l_1}^{(s)} \ldots x_{l_{k_s}}^{(s)}, \tag{3}$$

where $(x_i^{(1)}, x_i^{(2)}, \ldots, x_i^{(s)}) \in \mathbb{C}^s$.

Combining (2) and (3), we can get an analog of Newton's formula for block-symmetric polynomials on $\ell_1(\mathbb{C}^s)$.

Theorem 1. *The following formula is true for the algebraic bases of symmetric polynomials on $\ell_1(\mathbb{C}^s)$.*

$$\begin{aligned} nR_n^{k_1,k_2,\ldots,k_s} = & \sum_{\substack{|\mathbf{q}|=1 \\ k_r\geq q_r}} H_1^{q_1,q_2,\ldots,q_s} R_{n-1}^{k_1-q_1,k_2-q_2,\ldots,k_s-q_s} \\ & - \sum_{\substack{|\mathbf{q}|=2 \\ k_r\geq q_r}} \frac{2!}{q_1!q_2!\ldots q_s!} H_2^{q_1,q_2,\ldots,q_s} R_{n-2}^{k_1-q_1,k_2-q_2,\ldots,k_s-q_s} + \ldots \\ & + (-1)^{n-2} \sum_{\substack{|\mathbf{q}|=n-1 \\ k_r\geq q_r}} \frac{(n-1)!}{q_1!q_2!\ldots q_s!} H_{n-1}^{q_1,q_2,\ldots,q_s} R_1^{k_1-q_1,k_2-q_2,\ldots,k_s-q_s} \\ & + (-1)^{n-1} \frac{n!}{k_1!k_2!\ldots k_s!} H_n^{k_1,k_2,\ldots,k_s}, \end{aligned} \tag{4}$$

where $\mathbf{q} = (q_1, q_2, \ldots, q_s)$, $R_0^{k_1,k_2,\ldots,k_s} \equiv 1$ *and if* $k_r < q_r$ *for some* $r = 1, \ldots, s$, *then* $R_m^{k_1-q_1,k_2-q_2,\ldots,k_s-q_s} \equiv 0$.

Proof. Let us consider the polynomial $P(t_1x^{(1)} + t_2x^{(2)} + \ldots + t_sx^{(s)})$, which is symmetric on the space ℓ_1 with respect to simultaneously permutations of $t_1x_i^{(1)} + t_2x_i^{(2)} + \ldots + t_sx_i^{(s)}$, $i \geq 1$. Let us denote by $\widetilde{tx} = t_1x^{(1)} + t_2x^{(2)} + \ldots + t_sx^{(s)}$. For the algebraic bases $F_k(\widetilde{tx})$ and $G_k(\widetilde{tx})$ of this polynomial the Newton formula holds

$$\begin{aligned} nG_n(\widetilde{tx}) = & F_1(\widetilde{tx})G_{n-1}(\widetilde{tx}) - F_2(\widetilde{tx})G_{n-2}(\widetilde{tx}) \\ & + F_3(\widetilde{tx})G_{n-3}(\widetilde{tx}) - \ldots + (-1)^{n-2}F_{n-1}(\widetilde{tx})G_1(\widetilde{tx}) + (-1)^{n-1}F_n(\widetilde{tx}). \end{aligned} \tag{5}$$

Each of polynomials $F_m(\widetilde{tx})$ and $G_m(\widetilde{tx})$ can be represented as a linear combination of polynomials $H_m^{k_1,k_2,\dots,k_s}(x)$ and $R_m^{k_1,k_2,\dots,k_s}(x)$ respectively. Indeed,

$$\begin{aligned} G_n(\widetilde{tx}) &= G_n(t_1x^{(1)} + t_2x^{(2)} + \ldots + t_sx^{(s)}) \\ &= \sum_{i_1<\ldots<i_n} (t_1x^{(1)} + t_2x^{(2)} + \ldots + t_sx^{(s)})_{i_1} \ldots (t_1x^{(1)} + t_2x^{(2)} + \ldots + t_sx^{(s)})_{i_n} \\ &= \sum_{p_1+p_2+\ldots+p_s=n} t_1^{p_1} t_2^{p_2} \ldots t_s^{p_s} R_n^{p_1,p_2,\ldots,p_s}(x) \end{aligned} \tag{6}$$

and

$$\begin{aligned} F_n(\widetilde{tx}) &= F_n(t_1x^{(1)} + t_2x^{(2)} + \ldots + t_sx^{(s)}) = \sum_{i=1}^{\infty} (t_1x^{(1)} + t_2x^{(2)} + \ldots + t_sx^{(s)})_i^n \\ &= \sum_{k_1+k_2+\ldots+k_s=n} \frac{n!}{k_1!k_2!\ldots k_s!} t_1^{k_1} t_2^{k_2} \ldots t_s^{k_s} H_n^{k_1,k_2,\ldots,k_s}(x). \end{aligned} \tag{7}$$

So each term of equality (5) can be represented by polynomials $H_m^{k_1,k_2,\ldots,k_s}$ and $R_m^{p_1,p_2,\ldots,p_s}$. Then we obtain

$$\begin{aligned} F_1(\widetilde{tx}) G_{n-1}(\widetilde{tx}) &= \left(\sum_{k_1+k_2+\ldots+k_s=1} \frac{1!}{k_1!k_2!\ldots k_s!} t_1^{k_1} t_2^{k_2} \ldots t_s^{k_s} H_1^{k_1,k_2,\ldots,k_s}(x) \right) \\ &\quad \times \left(\sum_{p_1+p_2+\ldots+p_s=n-1} t_1^{p_1} t_2^{p_2} \ldots t_s^{p_s} R_{n-1}^{p_1,p_2,\ldots,p_s}(x) \right) \\ &= \sum_{\substack{k_1+k_2+\ldots+k_s=1 \\ p_1+p_2+\ldots+p_s=n-1}} \frac{1!}{k_1!k_2!\ldots k_s!} t_1^{k_1+p_1} t_2^{k_2+p_2} \ldots t_s^{k_s+p_s} H_1^{k_1,k_2,\ldots,k_s}(x) R_{n-1}^{p_1,p_2,\ldots,p_s}(x), \end{aligned}$$

. . . .

$$\begin{aligned} F_r(\widetilde{tx}) G_{n-r}(\widetilde{tx}) &= \left(\sum_{k_1+k_2+\ldots+k_s=r} \frac{r!}{k_1!k_2!\ldots k_s!} t_1^{k_1} t_2^{k_2} \ldots t_s^{k_s} H_r^{k_1,k_2,\ldots,k_s}(x) \right) \\ &\quad \times \left(\sum_{p_1+p_2+\ldots+p_s=n-r} t_1^{p_1} t_2^{p_2} \ldots t_s^{p_s} R_{n-r}^{p_1,p_2,\ldots,p_s}(x) \right) \\ &= \sum_{\substack{k_1+k_2+\ldots+k_s=r \\ p_1+p_2+\ldots+p_s=n-r}} \frac{r!}{k_1!k_2!\ldots k_s!} t_1^{k_1+p_1} t_2^{k_2+p_2} \ldots t_s^{k_s+p_s} H_r^{k_1,k_2,\ldots,k_s}(x) R_{n-r}^{p_1,p_2,\ldots,p_s}(x). \end{aligned}$$

If we substitute this equalities and equalities (6), (7) into (5) and equate multipliers at the all powers of $t_i, i = 1, \ldots, s$ we obtain the required formula. □

Note that equation (4) is invertible and so we have

$$\begin{aligned} \frac{n!}{k_1!\ldots k_s!} H_n^{k_1,\ldots,k_s} &= \sum_{\substack{|\mathbf{q}|=n-1 \\ k_r \ge q_r}} \frac{(n-1)!}{q_1!\ldots q_s!} H_{n-1}^{q_1,\ldots,q_s} R_1^{k_1-q_1,\ldots,k_s-q_s} + \ldots \\ &\quad + (-1)^{n-1} \sum_{\substack{|\mathbf{q}|=2 \\ k_r \ge q_r}} \frac{2!}{q_1!\ldots q_s!} H_2^{q_1,\ldots,q_s} R_{n-2}^{k_1-q_1,\ldots,k_s-q_s} \\ &\quad + (-1)^n \sum_{\substack{|\mathbf{q}|=1 \\ k_r \ge q_r}} H_1^{q_1,\ldots,q_s} R_{n-1}^{k_1-q_1,\ldots,k_s-q_s} + (-1)^{n+1} n R_n^{k_1,\ldots,k_s}. \end{aligned}$$

Let us rewrite formula (4) using multi-index notations. We denote by $\mathbf{k}\,! = k_1!k_2!\ldots k_s!$ and by $\mathbf{k} - \mathbf{q} = (k_1 - q_1, k_2 - q_2, \ldots k_s - q_s)$. Also, we say that $\mathbf{k} \geq \mathbf{q}$ if and only if $k_1 \geq q_1$, $k_2 \geq q_2, \ldots, k_s \geq q_S$. Then (4) can be expressed by

$$\begin{aligned} nR_n^{\mathbf{k}} &= \sum_{\substack{|\mathbf{q}|=1\\ \mathbf{k}\geq\mathbf{q}}} H_1^{\mathbf{q}} R_{n-1}^{\mathbf{k}-\mathbf{q}} - \sum_{\substack{|\mathbf{q}|=2\\ \mathbf{k}\geq\mathbf{q}}} \frac{|\mathbf{q}|!}{\mathbf{q}\,!} H_2^{\mathbf{q}} R_{n-2}^{\mathbf{k}-\mathbf{q}} + \cdots + (-1)^{n-2} \sum_{\substack{|\mathbf{q}|=n-1\\ \mathbf{k}\geq\mathbf{q}}} \frac{|\mathbf{q}|!}{\mathbf{q}\,!} H_{n-1}^{\mathbf{q}} R_1^{\mathbf{k}-\mathbf{q}} \\ &+ (-1)^{n-1}\frac{n!}{\mathbf{k}\,!} H_n^{\mathbf{k}} = \sum_{j=1}^{n} (-1)^{j-1} \sum_{\substack{|\mathbf{q}|=j\\ \mathbf{k}\geq\mathbf{q}}} \frac{|\mathbf{q}|!}{\mathbf{q}\,!} H_j^{\mathbf{q}} R_{n-j}^{\mathbf{k}-\mathbf{q}}, \quad \text{where} \quad n = |\mathbf{k}|. \end{aligned} \tag{8}$$

Comparing formula (8) with the classical Newton formula we can see that their are coincide if $s = 1$.

Let us turn out to the space $\ell_p(\mathbb{C}^s)$. Taking into account formula (2) we can see that by definition, $H_m^{\mathbf{k}} = 0$ in $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$ if $|\mathbf{k}| < \lceil p \rceil$. So, using (8), we can define *elementary block-symmetric polynomials on* $\ell_p(\mathbb{C}^s)$ by

$$nR_n^{\mathbf{k}} = \sum_{j=\lceil p \rceil}^{n-\lceil p \rceil} (-1)^{j-1} \sum_{\substack{|\mathbf{q}|=j\\ \mathbf{k}\geq\mathbf{q}}} \frac{|\mathbf{q}|!}{\mathbf{q}\,!} H_j^{\mathbf{q}} R_{n-j}^{\mathbf{k}-\mathbf{q}}, \quad \text{where} \quad n = |\mathbf{k}| \geq \lceil p \rceil. \tag{9}$$

Theorem 2. *Elementary block-symmetric polynomials on* $\ell_p(\mathbb{C}^s)$ *defined by* (9) *form an algebraic basis of* n*-homogeneous polynomials* $n \geq \lceil p \rceil$ *in* $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$.

Proof. It is easy to see that equation (9) is invertible. So we have a bijection between polynomials $H_n^{\mathbf{q}}$ and $R_n^{\mathbf{q}}$. Since $\{H_n^{\mathbf{q}}\}_{n\geq\lceil p \rceil}$ is an algebraic basis in $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$, so the set $\{R_n^{\mathbf{q}}\}_{n\geq\lceil p \rceil}$ is an algebraic basis in $\mathcal{P}_{vs}(\ell_p(\mathbb{C}^s))$ too. □

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Кравців В.В. *Аналог формули Ньютона для блочно-симетричних поліномів на $\ell_p(\mathbb{C}^n)$* // Карпатські матем. публ. — 2020. — Т.12, №1. — С. 17–22.

Класичні формули Ньютона задає рекурентні співвідношення між алгебраїчними базисами симетричних поліномів. Ці формули залишаються правильними і для симетричних поліномів на нескінченновимірних банахових просторах послідовностей.

В цій статті ми розглядаємо блочно-симетричні поліноми (або симетричні полінома Макмахона) на банахових просторах $\ell_p(\mathbb{C}^s)$, $1 \leq p \leq \infty$. Ми доводимо аналог формули Ньютона для блочно-симетричних поліномів у випадку $p = 1$. У випадку $1 < p$ немає класичних елементарних блочно-симетричних поліномів. Проте ми продовжили отриману формулу типу Ньютона для $\ell_1(\mathbb{C}^s)$ на випадок $\ell_p(\mathbb{C}^s)$, $1 < p \leq \infty$, і, в такий спосіб, запропонували природнє означення елементарних блочно-симетричних поліномів на $\ell_p(\mathbb{C}^s)$.

Ключові слова і фрази: симетричні поліноми, блочно-симетричні поліноми, алгебраїчний базис, формула Ньютона.