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ON THE CONVERGENCE OF MULTIDIMENSIONAL *S*-FRACTIONS WITH INDEPENDENT VARIABLES

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The paper investigates the convergence problem of a special class of branched continued fractions, i.e. the multidimensional S-fractions with independent variables, consisting of

$$\sum_{i_1=1}^{N}\frac{c_{i(1)}z_{i_1}}{1}+\sum_{i_2=1}^{i_1}\frac{c_{i(2)}z_{i_2}}{1}+\sum_{i_3=1}^{i_2}\frac{c_{i(3)}z_{i_3}}{1}+\cdots,$$

which are multidimensional generalizations of S-fractions (Stieltjes fractions). These branched continued fractions are used, in particular, for approximation of the analytic functions of several variables given by multiple power series. For multidimensional S-fractions with independent variables we have established a convergence criterion in the domain

$$H=\left\{\mathbf{z}=(z_1,z_2,\ldots,z_N)\in\mathbb{C}^N:\ |\arg(z_k+1)|<\pi,\ 1\le k\le N\right\}$$

as well as the estimates of the rate of convergence in the open polydisc

$$Q=\left\{\mathbf{z}=(z_1,z_2,\ldots,z_N)\in\mathbb{C}^N:\ |z_k|<1,\ 1\le k\le N\right\}$$

and in a closure of the domain Q.

Key words and phrases: branched continued fraction, convergence criterion, uniform convergence, estimates of the rate of convergence, continued fraction.

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INTRODUCTION

Special classes of branched continued fractions play an important role dealing with problem of the approximation of analytic functions of several variables. Among others, one of the classes is the class of multidimensional S-fractions with independent variables.

Let N be a fixed natural number and

$$\mathcal{I}_k=\{i(k):\ i(k)=(i_1,i_2,\ldots,i_k),\ 1\le i_p\le i_{p-1},\ 1\le p\le k,\ i_0=N\},\ k\ge1,$$

be the sets of multiindices. In addition, let $i(0)=0$ and $\mathcal{I}_0=\{0\}$.

Multidimensional S-fractions with independent variables are of the form

$$\sum_{i_1=1}^{N}\frac{c_{i(1)}z_{i_1}}{1}+\sum_{i_2=1}^{i_1}\frac{c_{i(2)}z_{i_2}}{1}+\sum_{i_3=1}^{i_2}\frac{c_{i(3)}z_{i_3}}{1}+\cdots, \tag{1}$$

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where the $c_{i(k)} > 0$ for all $i(k) \in \mathcal{I}_k$, $k \geq 1$, $\mathbf{z} = (z_1, z_2, \ldots, z_N) \in \mathbb{C}^N$.

Some of their important properties are studied in [2], convergent domains (here domain is an open connected set) and convergent regions (here region is a domain together with all, part or none of its boundary) are investigated in [2,5,13,17], estimates of the rate of convergence are established in [1–5,7,10].

The paper is a continuation of the article [17], where it was investigated the convergence domains of (1) with coefficients $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, satisfying the inequalities

$$c_{i(k)} \leq r_{i(k)}^{i_k} r_{i(k-1)}^{i_k-1}(1 - r_{i(k-1)}) \text{ for all } i(k) \in \mathcal{I}_k,\ k \geq 1, \tag{2}$$

where $\{r_{i(k)}\}_{i(k)\in\mathcal{I}_k,\ k\geq 0}$ is a sequence of real numbers such that

$$0 < r_{i(k)} < 1 \text{ for all } i(k) \in \mathcal{I}_k,\ k \geq 0. \tag{3}$$

For multidimensional S-fractions with independent variables (1) we have established a new convergence criteria and estimates of the rate of convergence in an open polydisc

$$Q = \left\{ \mathbf{z} \in \mathbb{C}^N :\ |z_k| < 1,\ 1 \leq k \leq N \right\} \tag{4}$$

and in a closure of the domain Q.

Various problems of the convergence of other classes have been studied in [2, 6, 7, 15] for multidimensional regular C-fractions with independent variables, in [20, 21] for multidimensional g-fractions with independent variables, and in [3, 4, 8, 11, 12, 18] for multidimensional A- and J-fractions with independent variables. Expansions of certain analytic functions of several variables in the above mentioned branched continued fractions may be found in [12,14, 16,19,21].

MAIN RESULTS

Let

$$G_{i(k)}^{(n)}(\mathbf{z}) = 1 + \sum_{i_{k+1}=1}^{i_k} \frac{c_{i(k+1)} z_{i_{k+1}}}{1} \,+\, \sum_{i_{k+2}=1}^{i_{k+1}} \frac{c_{i(k+2)} z_{i_{k+2}}}{1} \,+\, \cdots \,+\, \sum_{i_n=1}^{i_{n-1}} \frac{c_{i(n)} z_{i_n}}{1},$$

where $i(k) \in \mathcal{I}_k,\ 1 \leq k \leq n-1,\ n \geq 2$. Then

$$G_{i(k)}^{(n)}(\mathbf{z}) = 1 + \sum_{i_{k+1}=1}^{i_k} \frac{c_{i(k+1)} z_{i_{k+1}}}{G_{i(k+1)}^{(n)}(\mathbf{z})},\ i(k) \in \mathcal{I}_k,\ 1 \leq k \leq n-1,\ n \geq 2, \tag{5}$$

with the initial conditions $G_{i(n)}^{(n)}(\mathbf{z}) \equiv 1$, $i(n) \in \mathcal{I}_n$, $n \geq 1$. Thus, the n-th approximant of (1) we have written as

$$f_n(\mathbf{z}) = \sum_{i_1=1}^{N} \frac{c_{i(1)} z_{i_1}}{G_{i(1)}^{(n)}(\mathbf{z})},\ n \geq 1.$$

If $G_{i(k)}^{(n)}(\mathbf{z}) \not\equiv 0$ for all $i(k) \in \mathcal{I}$, $1 \le k \le n$, $n \ge 1$, then for each $n \ge 1$ and $m \ge 1$ the following formula is valid (see [5])

$$f_{n+m}(\mathbf{z}) - f_n(\mathbf{z}) = \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \dots \sum_{i_{n+1}=1}^{i_n} \frac{(-1)^n \prod_{k=1}^{n+1} c_{i(k)} z_{i_k}}{\prod_{k=1}^{n+1} G_{i(k)}^{(n+m)}(\mathbf{z}) \prod_{k=1}^{n} G_{i(k)}^{(n)}(\mathbf{z})}. \tag{6}$$

Theorem 1. *Let (1) be a multidimensional S-fraction with independent variables whose coefficients $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \ge 1$, satisfy the conditions (2)–(3). Then the multidimensional S-fraction with independent variables (1) converges to a holomorphic function in the domain*

$$H = \left\{ \mathbf{z} \in \mathbb{C}^N : \ |\arg(z_k + 1)| < \pi, \ 1 \le k \le N \right\}.$$

The convergence is uniform on every compact subset of H.

Proof. By [17, Theorem 1] the assertions of the theorem are valid for all $\mathbf{z}$ in the domain

$$R = \left\{ \mathbf{z} \in \mathbb{C}^N : \ |\arg(z_k + 1/4)| < \pi, \ 1 \le k \le N \right\}.$$

Therefore it suffices to show that these assertions are also valid in the domain (4). From [18, Corollary 2.1] it follows that the multidimensional S-fraction with independent variables (1) converges for all $\mathbf{z} \in Q$. Hence, by [9, Theorem 2.17] (see also [22, Theorem 24.2]), the convergence of (1) is uniform on every compact subset of Q. □

Theorem 2. *Let (1) be a multidimensional S-fraction with independent variables whose coefficients $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \ge 1$, satisfy conditions (2), where $\{r_{i(k)}\}_{i(k)\in\mathcal{I}_k,\, k\ge 0}$ is a sequence of real numbers such that*

$$1/2 \le r_{i(k)} < 1 \ \text{ for all } \ i(k) \in \mathcal{I}_k, \ k \ge 0.$$

Assume that the multidimensional S-fraction with independent variables converges to a function $f(\mathbf{z})$ holomorphic in the domain (4). If $f_n(\mathbf{z})$ denotes the n-th approximant of (1), then for each $\mathbf{z} \in Q$

$$|f(\mathbf{z}) - f_n(\mathbf{z})| \le \frac{r_0(1-r)^{n+1}}{r^{N+n-1}} \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \dots \sum_{i_{n+1}=1}^{i_n} \prod_{k=1}^{n+1} |z_{i_k}|, \quad n \ge 1, \tag{7}$$

where

$$r = \inf_{i(k)\in\mathcal{I}_k,\, k\ge 0} r_{i(k)}. \tag{8}$$

Proof. Let n be an arbitrary natural number and $\mathbf{z}$ be an arbitrary fixed point from the domain (4). By induction on k for each multiindex $i(k) \in \mathcal{I}_k$ we show that the following inequalities are valid

$$|G_{i(k)}^{(n)}(\mathbf{z})| > r_{i(k)}^{i_k}, \ \ 1 \le k \le n. \tag{9}$$

It is clear that for $k = n$, $i(n) \in \mathcal{I}_n$, relations (9) hold. By induction hypothesis that (9) hold for $k = s+1$, $s \le n-1$, $i(s+1) \in \mathcal{I}_{s+1}$, we prove (9) for $k = s$ and for each $i(s) \in \mathcal{I}_s$. Indeed, using of relations (2)–(5) for the arbitrary multiindex $i(s) \in \mathcal{I}_s$ leads to

$$|G_{i(s)}^{(n)}(\mathbf{z})| \ge 1 - \sum_{i_{s+1}=1}^{i_s} \frac{c_{i(s+1)}|z_{i_{s+1}}|}{|G_{i(s+1)}^{(n)}(\mathbf{z})|} > 1 - \sum_{i_{s+1}=1}^{i_s} \frac{r_{i(s+1)}^{i_{s+1}} r_{i(s)}^{i_{s+1}-1}(1 - r_{i(s)})}{|G_{i(s+1)}^{(n)}(\mathbf{z})|}.$$

By virtue of estimates (9), we have $G^{(n)}_{i(s+1)}(\mathbf{z}) \not\equiv 0$. Therefore, replacing $r^{i_{s+1}}_{i(s+1)}$ by $|G^{(n)}_{i(s+1)}(\mathbf{z})|$, we obtain inequalities (9) for $k = s$ and for all $i(s) \in \mathcal{I}_s$.

From (9) it follows that $G^{(n)}_{i(k)}(\mathbf{z}) \not\equiv 0$ for all indices. Applying the inequalities (2), (3), and (9) to the formula (6), for $n \geq 1$ and $m \geq 1$ we get

$$
\begin{aligned}
|f_{n+m}(\mathbf{z}) - f_n(\mathbf{z})| &\leq \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \frac{\prod_{k=1}^{n+1} c_{i(k)} |z_{i_k}|}{\prod_{k=1}^{n+1} |G^{(n+m)}_{i(k)}(\mathbf{z})| \prod_{k=1}^{n} |G^{(n)}_{i(k)}(\mathbf{z})|} \\
&< \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \frac{\prod_{k=1}^{n+1} r^{i_k}_{i(k)} r^{i_k-1}_{i(k-1)} (1 - r_{i(k-1)}) |z_{i_k}|}{\prod_{k=1}^{n+1} r^{i_k}_{i(k)} \prod_{k=1}^{n} r^{i_k}_{i(k)}} \\
&= \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \frac{\prod_{k=1}^{n+1} r^{i_k}_{i(k-1)} (1 - r_{i(k-1)}) |z_{i_k}|}{\prod_{k=1}^{n+1} r_{i(k-1)} \prod_{k=1}^{n} r^{i_k}_{i(k)}} \\
&\leq \frac{(1-r)^{n+1}}{r^n} \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \frac{\prod_{k=1}^{n+1} |z_{i_k}|}{\prod_{k=1}^{n} r^{i_k - i_{k-1}}_{i(k)}} \\
&\leq \frac{r_0 (1-r)^{n+1}}{r^{N+n-1}} \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \prod_{k=1}^{n+1} |z_{i_k}|.
\end{aligned}
$$

Hence, passing to the limit as $m \to \infty$, we obtain estimates (7). □

We remark that the expression in the right-hand side of inequality (7) tends to 0 as $n \to \infty$. Indeed, let $\mathbf{z}$ be an arbitrary fixed point from the domain (4). It is clear that for any $n \geq 1$

$$
\left(\frac{1-r}{r} \right)^n \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \prod_{k=1}^{n+1} |z_{i_k}| \leq \sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \prod_{k=1}^{n+1} |z_{i_k}|,
$$

where $1/2 \leq r < 1$. It is known (see [1]) that for an arbitrary natural N the following equality holds for each $n \geq 0$

$$
\sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} 1 = C^{N-1}_{N+n}.
$$

Let $1/L(\mathbf{z}) = \max\{|z_1|, |z_2|, \ldots, |z_N|\}$. Then for any $n \geq 1$

$$
\sum_{i_1=1}^{N} \sum_{i_2=1}^{i_1} \cdots \sum_{i_{n+1}=1}^{i_n} \prod_{k=1}^{n+1} |z_{i_k}| \leq \frac{C^{N-1}_{N+n}}{L^{n+1}(\mathbf{z})} = \prod_{k=0}^{n} \frac{N+k}{(k+1)L(\mathbf{z})} = \prod_{k=0}^{n} \left(1 - \frac{(k+1)L(\mathbf{z}) - N - k}{(k+1)L(\mathbf{z})} \right).
$$

Hence, since the series

$$
\sum_{k=0}^{\infty} \frac{(k+1)L(\mathbf{z}) - N - k}{(k+1)L(\mathbf{z})}
$$

diverges, it follows that the expression in the right-hand side of inequality (7) tends to 0 as $n \to \infty$.

From Theorem 2 we have the following assertion.

Corollary 1. *Let (1) be a multidimensional S-fraction with independent variables whose coefficients* $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, *satisfy*

$$c_{i(k)} \leq r^{2i_k-1}(1-r) \text{ for all } i(k) \in \mathcal{I}_k,\ k \geq 1, \tag{10}$$

where $1/2 \leq r < 1$, *and which converges to a function* $f(\mathbf{z})$ *holomorphic in the domain (4). If* $f_n(\mathbf{z})$ *denotes the* n*-th approximant of multidimensional S-fraction with independent variables (1), then for each* $\mathbf{z} \in Q$ *the estimates (7) hold.*

The following theorem can be proved in much the same way as Theorem 2.

Theorem 3. *Let (1) be a multidimensional S-fraction with independent variables whose coefficients* $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, *satisfy the conditions (2), where* $\{r_{i(k)}\}_{i(k)\in\mathcal{I}_k,\, k\geq 0}$ *is a sequence of real numbers such that*

$$1/2 + \varepsilon \leq r_{i(k)} < 1 \text{ for all } i(k) \in \mathcal{I}_k,\ k \geq 0,$$

where $0 < \varepsilon < 1/2$. *Then:*

(A) the multidimensional S-fraction with independent variables (1) converges to a finite value $f(\mathbf{z})$ *for each* $\mathbf{z} \in \overline{Q}$, *where* $\overline{Q}$ *is the closure of the domain* Q *defined by (4), and it converges uniformly on every compact subset of* Q*;*

(B) if $f_n(\mathbf{z})$ *denotes the* n*-th approximant of multidimensional S-fraction with independent variables (1), then for all* $\mathbf{z} \in \overline{Q}$

$$|f(\mathbf{z}) - f_n(\mathbf{z})| \leq \frac{r_0(1-r)^{n+1}}{r^{N+n-1}} C_{N+n}^{N-1}, \quad n \geq 1, \tag{11}$$

where r *is defined by (8).*

In view of Corollary 1, the following corollary follows directly from Theorem 3.

Corollary 2. *Let (1) be a multidimensional S-fraction with independent variables whose coefficients* $c_{i(k)}$, $i(k) \in \mathcal{I}_k$, $k \geq 1$, *satisfy the conditions (10), where* $1/2 < r < 1$. *Then:*

(A) the multidimensional S-fraction with independent variables (1) converges to a finite value $f(\mathbf{z})$ *for each* $\mathbf{z} \in \overline{Q}$, *where* $\overline{Q}$ *is the closure of the domain* Q *defined by (4), and it converges uniformly on every compact subset of* Q*;*

(B) if $f_n(\mathbf{z})$ *denotes the* n*-th approximant of multidimensional S-fraction with independent variables (1), then for all* $\mathbf{z} \in \overline{Q}$ *the estimates (11) hold.*

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Досліджується питання збіжності спеціального класу гіллястих ланцюгових дробів — багатовимірних S-дробів з нерівнозначними змінними

$$\sum_{i_1=1}^{N} \frac{c_{i(1)} z_{i_1}}{1} + \sum_{i_2=1}^{i_1} \frac{c_{i(2)} z_{i_2}}{1} + \sum_{i_3=1}^{i_2} \frac{c_{i(3)} z_{i_3}}{1} + \cdots,$$

що є багатовимірним узагальненням S-дробів (дробів Стілтьєса). Ці гіллясті ланцюгові дроби використовуються, зокрема, для наближення аналітичних функцій багатьох змінних, заданих кратними степеневими рядами. Для багатовимірних S-дробів з нерівнозначними змінними встановлено критерій збіжності в області

$$H = \left\{ \mathbf{z} = (z_1, z_2, \ldots, z_N) \in \mathbb{C}^N : \ |\arg(z_k + 1)| < \pi, \ 1 \le k \le N \right\}$$

та отримано оцінки швидкості збіжності у відкритому полікрузі

$$Q = \left\{ \mathbf{z} = (z_1, z_2, \ldots, z_N) \in \mathbb{C}^N : \ |z_k| < 1, \ 1 \le k \le N \right\}$$

та у замиканні області Q.

Ключові слова і фрази: гіллястий ланцюговий дріб, критерій збіжності, рівномірна збіжність, оцінки швидкості збіжності, неперервний дріб.