ISSN 2075-9827 **e-ISSN 2313-0210**
Carpathian Math. Publ. 2020, **12** (2), 360–367
doi:10.15330/cmp.12.2.360-367

https://journals.pnu.edu.ua/index.php/cmp
Карпатські матем. публ. 2020, Т.12, №2, С.360–367

POWER OPERATIONS AND DIFFERENTIATIONS ASSOCIATED WITH SUPERSYMMETRIC POLYNOMIALS ON A BANACH SPACE

CHERNEGA I.[1], FUSHTEI V.[1], ZAGORODNYUK A.[2]

We consider different approaches to constructing power operations on the ring of multisets associated with supersymmetric polynomials of infinitely many variables. Some relations between constructed power operations are established. Also, we study differential operators on algebras of symmetric and supersymmetric analytic functions of bounded type on the Banach space of absolutely summable sequences. We have proved the continuity of such operators and found their evaluations on basis polynomials.

Key words and phrases: supersymmetric polynomial, analytic function on a Banach space, power operation, differential operator, ring of multisets.

[1] Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, 3b Naukova str., 79060, Lviv, Ukraine
[2] Vasyl Stefanyk Precarpathian National University, 57 Shevchenka str., 76018, Ivano-Frankivsk, Ukraine
E-mail: icherneha@ukr.net (Chernega I.), v.i.fushtei@gmail.com (Fushtei V.),
azagorodn@gmail.com (Zagorodnyuk A.)

INTRODUCTION AND PRELIMINARIES

Let X be a complex Banach space. A (continuous) map $P\colon X \to \mathbb{C}$ is said to be a (continuous) n-homogeneous polynomial if there exists a (continuous) n-linear map $B_P\colon X^n \to \mathbb{C}$ such that $P(x) = B_P(x, \ldots, x)$. Here 0-homogeneous polynomials are just constant functions. A finite sum of homogeneous polynomials is a polynomial. We denote by $\mathcal{P}(^nX)$ the space of all continuous n-homogeneous polynomials on X and by $\mathcal{P}(X)$ the algebra of all polynomials on X. It is well known that $\mathcal{P}(^nX)$ is a Banach space with respect to any of the norms

$$\|P\|_r = \sup_{\|x\|\le r} |P(x)|, \qquad r > 0. \tag{1}$$

Let us denote by τ_b the topology on $\mathcal{P}(X)$ generated by the countable family of norms (1) for positive rational numbers r. It is clear that τ_b is metrisable. We denote by $H_b(X)$ the completion of $(\mathcal{P}(X), \tau_b)$. So $H_b(X)$ is a Fréchet algebra which consists of entire analytic functions on X which are bounded on all bounded subsets (so-called *entire functions of bounded type*). For details on polynomials and analytic functions on Banach spaces we refer the reader to [12]. The spectra (sets of continuous complex homomorphisms = sets of characters) of $H_b(X)$ and its subalgebras were investigated by many authors (see e.g., [1–3, 9, 23, 24]). For every $x \in X$, point evaluation functional $\delta_x\colon f \mapsto f(x)$, $f \in H_b(X)$, belongs to the spectrum of $H_b(X)$.

A sequence of polynomials $\{P_n\}_{n=1}^{\infty}$ is *algebraically independent* if for every nonzero polynomial q of m variables with $q(0) = 0$ and any finite subsequence $\{P_{n_1}, \ldots, P_{n_m}\}$, $m \in \mathbb{N}$, the

УДК 517.98
2010 *Mathematics Subject Classification:* 46J15, 46J20, 46E15.
This research was funded by the National Research Foundation of Ukraine, 2020.02/0025, 0120U103996

polynomial $q(P_{n_1}(x), \ldots, P_{n_m}(x))$ is nonzero. An algebraically independent sequence of polynomials $\{P_n\}_{n=1}^{\infty}$ is an *algebraic basis* of a subalgebra $A \in \mathcal{P}(X)$ if every element in A can be represented as an algebraic span of a finite number of polynomials from $\{P_n\}_{n=1}^{\infty}$.

Let $X = \ell_1$ and $\mathcal{G}$ be the group of permutations of basis vectors. A function f on ℓ_1 is said to be *symmetric* if it is invariant with respect to all permutations in G. We denote by $\mathcal{P}_s(\ell_1)$ the algebra of all continuous symmetric polynomials on ℓ_1 and by $H_{bs}(\ell_1)$ its closure in $H_b(\ell_1)$. There are a lot of algebraic bases in $\mathcal{P}_s(\ell_1)$. The following bases in the algebra of symmetric polynomials are interesting for us:

- the basis of *power series*:
$$F_k(x) = \sum_{n=1}^{\infty} x_n^k;$$
- the basis of *elementary symmetric polynomials*:
$$G_k(x) = \sum_{n_1 < n_2 < \cdots < n_k} x_{n_1} \ldots x_{n_k};$$
- *complete symmetric polynomials*:
$$H_k(x) = \sum_{n_1 \le n_2 \le \cdots \le n_k} x_{n_1} \ldots x_{n_k}.$$

Here $x = (x_1, x_2, \ldots, x_n, \ldots) \in \ell_1$. Note that basic properties of classical symmetric polynomials on finite-dimensional spaces are still true for symmetric polynomials on ℓ_1. In particular, the well known Newton formulas hold

$$mH_m = H_{m-1}F_1 + H_{m-2}F_2 + \cdots + H_{m-k}F_k + \cdots + F_m, \tag{2}$$
$$mG_m = G_{m-1}F_1 - G_{m-2}F_2 + \cdots + (-1)^{k+1}G_{m-k}F_k + \cdots + (-1)^{m+1}F_m.$$

Algebras of symmetric analytic functions with respect to various symmetry groups or semigroups of operators on Banach spaces were studied in [1,4–11,13–19,21].

Denote $\mathbb{Z}_0 = \mathbb{Z} \setminus \{0\}$. Let $\ell_1(\mathbb{Z}_0) = \ell_1 \bigoplus \ell_1$ be the Banach space of all absolutely summing complex sequences indexed by numbers in $\mathbb{Z}_0$. Any element z in $\ell_1(\mathbb{Z}_0)$ has the representation

$$z = (\ldots, z_{-n}, \ldots, z_{-2}, z_{-1}, z_1, z_2, \ldots, z_n, \ldots) = (y|x) = (\ldots, y_n, \ldots, y_2, y_1 | x_1, x_2, \ldots, x_n, \ldots)$$

with
$$\|z\| = \sum_{i=-\infty}^{\infty} |z_i|,$$

where $x = (x_1, x_2, \ldots, x_n, \ldots)$ and $y = (y_1, y_2, \ldots, y_n, \ldots)$ are in ℓ_1, $z_n = x_n$, $z_{-n} = y_n$ for $n \in \mathbb{N}$ and
$$x \longmapsto (0|x_1, x_2, \ldots, x_n, \ldots) \quad \text{and} \quad y \longmapsto (\ldots, y_{-n}, \ldots, y_{-2}, y_{-1}|0)$$

are natural isometric embeddings of the copies of ℓ_1 into $\ell_1(\mathbb{Z}_0)$.

Let us define the following polynomials on $\ell_1(\mathbb{Z}_0)$:

$$T_k(z) = F_k(x) - F_k(y) = \sum_{i=1}^{\infty} x_i^k - \sum_{i=1}^{\infty} y_i^k, \quad k \in \mathbb{N}.$$

A polynomial P on $\ell_1(\mathbb{Z}_0)$ is said to be *supersymmetric* if it can be represented as an algebraic combination of polynomials $\{T_k\}_{k=1}^{\infty}$. In other words, P is a finite sum of finite products of polynomials in $\{T_k\}_{k=1}^{\infty}$ and constants. We denote by $\mathcal{P}_{sup} = \mathcal{P}_{sup}(\ell_1(\mathbb{Z}_0))$ the algebra of all supersymmetric polynomials on $\ell_1(\mathbb{Z}_0)$. Let us denote by H_b^{sup} the closure of $\mathcal{P}_{sup}$ in $H_b(\ell_1(\mathbb{Z}_0))$.

Note that polynomials T_k are algebraically independent because F_k are so. Hence $\{T_k\}_{k=1}^{\infty}$ forms an algebraic basis in $\mathcal{P}_{sup}$. In [20] it was proved that

$$W_n(y|x) = \sum_{k=0}^{n} G_k(x)H_{n-k}(-y), \qquad n \in \mathbb{N}, \tag{3}$$

is another algebraic basis. From [20] it follows that the semigroup of symmetry of $\mathcal{P}_{sup}$ is a minimal semigroup that contains all permutations of coordinates on $(0|x_1, \ldots, x_n, \ldots)$, all permutations of coordinates on $(\ldots, y_n, \ldots, y_1|0)$ and operators of the form:

$$A_\lambda \colon (\ldots, y_n, \ldots, y_1|x_1, \ldots, x_n, \ldots) \mapsto (\ldots, y_n, \ldots, y_1, \lambda|\lambda, x_1, \ldots, x_n, \ldots), \quad \lambda \in \mathbb{C}.$$

We say that $z \sim w$ for some $z, w \in \ell_1(\mathbb{Z}_0)$ if $T_k(z) = T_k(w)$ for every $k \in \mathbb{N}$. Let us denote by $\mathcal{M}$ the quotient set $\ell_1(\mathbb{Z}_0)/\sim$ which is a natural domain for supersymmetric polynomials. For a given $z \in \ell_1(\mathbb{Z}_0)$, let $[z] \in \mathcal{M}$ be the class of equivalence which contains z. Also, we denote by $\mathcal{M}_+$ the subset of $\mathcal{M}$ consisting of elements $[(0|x_1, x_2, \ldots)]$. $\mathcal{M}_+$ can be considered as a set of multisets. It is clear that every function $f \in H_b^{sup}$ is well-defined on $\mathcal{M}$ and we will write $f(u) = f(y|x)$ for $u = [(y|x)] \in \mathcal{M}$. Note that $z \sim w$ if and only if $\delta_z = \delta_w$. Thus $\mathcal{M}$ can be embedded into the spectrum of H_b^{sup}.

In [20] algebraic operations "$\bullet$" and "$\diamond$" on $\ell_1(\mathbb{Z}_0)$ were introduced and extended to $\mathcal{M}$. Let $z = (y|x)$ and $w = (d|b)$ be in $\ell_1(\mathbb{Z}_0)$. Then

$$z \bullet w = (y \bullet d|x \bullet b) = (\ldots, d_n, y_n, \ldots, d_1, y_1|x_1, b_1, \ldots, x_n, b_n, \ldots).$$

Also, we denote $z^- = (y|x)^- = (x|y)$. Clearly, $(z^-)^- = z$ and $z \bullet z^- \sim (0|0)$. These operations can be naturally defined on $\mathcal{M}$ by

$$[z] \bullet [w] = [z \bullet w] \quad \text{and} \quad [z]^- = [z^-].$$

Let $x, y \in \ell_1$. By $x \diamond y$ (see [11]) we mean the resulting sequence of ordering the set $\{x_i y_j \colon i, j \in \mathbb{N}\}$ with one single index in some fixed order. If $u = [(0|x)]$ and $v = [(0|y)]$, then $u \diamond v = [(0|x \diamond y)]$. Let $u = [(y|x)]$ and $v = [(d|b)]$ be in $\mathcal{M}$. We define

$$u \diamond v = [((y \diamond b) \bullet (x \diamond d)|(y \diamond d) \bullet (x \diamond b))].$$

In [20] it is proved that $(\mathcal{M}, \bullet, \diamond)$ is a commutative ring (so-called the ring of multisets) with zero $0 = [(0|0)]$ and unity $\mathbb{I} = [(0|1, 0, \ldots)]$, and T_k, $k \in \mathbb{N}$, are ring homomorphisms from $\mathcal{M}$ to $\mathbb{C}$. Note that $(\mathcal{M}, \bullet, \diamond)$ is not a linear space but in [20] it was introduced a "norm" $\|\cdot\|$ on $\mathcal{M}$ satisfying natural conditions by

$$\|u\| = \inf\Big\{ \sum_i |x_i| + \sum_j |y_j| \colon (y|x) \in u \Big\}.$$

Using the norm, in [20] it was introduced a metric $\rho(u, v) = \|u \bullet v^-\|$ and proved that $(\mathcal{M}, \rho)$ is a complete metric space.

In the first section we consider different approaches to construct power operations on $\mathcal{M}$. In Section 2 we study some operators of differentiation on $H_{bs}(\ell_1)$ and H_b^{sup} associated with the symmetric shift $x \longmapsto x \bullet a$. For combinatorial theory of symmetric polynomials we refer the reader to [22].

1 POWER FUNCTIONS

Let $u_1, u_2, \ldots, u_n \in \mathcal{M}$. We will use notation

$$\bigodot_{k=1}^{n} u_k := u_1 \bullet u_2 \bullet \cdots \bullet u_n.$$

Using the operation "$\diamond$" we can consider for a given $k \in \mathbb{N}$ a natural power function on $\mathcal{M}$

$$u \longmapsto u^{\diamond k} = \underbrace{u \diamond \cdots \diamond u}_{k}.$$

However there are different ways to introduce power function on $\mathcal{M}$ or $\mathcal{M}_+$. Let us define

$$u^{[k]} = [(\ldots, y_n^k, \ldots, y_1^k | x_1^k, \ldots, x_n^k, \ldots)], \quad u = [(y|x)] \in \mathcal{M}, \quad k \in \mathbb{N}.$$

If $u = [(0|x)] \in \mathcal{M}_+$, then

$$u^{\langle k \rangle} = \bigodot_{i_1 < \cdots < i_k} [(0|x_{i_1} \ldots x_{i_k})] = [(0|x_1 x_2 \ldots x_k, x_2 x_3 \ldots x_{k+1}, \ldots)].$$

Finally, for $u = [(0|x)] \in \mathcal{M}_+$, we define $u^{\overline{\langle k \rangle}} = \bigodot_{i_1 \leq \cdots \leq i_k} [(0|x_{i_1} \ldots x_{i_k})]$.

Clearly that

$$T_1(u^{\diamond k}) = (T_1(u))^k, \quad T_1(u^{[k]}) = T_k(u)$$

and

$$T_1([(0|x)]^{\langle k \rangle}) = G_k(x), \quad T_1([(0|x)]^{\overline{\langle k \rangle}}) = H_k(x).$$

For the general case $u = [(y|x)]$ we set

$$u^{\langle k \rangle} = \bigodot_{m+n=k} \bigodot_{\substack{i_1 \leq \cdots \leq i_m \\ j_1 < \cdots < j_n}} [(\underbrace{y_{i_1} \cdots y_{i_m}}_{\text{if } m \text{ is odd}} x_{j_1} \cdots x_{j_n} | \underbrace{y_{i_1} \cdots y_{i_m}}_{\text{if } m \text{ is even}} x_{j_1} \cdots x_{j_n})].$$

Proposition 1. *Let $u = [(y|x)] \in \mathcal{M}$. Then $T_1(u^{\langle k \rangle}) = W_k(u)$.*

Proof. Using (3), we can see that

$$\begin{aligned} T_1(u^{\langle k \rangle}) &= \sum_{n+m=k,\, m=2s} H_m(y) G_n(x) - \sum_{n+m=k,\, m=2s+1} H_m(y) G_n(x) \\ &= \sum_{n+m=k} H_m(-y) G_n(x) = W_k(u). \end{aligned}$$

□

Theorem 1. *For every $k \in \mathbb{N}$, power operations $u \longmapsto u^{\diamond k}$, $u \longmapsto u^{[k]}$, $u \longmapsto u^{\langle k \rangle}$ are well-defined on $\mathcal{M}$ and continuous.*

Proof. Let us suppose that $u = [(y|x)]$, $v = [(d|b)]$ and $(y|x) \sim (d|b)$. Then

$$T_n(u^{\diamond k}) = \big(T_n(u)\big)^k = \big(T_n(v)\big)^k = T_n(v^{\diamond k}), \quad n \in \mathbb{N},$$

that is, $u^{\diamond k} = v^{\diamond k}$. The continuity of $u \longmapsto u^{\diamond k}$ is proved in [20]. By the similar reason,

$$T_n(u^{[k]}) = T_{nk}(u) = T_{nk}(v) = T_n(v^{[k]}), \quad n \in \mathbb{N},$$

and so $u^{[k]} = v^{[k]}$.

Finally,

$$T_n(u^{\langle k\rangle}) = T_1\Big(\big(u^{\langle k\rangle}\big)^{[n]}\Big) = T_1\Big(\big(u^{[n]}\big)^{\langle k\rangle}\Big) = W_k(u^{[n]}) = W_k(v^{[n]}) = T_n(v^{\langle k\rangle}).$$

The continuity of $u \longmapsto u^{[k]}$ and $u \longmapsto u^{\langle k\rangle}$ follows from the simple fact that if $\|u_j\| \to 0$, then $\|u^{[k]}\| \to 0$ and $\|u^{\langle k\rangle}\| \to 0$ as $j \to \infty$. $\square$

In [20] it was prowed that there exists a continuous homomorphism $\Phi\colon H_b^{sup} \to H_{bs}(\ell_1)$ with a dense range such that $\Phi(T_n) = F_n$ and $\Phi(W_n) = G_n$. Thus the restriction of the inverse map $\Lambda = \Phi^{-1}$ on the subspace of supersymmetric polynomials $\mathcal{P}_{sup}$ is an algebra isomorphism from $\mathcal{P}_{sup}$ onto $\mathcal{P}_s(\ell_1)$. Applying Λ to the Newton formula, we have

$$kW_k(u) = \sum_{i=1}^{k}(-1)^{i+1}T_i(u)W_{k-i}(u), \quad u \in \mathcal{M}, \quad k \in \mathbb{N}. \tag{4}$$

Proposition 2. *For every $k \in \mathbb{N}$ and $u \in \mathcal{M}$ the following equality holds*

$$\underbrace{u^{\langle k\rangle} \bullet \cdots \bullet u^{\langle k\rangle}}_{k} = \bigodot_{i=1}^{k}[(1|0)]^{\diamond(i+1)} \diamond u^{[i]} \diamond u^{\langle k-i\rangle}.$$

Proof. Using (4), we can write

$$\begin{aligned} T_n\Big(\underbrace{u^{\langle k\rangle} \bullet \cdots \bullet u^{\langle k\rangle}}_{k}\Big) &= kT_n(u^{\langle k\rangle}) = kW_k(u^{[n]}) = \sum_{i=1}^{k}(-1)^{i+1}T_i(u^{[n]})W_{k-i}(u^{[n]}) \\ &= \sum_{i=1}^{k}(-1)^{i+1}T_n(u^{[i]})T_n(u^{\langle k-i\rangle}) \\ &= T_n\Big(\bigodot_{i=1}^{k}[(1|0)]^{\diamond(i+1)} \diamond u^{[i]} \diamond u^{\langle k-i\rangle}\Big). \end{aligned}$$

Since it is true for every $n \in \mathbb{N}$, we have our equality. $\square$

Let us denote by $\nu\colon \mathbb{Z} \to \mathcal{M}$ the following mapping: $\nu(n) = [(0|\underbrace{1,\ldots,1}_{n},0,\ldots)]$ if $n \geq 0$ and $\nu(n) = [(\ldots,0\underbrace{1,\ldots,1}_{-n}|0)]$ if $n < 0$. Clearly, ν is a ring homomorphism. Let

$$q(t) = \sum_{k=0}^{m} n_k t^k$$

be a polynomial with integer coefficients. For a given $u \in \mathcal{M}$ we consider the following transformations from the set $\mathbb{Z}[t]$ of polynomials with integer coefficients to $\mathcal{M}$:

$$q_\diamond(u) = \bigodot_{k=0}^{m} \nu(n_k)u^{\diamond k};$$

$$q_{[\cdot]}(u) = \bigodot_{k=0}^{m} \nu(n_k) u^{[k]};$$

$$q_{\langle\cdot\rangle}(u) = \bigodot_{k=0}^{m} \nu(n_k) u^{\langle k\rangle}.$$

From the proved properties of the power operations we have the following proposition.

Proposition 3. *All mappings $q \longmapsto q_\diamond$, $q \longmapsto q_{[\cdot]}$ and $q \longmapsto q_{\langle\cdot\rangle}$ are additive maps and $q \longmapsto q_\diamond$ is a multiplicative map between the rings $\mathbb{Z}[t]$ and $\mathcal{M}$.*

2 DERIVATIVES

Let us consider the following operator on H_{bs} (c. f. [8])

$$\partial f(x) = \lim_{t\to 0} \frac{f(x \bullet (t, 0, 0, \ldots)) - f(x)}{t}.$$

Note first that ∂ is linear and satisfies the Leibnitz rule for differentiation

$$\partial(fg) = \partial(f)g + f\partial(g)$$

for all f, g in the domain of ∂. Also, ∂ is well defined on polynomials in H_{bs}. Indeed

$$F_m(x \bullet (t, 0, 0, \ldots)) - F_m(x) = F_m((t, 0, 0, \ldots)) = t^m.$$

Thus $\partial F_1 = 1$ and $\partial F_m(x) = 0$ for $m > 0$. Since polynomials F_k, $k \in \mathbb{N}$, form an algebraic basis in the algebra of all symmetric polynomials $\mathcal{P}_s(\ell_1)$, operator ∂ is defined on $\mathcal{P}_s(\ell_1)$ and $\partial P \in \mathcal{P}_s(\ell_1)$ for every $P \in \mathcal{P}_s(\ell_1)$.

Proposition 4. *The differential operator ∂ is continuous on $H_{bs}(\ell_1)$.*

Proof. Let $A_0 \colon \ell_1 \to \ell_1$ be the forward shift operator on ℓ_1,

$$A_0(x_1, \ldots x_n, \ldots) = (0, x_1, \ldots x_n, \ldots)$$

and d_1 be the Gâteaux derivative in direction $(1, 0, 0 \ldots)$ on $H_b(\ell_1)$. Then ∂ is the restriction to $H_{bs}(\ell_1)$ of the continuous operator $d_1 \circ C_{A_0}$, where $C_{A_0}(f)(x) = f(A_0(x))$, $f \in H_b(\ell_1)$. Thus ∂ is continuous on $H_{bs}(\ell_1)$. □

Proposition 5. *For every $m \in \mathbb{N}$ we have $\partial G_m = G_{m-1}$ and $\partial H_m = H_{m-1}$.*

Proof. Since

$$G_m(x \bullet z) = \sum_{k+n=m} G_k(x) G_m(z), \quad x, z \in \ell_1,$$

we have

$$G_m(x \bullet (t, 0, 0 \ldots)) = \sum_{k=0}^{m} t^k G_k(x)$$

and so $\partial G_m = G_{m-1}$. The second equality can be obtained by the similar way or using Newton formulas (2) and simple induction. □

The differential operator ∂ can be extended to the supersymmetric analytic functions. Let $\mathbb{I} = [(0|1)]$ and $\mathbb{I}^- = [(1|0)]$. We set

$$\partial_+ f(u) = \lim_{t\to 0} \frac{f(u \bullet t\mathbb{I}) - f(u)}{t} \quad \text{and} \quad \partial_- f(u) = \lim_{t\to 0} \frac{f(u \bullet t\mathbb{I}^-) - f(u)}{t},$$

where $u \in \mathcal{M}$. Similarly to the symmetric case, both ∂_+ and ∂_- are continuous and well-defined on the whole space H_b^{sup}.

Theorem 2. *For every $m \in \mathbb{N}$*

$$\partial_+ W_m = W_{m-1} \quad \textit{and} \quad \partial_- W_m = -W_{m-1},$$

where $W_0 = 1$.

Proof. According to (3), we can write

$$\begin{aligned}\partial_+ W_m(y|x) &= \partial_+ \sum_{k=0}^{n} G_k(x) H_{m-k}(-y) = \sum_{k=0}^{n} G_{k-1}(x) H_{m-k}(-y) \\ &= \sum_{k=0}^{m-1} G_k(x) H_{m-k-1}(-y) = W_{m-1}.\end{aligned}$$

The second equality can be proved by the same way, taking into account that $H_{m-k}(-y) = (-1)^{m-k} H_{m-k}(y)$. $\square$

References

[1] Alencar R., Aron R., Galindo P., Zagorodnyuk A. *Algebra of symmetric holomorphic functions on ℓ_p*. Bull. Lond. Math. Soc. 2003, **35**, 55–64. doi:10.1112/S0024609302001431

[2] Aron R.M., Cole B.J., Gamelin T.W. *Spectra of algebras of analytic functions on a Banach space*. J. Reine Angew. Math. 1991, **415**, 51–93. doi:10.1515/crll.1991.415.51

[3] Aron R.M., Cole B.J., Gamelin T.W. *Weak-star continuous analytic funtions*. Canad. J. Math. 1995, **47** (4), 673–683. doi:10.4153/CJM-1995-035-1

[4] Aron R.M., Falcó J., García D., Maestre M. *Algebras of symmetric holomorphic functions of several complex variables*. Rev. Mat. Complut. 2018, **31**, 651–672. doi: 10.1007/s13163-018-0261-x

[5] Aron R., Falcó J., Maestre M. *Separation theorems for group invariant polynomials*. J. Geom. Anal. 2018, **28** (1), 393–404. doi:10.1007/s12220-017-9825-0

[6] Aron R., Galindo P., Pinasco D., Zalduendo I. *Group-symmetric holomorphic functions on a Banach space*. Bull. Lond. Math. Soc. 2016, **48** (5), 779–796. doi:10.1112/blms/bdw043

[7] Chernega I.V. *A semiring in the spectrum of the algebra of symmetric analytic functions in the space ℓ_1*. J. Math. Sci. 2016, **212**, 38–45. doi:10.1007/s10958-015-2647-3 (translation of Mat. Metody Fiz.-Mekh. Polya 2014, **57**, 35–40. (in Ukrainian))

[8] Chernega I., Holubchak O., Novosad Z., Zagorodnyuk A. *Continuity and hypercyclicity of composition operators on algebras of symmetric analytic functions on Banach spaces*. Eur. J. Math. 2020, **6**, 153–163. doi:10.1007/s40879-019-00390-z

[9] Chernega I., Galindo P., Zagorodnyuk A. *Some algebras of symmetric analytic functions and their spectra*. Proc. Edinb. Math. Soc. (2) 2011, **55** (1), 125–142. doi:10.1017/S0013091509001655

[10] Chernega I., Galindo P., Zagorodnyuk A. *The convolution operation on the spectra of algebras of symmetric analytic functions*. J. Math. Anal. Appl. 2012, **395** (2), 569–577. doi:10.1016/j.jmaa.2012.04.087

[11] Chernega I., Galindo P., Zagorodnyuk A. *A multiplicative convolution on the spectra of algebras of symmetric analytic functions*. Rev. Mat. Complut. 2014, **27** (2), 575–585. doi:10.1007/s13163-013-0128-0

[12] Dineen S. Complex analysis on infinite-dimensional spaces. Springer-Verlag, London, 1999.

[13] Falcó J., García D., Jung M., Maestre M. *Group invariant separating polynomials on a Banach space*. Publ. Mat. To appear.

[14] Galindo P., Vasylyshyn T., Zagorodnyuk A. *Analytic structure on the spectrum of the algebra of symmetric analytic functions on L_∞*. Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Math. RACSAM 2020, **114** (56). doi:10.1007/s13398-020-00791-w

[15] Galindo P., Vasylyshyn T., Zagorodnyuk A. *Symmetric and finitely symmetric polynomials on the spaces ℓ_∞ and $L_\infty[0,+\infty)$*. Math. Nachr. 2018, **291**, 1712–1726. doi:10.1002/mana.201700314

[16] Galindo P., Vasylyshyn T., Zagorodnyuk A. *The algebra of symmetric analytic functions on L_∞*. Proc. Roy. Soc. Edinburgh Sect. A 2017 **147** (4), 743–761. doi:10.1017/S0308210516000287

[17] García D., Maestre V., Zalduendo I. *The spectra of algebras of group-symmetric functions*. Proc. Edinb. Math. Soc. (2) 2019, **62** (3), 609–623. doi:10.1017/S0013091518000603

[18] Gonzaléz M., Gonzalo R., Jaramillo J. *Symmetric polynomials on rearrangement invariant function spaces*. J. Lond. Math. Soc. (2) 1999, **59**, 681–697. doi:10.1112/S0024610799007164

[19] Jawad F., Karpenko H., Zagorodnyuk A., *Algebras generated by special symmetric polynomials on ℓ_1*. Carpathian Math. Publ. 2019, **11** (2), 335–344. doi:10.15330/cmp.11.2.335-344

[20] Jawad F., Zagorodnyuk A. *Supersymmetric Polynomials on the Space of Absolutely Convergent Series*. Symmetry 2019, **11** (9), 1111. doi:10.3390/sym11091111

[21] Kravtsiv V. *Analogues of the newton formulas for the block-symmetric polynomials on $\ell_p(\mathbb{C}^s)$*. Carpathian Math. Publ. 2020, **12** (1), 17–22. doi: 10.15330/cmp.12.1.17-22

[22] Macdonald I.G. Symmetric Functions and Orthogonal Polynomials. University Lecture Series, **12**, AMS, Providence, RI, 1997.

[23] Mujica J. *Ideals of holomorphic functions on Tsirelson's space*. Arch. Math. (Basel) 2001, **76**, 292–298. doi: 10.1007/s000130050571

[24] Zagorodnyuk A. *Spectra of Algebras of Entire Functions on Banach Spaces*. Proc. Amer. Math. Soc. 2006, **134**, 2559–2569. doi:10.1090/S0002-9939-06-08260-8

Received 05.11.2020

Чернега І., Фуштей В., Загороднюк А. *Степеневі операції та диференціювання, асоційовані з суперсиметричними поліномами на банаховому просторі* // Карпатські матем. публ. — 2020. — Т.12, №2. — С. 360–367.

Розглядаються різні підходи до побудови степеневих операцій в кільці мультимножин, асоційованому з суперсиметричними поліномами від нескінченної кількості змінних. Встановлено деякі співвідношення між побудованими степеневими операціями. Також досліджено оператори диференціювання на алгебрах симетричних і суперсиметричних аналітичних функцій обмеженого типу на банаховому просторі абсолютно збіжних послідовностей.

Ключові слова і фрази: суперсиметричний поліном, аналітична функція на банаховому просторі, степеневий оператор, оператор диференціювання, кільце мультимножин.