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On the structure of some non-periodic groups whose subgroups of infinite special rank are transitively normal

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A group G has a finite special rank r if every finitely generated subgroup of G is generated by at most r elements and there is a finitely generated subgroup of G which has exactly r generators. If there is not such r, then we say that G has infinite special rank. In this paper, we study generalized radical non-abelian groups of infinite special rank whose subgroups of infinite special rank are transitively normal.

Key words and phrases: special rank, transitively normal subgroup.

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Introduction

The groups with certain prescribed properties of subgroups are among central research subjects in group theory. The study of such groups led to the emergence of many concepts, such as the finiteness conditions, local nilpotency, local solubility, subnormality, permutability, some important numerical invariants of groups (as, for example, distinct group ranks), and others. Choosing specific prescribed properties and concrete families of subgroups that possess these properties, we come to the distinct classes of groups.

A group G has finite special rank r if every finitely generated subgroup of G is generated by at most r elements and r is the least integer with this property. If there is not such r, then we say that G has infinite special rank [10]. The theory of the groups of finite special rank is one of the most developed branches of the group theory (see, for example, [1, 3–5]). In [2], M.R. Dixon, M.J. Evans and H. Smith initiated the investigation of the groups whose subgroups of infinite special rank have some fixed property $\mathcal{P}$. These investigations have been continuing by many authors for various properties $\mathcal{P}$ (see, for example, [3]).

A subgroup H of a group G is said to be transitively normal in G if H is normal in every subgroup $K \geqslant H$ in which H is subnormal [8]. There are many natural types of subgroups that are transitively normal. For example, pronormal subgroups and their generalizations are transitively normal [7].

In [9, 12], the study of the groups in which every subgroup of infinite special rank is transitively normal was initiated. In [12], the structure of periodic soluble groups of infinite special rank with this property has been described.

A group G is called radical if G has an ascending series whose factors are locally nilpotent. If G is a radical group, then a locally nilpotent radical of G is non-trivial. It follows that a radical

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group has an ascending series of normal subgroups whose factors are locally nilpotent.

A group G is called generalized radical if G has an ascending series whose factors are locally nilpotent or locally finite. If G is a generalized radical group, then either a locally nilpotent radical of G is non-trivial or a locally finite radical of G is non-trivial. Therefore, a generalized radical group has an ascending series of normal subgroups whose factors are locally nilpotent or locally finite.

In [9], the study of some non-periodic groups in which every subgroup of infinite special rank is transitively normal was initiated. More precisely, the authors proved that if G is a non-periodic locally generalized radical group with this property and G includes an ascendant locally nilpotent subgroup of infinite special rank, then G is abelian. In the current paper, we continue the study of such groups with some additional restrictions on the locally nilpotent radical.

Result of this paper is the following theorem.

Theorem 1. *Let G be a generalized radical non-abelian group of infinite special rank whose subgroups of infinite special rank are transitively normal. Suppose that $Tor(G) = \langle 1 \rangle$ and a locally nilpotent radical L of G is abelian. Then, the following assertions hold:*

(i) L includes a G-invariant pure subgroup A, having a finite series

$$\langle 1 \rangle = A_0 \leqslant A_1 \leqslant \ldots \leqslant A_j \leqslant A_{j+1} \leqslant \ldots \leqslant A_n = A$$

of G-invariant pure subgroups whose factors A_{j+1}/A_j are G-chief and G-eccentric for all $j \in \{0, \ldots, n-1\}$;

(ii) $G = AC$ for some subgroup C, so that $A \cap C = \langle 1 \rangle$ and every complement to A in G is conjugate to C;

(iii) $C = S \times T$, where S is a free abelian subgroup, having infinite 0-rank, and T is a finite abelian subgroup.

1 Preliminary results

Lemma 1. *Let G be a soluble group of infinite special rank whose subgroups of infinite special rank are transitively normal. Suppose that $T := Tor(G)$ has infinite special rank. If G is not periodic, then G is abelian.*

Proof. Let

$$\langle 1 \rangle = T_0 \leqslant T_1 \leqslant \ldots \leqslant T_n = T$$

be the derived series of T. We proceed by induction on n. If $n = 1$, then T is abelian. Therefore, G is also abelian [9, Corollary 1].

Suppose now that $n > 1$. If T_1 has infinite special rank, then again G is abelian [9, Corollary 1]. Thus, suppose that T_1 has finite special rank. Then, T/T_1 has infinite special rank, so that G/T_1 is abelian by induction hypothesis. We have $T_1 = \mathrm{Dr}_{p\in\pi} S_p$, where $\pi = \Pi(T_1)$ and S_p is a Sylow p-subgroup of T_1. Since T_1 has a finite rank, S_p is a Chernikov p-subgroup for each $p \in \pi$.

Put $Q_p = \mathrm{Dr}_{q\in\pi, q\neq p} S_q$, then Q_p is G-invariant and T_1/Q_p is a Chernikov p-group. If we put $C_p/Q_p = C_{T/Q_p}(T_1/Q_p)$, then $(T/Q_p)(C_p/Q_p)$ is a Chernikov group [1, Theorem 1.5.16].

It follows that C_p/Q_p has infinite special rank. Since T_1/Q_p is abelian, $T_1/Q_p \leqslant \zeta(C_p/Q_p)$, so that C_p/Q_p is nilpotent. It follows that locally nilpotent radical of G/Q_p has infinite special rank. Therefore, G/Q_p is abelian [9, Theorem 1]. Then, $[G,G] \leqslant Q_p$ for all $p \in \pi$. Hence, $[G,G] \leqslant \bigcap_{p\in\pi} Q_p = \langle 1 \rangle$. □

This lemma shows that the base case is the case when $Tor(G)$ has finite rank. However, first consider the case when $Tor(G) = \langle 1 \rangle$.

Let G be a group and A be a normal abelian torsion-free subgroup of G. Recall that A is called G-rationally irreducible if A/B is periodic for every non-identity G-invariant subgroup B of A.

Proposition 1. *Let G be a generalized radical group of infinite special rank whose subgroups of infinite special rank are transitively normal. Suppose that $Tor(G) = \langle 1 \rangle$. If G is non-abelian, then locally nilpotent radical L of G is a nilpotent group of finite special rank, $G/L = A \times T$, where A is a free abelian group of infinite rank, T is a finite abelian group.*

Proof. Since $Tor(G) = \langle 1 \rangle$, L is torsion-free. If we suppose that L has infinite special rank, then G must be abelian [9, Theorem 1]. This contradiction shows that L has finite special rank.

Therefore, L is nilpotent [5, Proposition 6.2.4]. Let

$$\langle 1 \rangle = Z_0 \leqslant Z_1 \leqslant \ldots \leqslant Z_n = L$$

be the upper central series of L. Then, every subgroup Z_j is a G-invariant, every factor Z_j/Z_{j-1} is abelian, torsion-free and has finite special rank for all $j \in \{1,\ldots,n\}$ [5, Proposition 6.2.4]. Therefore, this series has a refinement

$$\langle 1 \rangle = K_0 \leqslant K_1 \leqslant \ldots \leqslant K_m = L$$

consisting of G-invariant subgroups whose factors are abelian, torsion-free, have finite rank and G-rationally irreducible.

Every factor-group $G/C_G(K_j/K_{j-1})$ is isomorphic to some irreducible subgroup of $GL_t(\mathbb{Q})$, where $t = r(K_j/K_{j-1})$ for all $j \in \{1,\ldots,m\}$. Then, $G/C_G(K_j/K_{j-1})$ includes a normal free abelian subgroup $S_j/C_G(K_j/K_{j-1})$ of finite or countable rank, so that G/S_j is finite for all $j \in \{1,\ldots,m\}$ [5, Corollary 1.4.12].

Let $S = \bigcap_{1\leqslant j\leqslant t} S_j$. By Remak's theorem, G/S is embedded in $\mathrm{Dr}_{1\leqslant j\leqslant t} G/S_j$. Therefore, G/S is finite. Put $C = \bigcap_{1\leqslant j\leqslant m} C_G(K_j/K_{j-1})$. Applying again Remak's theorem, we obtain that $S/C \hookrightarrow \mathrm{Dr}_{1\leqslant j\leqslant m} S_j/C_G(K_j/K_{j-1})$, which shows that S/C is a free abelian group of countable rank. Since every element of C acts trivially on every factor K_j/K_{j-1} of a locally nilpotent radical L, then $C \leqslant L$ [11, Theorem 12].

Thus, G/L is an extension of free abelian subgroup of infinite 0-rank by finite group. Therefore, G/L is abelian [9, Corollary 1]. Since $T = Tor(G/L)$ is finite, then $G/L = T \times A$, where A is a torsion-free subgroup [6, Theorem 27.5]. □

Lemma 2. *Let G be a non-abelian group of infinite special rank, so that $Tor(G) = \langle 1 \rangle$. Suppose that G includes such a normal abelian torsion-free subgroup A of finite special rank that G/A is abelian and (free abelian)-by-finite. If A is a non-minimal normal subgroup of G, but A is G-rationally irreducible, then G includes a subgroup of infinite special rank which is not transitively normal.*

Proof. Suppose the contrary, let every subgroup of infinite special rank is transitively normal. Suppose that G is nilpotent-by-finite. Then, its locally nilpotent radical has infinite special rank. Therefore, G is abelian [9, Theorem 1]. This contradiction shows that G is not nilpotent-by-finite. Then, G includes a subgroup C, so that $A \cap C = \langle 1 \rangle$ and AC has finite index [13].

Suppose that there is a prime number p, so that $A \neq A^p = A_1$. Since A is torsion-free, the mapping $a \to a^p$, $a \in A$, is a monomorphism. It follows that $A \cong A_1$ and so $A_1^p \neq A_1$. Put $A_2 = A_1^p$ and by induction $A_{n+1} = A_n^p$, $n \in \mathbb{N}$. Since A has finite special rank, A/A_n are finite for all $n \in \mathbb{N}$. Since A_n are G-invariant for each $n \in \mathbb{N}$, then its intersection $J = \bigcap_{n\in\mathbb{N}} A_n$ is a G-invariant subgroup. Clearly, A/J is non-periodic. Since A is G-rationally irreducible $\bigcap_{n\in\mathbb{N}} A_n = \langle 1 \rangle$. The fact that A/A_n is finite implies that G/A_n includes as a subgroup of finite index the product of finite G-invariant subgroup A/A_n and free abelian subgroup CA_n/A_n. In particular, CA_n/A_n has finite index in G/A_n. Then, it includes a G-invariant subgroup X_n/A_n having a finite index. Since X_n/A_n has infinite special rank, then G/A_n is abelian [9, Corollary 1]. Since it is true for each $n \in \mathbb{N}$, an equality $\bigcap_{n\in\mathbb{N}} A_n = \langle 1 \rangle$ implies that G is abelian.

Suppose now that $A = A^p$ for every prime p. It follows that A is divisible. Since A is a non-minimal normal subgroup of G, A includes a proper non-trivial G-invariant subgroup S. Then, A/S is a periodic divisible group, so that $A/S = \mathrm{Dr}_{p\in\Pi(A/S)} D_p/S$ where D_p/S is a Sylow p-subgroup of A/S. Note that D_p/S is a divisible Chernikov p-subgroup, $p \in \Pi(A/S)$. Then, its subgroup $D_{p,n}/S = \Omega_n(D_p/S)$ is finite and G-invariant for every $n \in \mathbb{N}$. It is not hard to see that $(D_{p,n}/S)(CS/S)$ includes a normal abelian subgroup of finite index, which has infinite special rank. Therefore, $(D_{p,n}/S)(CS/S)$ is abelian [9, Corollary 1]. Since it is true for each $n \in \mathbb{N}$, $(D_p/S)(CS/S)$ is abelian. In other words, $[D_p, C] \leqslant S$. It is true for each prime $p \in \Pi(A/S)$, so that $[A, C] \leqslant S$. Since A is torsion-free, S cannot be divisible. Hence, there is a prime number p, so that $S \neq S^p = S_1$. Put again $S_2 = S_1^p$ and by induction $S_{n+1} = S_n^p$, $n \in \mathbb{N}$. As above, every subgroup S_n is G-invariant and S/S_n is finite for each $n \in \mathbb{N}$. Furthermore, $\bigcap_{n\in\mathbb{N}} S_n = \langle 1 \rangle$. Using the above arguments, we obtain that every factor–group G/S_n is abelian. Since $\bigcap_{n\in\mathbb{N}} S_n = \langle 1 \rangle$, G is abelian. □

Let G be a group, A, B are normal abelian subgroups of G, so that $B \leqslant A$. The factor A/B is called G-central if $C_G(A/B) = G$. The factor A/B is called G-eccentric if $C_G(A/B) \neq G$.

Lemma 3. *Let G be a generalized radical non-abelian group of infinite special rank whose subgroups of infinite special rank are transitively normal. Suppose that $Tor(G) = \langle 1 \rangle$ and a locally nilpotent radical L of G is abelian. Then, L includes a G-invariant pure subgroup A having a finite series*

$$\langle 1 \rangle = A_0 \leqslant A_1 \leqslant \ldots \leqslant A_j \leqslant A_{j+1} \leqslant \ldots \leqslant A_n = A$$

of G-invariant subgroups that satisfy the following conditions:

(i) *A_j is pure in L for all $j \in \{1, \ldots, n\}$;*

(ii) *A_{j+1}/A_j is G-rationally irreducible and G-eccentric for all $j \in \{0, \ldots, n-1\}$;*

(iii) *G/A is abelian and has a finite periodic part.*

Proof. Applying Proposition 1 we obtain that L has finite special rank, G/L is abelian and it has a finite periodic part. We can consider L as a $\mathbb{Z}H$-module where $H = G/C_G(L)$. Let V be a divisible envelope of L. Then the action of H on L can be extended in a natural way to the action of H on V.

Thus, we can consider V as a $\mathbb{Q}H$-module. Since L has finite special rank, V has finite dimension over $\mathbb{Q}$. Then, V is a direct sum of two $\mathbb{Q}H$-submodules U and Z where U has a finite series of $\mathbb{Q}H$-submodules

$$\langle 1 \rangle = U_0 \leqslant U_1 \leqslant \ldots \leqslant U_j \leqslant U_{j+1} \leqslant \ldots \leqslant U_n = U,$$

whose factors are H-eccentric simple $\mathbb{Q}H$-modules, Z has a finite series of $\mathbb{Q}H$-submodules whose factors are H-central [5, Corollary 7.1.30].

Put $A = L \cap U$, $A_j = L \cap U_j$ for all $j \in \{1, \ldots, n\}$. Then, A has a finite series of G-invariant pure subgroups

$$\langle 1 \rangle = A_0 \leqslant A_1 \leqslant \ldots \leqslant A_j \leqslant A_{j+1} \leqslant \ldots \leqslant A_n = A,$$

whose factors are G-eccentric and G-rationally irreducible, L/A has a finite series of G-invariant pure subgroups whose factors are G-central. Thus, G/A is abelian [9, Theorem 1]. Since L/A is torsion-free, the periodic part of G/A is finite. □

Lemma 4. *Let G be a generalized radical non-abelian group of infinite special rank whose subgroups of infinite special rank are transitively normal. Suppose that $Tor(G) = \langle 1 \rangle$ and a locally nilpotent radical L of G is abelian. Then, L includes a G-invariant pure subgroup A having a finite series*

$$\langle 1 \rangle = A_0 \leqslant A_1 \leqslant \ldots \leqslant A_j \leqslant A_{j+1} \leqslant \ldots \leqslant A_n = A$$

of G-invariant subgroups that satisfy the following conditions:

(i) A_j is pure in L for all $j \in \{1, \ldots, n\}$;

(ii) A_{j+1}/A_j is G-chief and G-eccentric for all $j \in \{0, \ldots, n-1\}$;

(iii) G/A is abelian and it has a finite periodic part.

Proof. Lemma 3 implies that L includes a G-invariant pure subgroup A having a finite series

$$\langle 1 \rangle = A_0 \leqslant A_1 \leqslant \ldots \leqslant A_j \leqslant A_{j+1} \leqslant \ldots \leqslant A_n = A$$

of G-invariant pure subgroups, so that every factor A_{j+1}/A_j is G-rationally irreducible and G-eccentric for all $j \in \{0, \ldots, n-1\}$, G/A is abelian and it has a finite periodic part. By Lemma 2, A_n/A_{n-1} is G-chief. Since A_n/A_{n-1} is G-eccentric, $G \neq C_G(A_n/A_{n-1})$. Choose an element $z \notin C_G(A_n/A_{n-1})$. Then $C_{A_n/A_{n-1}}(z) \neq A_n/A_{n-1}$. Since $G/C_G(A_n/A_{n-1})$ is abelian, then $C_{A_n/A_{n-1}}(z)$ is G-invariant. The fact that A_n/A_{n-1} is G-chief implies that $C_{A_n/A_{n-1}}(z)$ is trivial. The choice of z implies that $[A_n/A_{n-1}, z]$ is non-trivial. Since $G/C_G(A_n/A_{n-1})$ is abelian, $[A_n/A_{n-1}, z]$ is G-invariant. The fact that A_n/A_{n-1} is G-chief implies that $[A_n/A_{n-1}, z] = A_n/A_{n-1}$. Then, A_n/A_{n-1} has in G/A_{n-1} a complement C/A_{n-1}, that is

$$G/A_{n-1} = (A_n/A_{n-1})(C/A_{n-1})$$

and $(A_n/A_{n-1}) \cap (C/A_{n-1})$ is trivial [5, Theorem 8.2.7].

Consider now a subgroup C. Since A_{n-1}/A_{n-2} is G-eccentric, an equality $G/A_{n-1} = (A_n/A_{n-1})(C/A_{n-1})$ shows that this factor is C-eccentric. By the same reason, A_{n-1}/A_{n-2} is C-rationally irreducible.

Note that every subgroup of C/A_{n-2} having infinite special rank is transitively normal [12, Lemma 1.1].

Applying Lemma 2 we obtain that A_{n-1}/A_{n-2} is C-chief. Then, this factor is also G-chief. Repeating the above arguments, we obtain that C/A_{n-2} includes a subgroup D/A_{n-2}, so that $C/A_{n-2} = (A_{n-1}/A_{n-2})(D/A_{n-2})$ and $(A_{n-1}/A_{n-2}) \cap (D/A_{n-2})$ is trivial. Repeating the above arguments finitely many times, we obtain that every factor A_{j+1}/A_j is G-chief. □

2 Proof of main theorem

Lemma 4 implies that L includes a G-invariant pure subgroup A having a finite series

$$\langle 1 \rangle = A_0 \leqslant A_1 \leqslant \ldots \leqslant A_j \leqslant A_{j+1} \leqslant \ldots \leqslant A_n = A$$

of G-invariant pure subgroups, so that every factor A_{j+1}/A_j is G-chief and G-eccentric for all $j \in \{0, \ldots, n-1\}$, G/A is abelian and it has a finite periodic part. Thus, we must prove only *(ii)*.

We proceed an induction by n. If $n = 1$, then we can repeat the arguments from the proof of Lemma 4. Furthermore, from [5, Theorem 8.2.7] shows that $G = AC$ where $A \cap C$ is trivial and every other complement to A in G is conjugate to C.

Suppose now that $n > 1$ and we have already proved that $G/A_1 = (A/A_1)(D/A_1)$, $(A/A_1) \cap (D/A_1)$ is trivial and every complement to A/A_1 is conjugate to D/A_1. Since A is abelian and $G \neq C_G(A_1)$, then $D \neq C_D(A_1)$. Hence, we can choose an element $z \notin C_D(A_1)$. Then, $C_{A_1}(z) \neq A_1$. Since D/A_1 is abelian, $D/C_D(A_1)$ is also abelian. Therefore, $C_{A_1}(z)$ is D-invariant. The fact that A is abelian together with an equality $G = AD$ implies that $C_{A_1}(z)$ is G-invariant. Since A_1 is G–chief, $C_{A_1}(z)$ is trivial. The choice of z implies that $[A_1, z]$ is non-trivial. Since $D/C_D(A_1)$ is abelian, then $[A_1, z]$ is D-invariant. The fact that A is abelian together with equality $G = AD$ implies that $[A_1, z]$ is G-invariant. Since A_1 is a minimal G-invariant subgroup, then $[A_1, z] = A_1$. Therefore, A_1 has a complement C in D, that is $D = A_1C$ for some subgroup C, so that $A_1 \cap C$ is trivial. Moreover, every other complement to A_1 in D is conjugate to C [5, Theorem 8.2.7]. The equalities $G = AD$ and $D = A_1C$ imply that $G = AC$. Furthermore, $C \cap A = C \cap D \cap A = C \cap A_1 = \langle 1 \rangle$.

Let V be a subgroup of G, so that $G = AV$ and $A \cap V = \langle 1 \rangle$. Then,

$$G/A_1 = (A/A_1)(VA_1/A_1).$$

Since $VA_1 \cap A = A_1(V \cap A) = A_1$, then $(A/A_1) \cap (VA_1/A_1)$ is trivial. This means that VA_1/A_1 is a complement to A/A_1 in G/A_1. Note that VA_1/A_1 is conjugate to D/A_1. In other words, there is such an element x that $(VA_1)^x = D$. Then

$$D = (VA_1)^x = V^x A_1^x = V^x A_1$$

and

$$V^x \cap A_1 = V^x \cap A_1^x = (V \cap A_1)^x = \langle 1 \rangle.$$

These equalities show that V^x is a complement to A_1 in D. Then, there is such an element $y \in D$ that $C = (V^x)^y = V^{xy}$, so *(ii)* is proved.

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Величко Т.В. *Про будову деяких неперіодичних груп, підгрупи нескінченного спеціального рангу яких транзитивно нормальні* // Карпатські матем. публ. — 2021. — Т.13, №2. — С. 515–521.

Група G має скінченний спеціальний ранг r, якщо кожна скінченно породжена підгрупа групи G породжена щонайбільше r елементами та існує скінченно породжена підгрупа групи G, яка має рівно r породжуючих елементів. Якщо такого r не існує, то говоритимемо, що G має нескінченний спеціальний ранг. У цій статті вивчаються узагальнено радикальні неабелеві групи нескінченного спеціального рангу, підгрупи нескінченного спеціального рангу яких транзитивно нормальні.

Ключові слова і фрази: спеціальний ранг, транзитивно нормальна підгрупа.