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Mikhail Ostrovskii (for his 60$^{\text{th}}$ birthday)

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Mikhail Ostrovskii, a well-known Ukrainian-American mathematician, turned 60 on December 26, 2020. This note aims to describe his professional activities from the point of view of his friends and co-authors.

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Some biographical information

Mikhail Ostrovskii (Михайло Йосипович Островський (Ukrainian), Михаил Иосифович Островский (Russian)) was born on December 26, 1960 in Kharkiv to a family of mathematicians. His parents, Larisa Kudina (1932–2011) and Iosif Ostrovskii (1934–2020), worked for many years in the School of Mathematics and Mechanics of the Kharkiv University. His elder sister – Sofiya Ostrovska – also became a mathematician[1].

Larisa Kudina worked at Kharkiv University for about 30 years and is remembered by her former students as a very enthusiastic Calculus teacher for all of those years. Some of the Kharkiv University graduates remember her as a person thanks to whom they were accepted into Kharkiv University. This happened because Larisa Kudina was invited to work on the admissions committee at the beginning of her career. During her work there, she did not tolerate the widespread policy at the time of using substantially higher standards for admitting students of Jewish descent. During the rest of her career, Larisa Kudina was not invited to work on the admission committee.

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[1] Lists of publications of L.S. Kudina, I.V. Ostrovskii, and S. Ostrovska, as well as links to information about them at other databases can be found at https://www.zbmath.org/

Iosif Ostrovskii is remembered as one of the most influential Kharkiv mathematicians. He was a corresponding member of the Ukrainian Academy of Sciences and the author of two well-known monographs [1, 2]. He is also remembered for his ability to confront injustice. One such instance was a failed Habilitation defense in Kharkiv at the end of the 1980s. At the time of the defense, the person who was trying to defend the Habilitation (whom we will call Mr. L) was a director of one of the research institutes in Kharkiv after having been the Dean of the School of Mathematics and Mechanics at Kharkiv University for many years. Many members of the mathematical community in Kharkiv, including Iosif Ostrovskii, were sure that Mr. L could not produce a Habilitation thesis by himself. He read most of the voluminous Habilitation thesis. Although he did not find any mistakes (he did not expect to find serious mistakes because he was sure that qualified people had prepared the Habilitation thesis), he found several places where the explanation was incomprehensible. During the defense, after the praise by several speakers on behalf of Mr. L and his Habilitation thesis, Iosif Ostrovskii asked Mr. L to explain those impenetrable places in his Habilitation thesis. At that moment, it became clear that Mr. L did not understand the material in his Habilitation thesis. After that leadership of the committee tried to convince the other members of the committee that it is normal that the candidate does not remember the contents of their Habilitation thesis. Iosif Ostrovskii responded that a person who cannot explain the results of their Habilitation thesis should not receive the Habilitation degree. The vote by secret ballot showed that the number of committee members who agreed with Iosif Ostrovskii was barely enough to fail the defense (according to the rules, to be successful the candidate must receive at least 2/3 of the votes "for"). It is clear that after this event Iosif Ostrovskii had some problems with the administration. What was more important to him, however, was that he had the support of the Kharkiv mathematical community. For example, a group of students who had attended Kharkiv University when Mr. L was the Dean brought flowers to Iosif Ostrovskii on the day of the failed defense and said that they are thrilled that in this case justice had been done.

Mikhail's parents supported his interest in mathematics but did not pressure him. In 1973 they encouraged him to transfer to the 27$^{\text{th}}$ school, well-known for its emphasis on Mathematics and Physics. After graduating from this school in 1977, Mikhail entered Kharkiv University, majoring in Mathematics.

At the University his favorite professors were Boris Levin[2] (1906–1993) and Iosif Ostrovskii. He also liked the lectures of Mikhail Kadets[3] (1923–2011) who did not work at Kharkiv University but was giving lecture courses there from time to time.

When picking an area of research for his Ph. D., Mikhail was influenced by his belief that he should not work in an area close to his father's research. He decided to work in Banach space theory under the supervision of Mikhail Kadets. In 1982 Mikhail Ostrovskii graduated from Kharkiv University and entered the Ph. D. program at the Kharkiv Institute of Municipal Engineering, where Mikhail Kadets was a professor. In 1985 Mikhail Ostrovskii defended his Ph. D. [3].

In 1985–1998, Mikhail worked as a researcher at the Mathematical Division of the Institute for Low Temperature Physics and Engineering (Kharkiv).

During the 'perestroika' years, Mikhail read a lot of political, historical, and social literature (books and articles), which had suddenly become available. According to him, the amount of

[2] See his biography at https://mathshistory.st-andrews.ac.uk/Biographies/Levin/

[3] See his biography at https://mathshistory.st-andrews.ac.uk/Biographies/Kadets/

such literature he read in the period 1987–1990 exceeds the amount he read during the rest of his life. In 1990 he actively participated in the political process, serving as an "official trusted person" (довірена особа; доверенное лицо) of a well-known dissident Genrikh Altunyan[4] (1933-2005), who was elected a deputy of Verhovna Rada of Ukraine for 1990–1994. He also served as an "official trusted person" of a well-known Ukrainian human rights activist Yevgen Zakharov in a local government election.

In 1991 Mikhail married his wife Marina (née Likhosherst) and shortly thereafter they had two children (son Stanislav, 1992, and daughter Kseniya, 1993). The necessity of making a living in the country, which was in a deep economic crisis after the collapse of the Soviet Union, encouraged him to make many research visits to other countries. During these visits, he tried to save as much money as possible, even when he did not receive any salary. For example, during his research trip to the University of Cape Town (South Africa) in Fall 1993, Mikhail's forty day visit was funded by his hosts who offered to pay for him to stay at a relatively expensive hotel and to dine in its restaurant. Instead, Mikhail used the money that was allocated for his visit to stay in a dormitory and eat food from a supermarket and was able to save $2000. Meanwhile his salary in Ukraine at that time was equivalent to $7 per month. According to Mikhail, he never felt as rich as after returning home with those $2000. Other countries Mikhail visited in 1992–1997 were Spain, Turkey, Israel, France, Italy, the USA, Denmark, and Germany.

In 1997 Mikhail defended his Habilitation [5]. In 1998 he went to the US with his family for a 2-year visiting position (1998–2000) at the University of California, Riverside. During this visit, he and his wife decided they wanted to move to the United States. He describes his decision to move to the United States as follows: "By 1998 we lost hope that Ukraine will get out of the state of wild capitalism combined with widespread corruption in our lifetime." In 2000–2005, Mikhail worked at the Catholic University of America in Washington, DC. In 2005 he moved to St. John's University in New York City.

Some mathematical results

Early results

Mikhail's first publications were devoted to the notion of *opening* (also called *gap*) between subspaces of a Banach space. The study of this notion was the topic suggested by Mikhail Kadets when Mikhail Ostrovskii started his Ph. D. studies. In his first publication on this topic [6], Mikhail, among other results, answered an open question posed by M. G. Krein, M. A. Krasnoselskii, and D. P. Milman in 1948. Namely, he constructed two subspaces of a Banach space with different density characters, but with a gap strictly less than one.

In his 1985 paper [7], Mikhail invented an elegant construction that gave a negative answer to the three-space problem for the weak Banach-Saks property (posed by several people: Partington (1977), Godun-Rakov (1982)). Namely, he proved that there are a Banach space X

[4] See his official biography at
https://web.archive.org/web/20071018121634/http://rada.gov.ua/zakon/new/DEPUTAT1/370.htm

without the weak Banach-Saks property and a subspace Y of X such that both Y and X/Y do have the weak Banach-Saks property.

Sufficient enlargements and operator theory

In 1996–2011, Mikhail wrote a series of papers on sufficient enlargements. Mikhail introduced this notion, which was implicit in the work of Grünbaum (1960), in the following way: a symmetric bounded closed convex subset A of a finite-dimensional normed space X is called a *sufficient enlargement*, provided that for any isometric embedding of X into a Banach space Y there is a linear projection $P\colon Y \to X$ with $P(B_Y) \subseteq A$, where B_Y is the closed unit ball of Y.

The most profound work of Mikhail [8, 9, 10] on sufficient enlargements is devoted to sufficient enlargements of minimum volume. He described the possible shapes of such sufficient enlargements. In addition to usual parallelepipeds, the set of possible shapes contains more complex bodies generated by totally unimodular matrices. Also, he described spaces for which more complex bodies can occur. See [11] for his survey of this topic.

Under the influence of Viktor Shulman, Mikhail wrote a series of papers on Operator Theory, see [12, 13, 14].

Recent results: Metric embeddings and related topics

Around 2010, the main direction of Mikhail's research's moved to the theory of Metric Embeddings. His interest in this direction was ignited by the famous Chapter 15 of the textbook by Matoušek [15]. According to Mikhail, it was the only textbook for which he was unable to resist the temptation to solve all of the exercises, except those marked as trivial (he also says that he usually does not have such temptation).

Essential for Mikhail was also his interaction with mathematicians W. B. Johnson, A. Naor, G. Schechtman, and their interest in the theory of Metric Embeddings. In particular, thanks to Johnson, Mikhail got invitations and funding for two significant conferences on Metric Embeddings (Princeton, 2003 and College Station, TX, 2006).

Assaf Naor became a leading figure in the theory of Metric Embeddings. Naor's lectures, papers, surveys, and notes from his lecture courses became the primary source of inspiration for Mikhail. According to Mikhail, essential reading for him are Naor's very detailed introductions to almost all of his papers, which describe not only the contents of the paper, but also the history of the topic, its perspectives, alternative approaches to the problems considered in the paper, and the difficulties one may encounter along the way.

According to Mikhail, his most important collaboration in Metric Embeddings's theory is the one with Beata Randrianantoanina. He considers the joint paper [16] as his (at least so far) deepest work on Metric Embeddings. His other works on Metric Embeddings, which he values most, are [17], [18].

In 2013 Mikhail published the monograph "Metric Embeddings" [19]. This book is used as a reference and a text for advanced courses by some of the leading experts in the area. It got positive reviews by Baudier and Johnson [20], and by Werner (https://www.zbmath.org/). Recently Mikhail started an extensive work on transportation cost spaces, see [21], [22], [23].

For many years Mikhail led a Seminar on Metric Embeddings at St. John's University. Even though none of his colleagues in the department was a Banach space theorist, Mikhail could find a "middle ground" to do joint research with them. He wrote a paper [24] with David Rosenthal on minor exclusion in groups (which has recently attracted significant interest), and a paper [25] with Florin Catrina that answered a question that Alessio Figalli (who received the Fields Medal in 2018) emailed to Mikhail Ostrovskii in 2016.

Interest in the history of mathematics

Mikhail often says that although almost every mathematician wants others to reflect correctly and adequately her/his contributions to the subject, virtually nobody does this for other mathematicians. He tries to avoid this tendency in his work (although he admits that sometimes his attributions were erroneous). In this connection, he has some interest in the history of those subjects on which he works. One can see this interest in some of his publications [26, 19], [27, Section 1.6]. Jointly with Plichko, he wrote a purely historical publication [28].

Some personal notes

1. Mikhail's two favorite subjects in school were English and Mathematics. Even though he comes from a family of mathematicians, he tells the following story on how he decided on the latter. One day an enthusiastic middle school substitute math teacher presented to the class a list of challenging problems. As the student Mikhail started to solve the problems one by one, the teacher became more and more jubilant. Seeing how happy the teacher became, convinced Mikhail to dedicate himself to math.

We are very fortunate at St. John's to have him as a colleague and also currently as a chair. I will not give here a long list of reasons why this is so, other than to say that his clarity of thought and focus motivate all who work with him.

Before our paper together, we had only one (unfortunately unsuccessful) attempt to solve a serious math problem. It was impressive to see Mikhail throw himself fully into following every idea if it was even slightly promising or launch on broad literature searches. When he decided it was enough, he proposed we try something different.

It is known that the Lebesgue space $L^1([0,1])$ does not have the Radon-Nikodým property, that is, Lipschitz functions from $\mathbb{R}$ to $L^1([0,1])$ are not necessarily a.e. differentiable. A couple of years before being awarded the Fields medal, Alessio Figalli had sent Mikhail an e-mail asking whether one should expect a.e. differentiability for Lipschitz functions from $\mathbb{R}$ into a subclass of $L^1([0,1])$, say of functions in $L^1([0,1])$ which themselves are 1-Lipschitz. In [25] we were able to construct examples of nowhere differentiable isometric embeddings of $[0,1]$ into $L^1([0,1])$, where the images in $L^1([0,1])$ have any degree of regularity one may desire.

I join all colleagues and friends in wishing Mikhail good health and a long career ahead, as successful as it has been thus far.

Florin Catrina

2. It is a great pleasure and an honor to have this opportunity to express my gratitude and appreciation to my friend and colleague, Dr. Mikhail Ostrovskii, on the occasion of the celebration of his sixtieth birthday. Mikhail and I are roughly contemporaries, both graduating in our respective home countries in the mid-eighties, and both coming to the United States in due course to pursue our careers in mathematics. Having followed his important work in Banach space theory with great interest over the years, it was not until Mikhail came to the United States, however, that we met in person at the Workshop in Analysis and Probability at Texas A&M University and at various meetings of the American Mathematical Society.

As this article clearly indicates, the influence of Mikhail's work is very great. Let me cite just one example from my personal experience. In a joint work with Pete Casazza, Ted Odell, Thomas Schlumprecht, and András Zsák entitled 'Quantization of Frames in Banach spaces' (J. Math. Anal Appl. 346 (2008), 66-86), one question which we investigated was the possibility of finding a normalized frame for a given n-dimensional normed space X with the property that every vector in the unit ball can be well-approximated by a linear combination of frame vectors with discretized coefficients. Our main result was that the size of the frame has to be at least $Cn \log n$ (where C depends on the cotype q constant of X for $q < \infty$), which was essentially best possible. To prove this, however, the crucial ingredient which we needed was a result on minimal-volume projections of cubes from Mikhail's beautiful paper [8].

Recently I had the good fortune to be invited by Mikhail and Denka Kutzarova to join them on a research project to investigate the structure of transportation cost spaces (also known as Lipschitz-free spaces) of finite metric spaces. This is the topic of a chapter of Mikhail's authoritative monograph on nonlinear embeddings [19]. It has a long history dating back over several decades to pioneering work of Kantorovich, Rubinstein and Wasserstein. The research proceeded rather slowly at first, but with perseverance we eventually obtained some interesting results on diamond graphs and other families of iteratively defined graphs endowed with the graph metric (see [21]). During the course of this work, Mikhail kindly invited me to St. John's University to work on these questions and to enjoy his generous hospitality. I hope that our collaboration can continue into the future.

A second result in [21] asserted that the transportation cost space $\mathrm{TC}(M)$ associated to any metric space M consisting of $2n$ points contains a 2-complemented subspace 2-isomorphic to ℓ_1^n. We left open the question as to whether this could be improved by replacing 2-complemented by 1-complemented, etc. This would certainly require more powerful methods than those at our disposal. Subsequently, Mikhail and his students Seychelle S. Khan and Mutasim Mim answered the question positively in a beautiful paper [22]. Using deep results from graph theory, namely the Edmonds algorithm for the minimum weight matching problem, they proved that $TC(M)$ does indeed contain a 1-complemented subspace isometric to ℓ_1^n.

Let me conclude these brief remarks on a personal note by congratulating Mikhail on this milestone anniversary and thanking him for his friendship and collaboration. I wish him all the best in the years to come.

Stephen Dilworth

3. Mikhail and I were classmates both in secondary school and during our university studies. My father was the scientific adviser for both of us and we attended the same Kharkiv Banach space seminar. It is maybe strange that although in the early stages of our research careers we were interested in similar problems and were speaking a lot about mathematics (and even more about non-mathematical subjects) our friendship did not lead to any joint mathematical result.

One of the subjects of common interest for us was rearrangements of series in Banach spaces. Let $\sum_{k=1}^{\infty} x_k$ be a series in a Banach space X. By definition, a point $x \in X$ belongs to the sum-set of the series if there is a rearrangement $\pi : \mathbb{N} \to \mathbb{N}$ such that $\sum_{k=1}^{\infty} x_{\pi(k)} = x$. The Riemann Theorem states that the sum-set of a convergent series of real numbers must be either a single point or the whole real line. A finite-dimensional analogue of this result was obtained in 1913 by E. Steinitz: *the sum-set of a convergent series in $\mathbb{R}^n$ is a shifted subspace of $\mathbb{R}^n$*

(in dimension 2 this was demonstrated earlier by P. Lévy).

It is natural to ask whether the above theorem remains true in infinite-dimensional spaces. The corresponding question was asked by S. Banach in the Scottish Book (Problem 106). The negative answer to this question given by J. Marcinkiewicz can be found in the Scottish Book as well. Marcinkiewicz's construction is as follows. Consider in $L_2[0,1]$ the series

$$\mathbb{1}_{[0,1]} - \mathbb{1}_{[0,1]} + \mathbb{1}_{[0,\frac{1}{2}]} - \mathbb{1}_{[0,\frac{1}{2}]} + \mathbb{1}_{[\frac{1}{2},1]} - \mathbb{1}_{[\frac{1}{2},1]} + \mathbb{1}_{[0,\frac{1}{4}]} - \dots,$$

where $\mathbb{1}_A$ denotes the characteristic function of the set A. This series evidently converges to 0. Its reordering

$$\mathbb{1}_{[0,1]} + \mathbb{1}_{[0,\frac{1}{2}]} + \mathbb{1}_{[\frac{1}{2},1]} - \mathbb{1}_{[0,1]} + \mathbb{1}_{[0,\frac{1}{4}]} + \mathbb{1}_{[\frac{1}{4},\frac{1}{2}]} - \mathbb{1}_{[0,\frac{1}{2}]} + \dots$$

converges to $\mathbb{1}_{[0,1]}$. On the other hand, all the terms take only integer values, so there is no rearrangement in which the series could converge to $\frac{1}{2}\mathbb{1}_{[0,1]}$.

The example has a clear idea and looks simple, but it is highly non-trivial. A good demonstration of its non-triviality is the following: since the Scottish Book was not available for Soviet mathematicians, in the Soviet Union the question of the Lévy - Steinitz Theorem's validity in infinite-dimensional spaces was considered open and it took decades until an independent solution was found by E.M. Nikishin (1973).

A question that both Mikhail and I were interested in (and which remains unsolved) was: What are the topological restrictions on a sum-set? The first important step toward its solution was made by Mikhail in [4]. He constructed an example of a series with non-closed sum-set. The example was given in the Hilbert space $L_2([0,1] \times [0,1])$ of functions of two variables. Denoting by $f_n \in L_2[0,1]$ the elements of Marcinkiewicz's series, Mikhail considered the following series of functions in variables t, τ:

$$f_1(t) + \sqrt{2}f_1(\tau) + f_2(t) + \sqrt{2}f_2(\tau) + \dots$$

The sum-set of this series intersects the subspace of all constants by the dense set of constants of the form $n + m\sqrt{2}$, $m, n \in \mathbb{Z}$, which is not closed, so the sum-set is not closed either.

I am still impressed by the elegance of that example.

Vladimir Kadets

4. It doesn't often happen a niece to be older than her uncle, yet that's exactly what happened to me. I was a student of Troyanski, who in turn was a student of Mikhail Kadets (Kadec), the PhD adviser of Mikhail Ostrovskii. In September 1989, just two months before the political changes in Bulgaria, Troyanski organized a conference in Banach spaces at the Black Sea, near Varna. Naturally, Kadets came to our conference and so did Mikhail Ostrovskii. I recall Troyanski telling me that, as a PhD student, he took a class with Mikhail's father. I talked to Mikhail at that conference, but not much, being busy with organizational work.

My first true conversation with him happened in the USA in 1996 when we both attended a concentration week on infinite dimensional Banach spaces at MSRI, Berkeley. It was an informative conversation. Mikhail already had strong opinions and ideas about what was important in mathematics and what he wanted to do next. Fastforward more than fifteen years, and we met again at a conference in College Station, Texas. What a significant change had occurred in our lives - we were both living in America. And yet, talking in the inner garden of our hotel in the heat of the Texan summer, gave me the temporary impression that we were still back in our native countries. Mikhail suggested we start a joint project. By that time, his creative mathematical talent was in full bloom. He sent me a file with math questions, I looked at them and they seemed further away from the topics of my recent papers, so I didn't work on them. I suppose I was afraid of disappointing my "uncle" in case I tried and didn't solve anything.

Fortunately, we met again in Texas in 2016. On the way back, we had the same flight from College Station to Dallas and more than an hour to our next flights. "You didn't want to work with me", Misha said suddenly. I explained to him why I didn't think about his questions, so he took out a piece of paper and said: "You have worked with the Haar basis. Let's consider the signed indicator functions of the cycles of the diamond graphs. See, they look like Haar functions." That did it for me. I made my first observations, then Steve Dilworth also joined and the three of us wrote our first joint paper. Misha is a tough coauthor; he once complained that I could't work eight hours per day. At the same time, he is a great coauthor, possessing broad, far reaching vision and phenomenal knowledge of mathematics. In just one word, Mikhail Ostrovskii is a leader.

Denka Kutzarova

5. I will describe some of the results from Mikhail's Habilitation paper "Класифікація тотальних підпросторів спряженого банахового простору та її застосування" (Ukrainian. English translation: "Classification of total subspaces of dual Banach spaces and its applications"), Kharkiv, 1997, concerning total subspaces of dual Banach spaces and which is related to my mathematical interests.

A subspace M of a dual Banach space X^* is said to be *total* if for every $x \in X$, $x \neq 0$, there exists $f \in M$ such that $f(x) \neq 0$.

The question of how "small" a total subspace can be traced back to mathematicians from the well-known Lviv Mathematical School of Banach. They showed that a total subspace can be *nonnormimg*, i.e., the original norm $\|x\|$ need not be equivalent to the norm $|||x||| = \sup\{|f(x)| : f \in M, \|f\| = 1\}$, and also that a subspace M in the dual of a separable Banach space X is norming if and only if its weak* sequential closure $M_{(1)}$ coincides with X^*.

After the war, the interest in the study of total subspaces was revived in connection with various problems in other areas of mathematics. Ukrainian mathematicians played an active role in this revival. Mikhail joined the work in this direction after defending Ph.D. In 1930 Mazurkiewicz proved that the weak* sequential closure of a linear subspace $M \subset X^*$ need not be weak* closed. So, one can introduce the weak* sequential closure of the second order $M_{(2)}$ as the weak* sequential closure of $M_{(1)}$ and so on, up to the first uncountable ordinal ω_1. The first of Mikhail's results [29] on total subspaces asserts that, if a separable Banach space X is nonquasireflexive (i.e., if the quotient X^{**}/X is infinite-dimensional), then for any ordinal $\alpha < \omega_1$ there is a total subspace $M \subset X$ such that $M_{(\alpha)} \neq M_{(\alpha+1)} = X^*$. This result concluded the long line of investigations of many mathematicians in this direction. See [26] for a story of the problem. This story is interesting, particularly because of a substantial number of erroneous attempts, one of which was due to S. Banach himself.

This result implies the well-known fact that for a nonquasireflexive Banach space X, the dual space X^* contains a total nonnorming subspace M of X^*. Davis and Johnson (1973) discovered that there are examples of Banach spaces X and total subspaces M of X^* which norm no infinite-dimensional subspace of X (property *TNNS*). On the other hand, there are examples of nonquasireflexive Banach spaces without *TNNS*. Mikhail [30] found a characterization of separable Banach spaces with *TNNS*: *X has TNNS if and only if, for some nonquasireflexive infinite-dimensional Banach space Y there exists a surjective strictly singular operator* $T : X \to Y$.

I have mentioned only a small part of Mikhail's results on total subspaces. A more complete description of these results can be found in [5, 26].

Anatolij Plichko

6. Those of us who were involved in conferences organized by Mikhail, either as participants or as co-organizers, noticed Mikhail's following qualities, which become apparent during the process: (1) Definite opinions on almost everything related to the organization of the conference. (2) Ability to adjust his decisions according to other co-organizers' views, but only in cases when he finds them acceptable. (3) Willingness to cover unusual tasks. For example, during the Lviv conference in 2019, he was organizing toasts during the banquet. (4) Fast response time. For example, I succeeded in getting funding and attending the annual AMS meeting in Baltimore (2003) only because Mikhail filed the corresponding application within an hour after the opportunity (on a first-come, first-served basis) was announced.

Mikhail was also one of the main organizers of the conference "Banach Spaces and their Applications" dedicated to the 70th anniversary of Anatolij Plichko held on June 26-29, 2019 in Lviv. It was one of the most successful mathematical conferences in Ukraine in recent years, attracting over a hundred participants from 19 countries and having a solid list of invited speakers.

Mikhail Popov

7. I have had a pleasure of knowing Mikhail for over 30 years. When I first met him I thought that he was a very quiet and peaceful person. As I got to know him better, I learned that he indeed is one of the most peaceful people I know, but not necessarily very quiet. He always stands for his very carefully considered, thoughtful principles that guide his opinions and life. He is a very upright person. He can also be very funny and has a somewhat peculiar, but keen sense of humor.

I had a great pleasure to collaborate with Mikhail on quite a lot of mathematics. So far we have written 5 joint papers, and I hope that this number will continue to grow. We worked together on a number of problems related to metric embeddings of families of graphs into Banach spaces and we succeeded in solving some of the problems which Mikhail considered very important.

Collaboration with Mikhail is an exhilarating and stimulating experience. He has a vast knowledge of all literature related to all topics that have ever attracted his mathematical interest. He is always full of new ideas to explore and he can speak about them passionately for hours (!). He never runs out of interesting questions to investigate. I feel very fortunate that I got to know Mikhail and that I was able to work with him, and especially to become his friend. On the occasion of this anniversary, I send Mikhail all the best wishes for many more years of exciting and far reaching mathematical explorations and fulfillment and joy in his professional and personal life.

Beata Randrianantoanina

8. Mikhail is first and foremost an outstanding mathematician, producing many fine results and instigating many interesting lines of research. He is a great problem solver and has a knack for finding nice problems to solve. He is also a pleasure to work with and has had numerous coauthors - I feel honored to be one of them.

Mikhail is a tremendous asset to our department at St. John's. His focus and hard work inspire those around him. He shares his mathematical expertise with both colleagues and students, leading seminars and sponsoring student research projects. His problem solving ability and clarity of thought extend beyond mathematics. His leadership, both as a colleague and as department chair, has provided invaluable guidance during these challenging times.

On a more personal level, I have benefited greatly from spending time with Mikhail, whether it be discussing research mathematics, working on program development or discussing the nuances of giving a good talk. I value his honesty and admire his commitment to the profession. We are lucky to have him as a colleague.

David Rosenthal

9. The central part of my joint work with Mikhail Ostrovskii is devoted to operator linear fractional functions and quadratic operator inequalities. The number of exciting problems and objects appearing in the study of these seemingly well-developed topics is amazing if one works with Misha! For instance, the first problem we worked on together asked, for which Banach spaces X and Y is it true that for every operator $T\colon X \to Y$ there is a norm one operator $S\colon X \to X$ with $\|T\| = r(TS)$? Trying not to count the uncountable, I will focus on my joint work with Lyuda Turowska and Misha [13, 14]. This work was devoted to a fixed point problem for groups of linear fractional functions acting in the open unit ball of the space of operators on Hilbert spaces with k-dimensional domain and an infinite-dimensional range space (this problem is significant for representation group theory in Pontryagin's space). Long ago, I found a solution for $k = 1$, but the general case remained unsolved. Almost immediately, Misha offered an idea to explore a normal structure of the corresponding metric space and this idea turned out to be crucial. I should add that Misha is an ideal co-author: active, precise, correct, and delicate. I am delighted that fortune (through V.Å. Khatskevich) has connected us.

Viktor Shulman

Afterword

We know Mikhail to be a very pleasant, honest, generous, and fair person. We wish him many more years of active work in mathematics and happiness in his private life.

References

[1] Goldberg A.A., Ostrovskii I.V. Value distribution of meromorphic functions. In: Translations of Mathematical Monographs, **236**. Amer. Math. Soc., Providence, RI, 2008.

[2] Linnik Yu.V., Ostrovskii I.V. Decomposition of random variables and vectors. In: Rosenblatt J. (Ed.) Translations of Mathematical Monographs, **48**. Amer. Math. Soc., Providence, RI, 1977.

[3] Ostrovskii M.I. Distances between Banach spaces induced by the opening between subspaces. Donetsk, 1985. (in Russian)

[4] Ostrovskii M.I. *Domains of sums of conditionally convergent series in Banach spaces*. Teor. Funkts., Funkts. Anal. Prilozh. 1986, **46**, 77–85. (in Russian)

[5] Ostrovskii M.I. Classification of total subspaces of dual Banach spaces and its applications. Kharkiv, 1997. (in Ukrainian)

[6] Ostrovskii M.I. *On properties of the opening and related closeness characterizations of Banach spaces*. Amer. Math. Soc. Transl. 1987, **136** (2), 109–119. doi:10.1090/trans2/136 (translation of Teor. Funkts., Funkts. Anal. Prilozh. 1984, **42**, 97–107. (in Russian))

[7] Ostrovskii M.I. *The three space problem for the weak Banach-Saks property*. Math. Notes 1985, **38** (5-6), 905–907. doi:10.1007/BF01157537 (translation of Mat. Zametki 1985, **38** (5), 721–725. (in Russian))

[8] Ostrovskii M.I. *Minimal-volume shadows of cubes*. J. Funct. Anal. 2000, **176** (2), 317–330. doi: 10.1006/jfan.2000.3641

[9] Ostrovskii M.I. *Sufficient enlargements of minimal volume for finite dimensional normed linear spaces*. J. Funct. Anal. 2008, **255** (3), 589–619. doi:10.1016/j.jfa.2008.04.012

[10] Ostrovskii M.I. *Auerbach bases and minimal volume sufficient enlargements*. Canad. Math. Bull. 2011, **54**, 726–738. doi:10.4153/CMB-2011-043-3

[11] Ostrovskii M.I. *Sufficient enlargements in the study of projections in normed linear spaces*. Indian J. Math., Golden Jubilee Year Volume, 2008 (Supplement), Proceedings, Dr. George Bachman Memorial Conference, The Allahabad Mathematical Society, 105–122.

[12] Ostrovskii M.I., Shulman V.S. *Weak operator topology, operator ranges and operator equations via Kolmogorov widths*. Integral Equations Operator Theory 2009, **65**, 551–572. doi:10.1007/s00020-009-1691-0

[13] Ostrovskii M.I., Shulman V.S., Turowska L. *Unitarizable representations and fixed points of groups of biholomorphic transformations of operator balls*. J. Funct. Anal. 2009, **257**, 2476–2496. doi:10.1016/j.jfa.2009.06.021

[14] Ostrovskii M.I., Shulman V.S., Turowska L. *Fixed points of holomorphic transformations of operator balls*. Q. J. Math. 2011, **62** (1), 173–187. doi:10.1093/qmath/hap031

[15] Matoušek J. Lectures on discrete geometry. In: Graduate Texts in Mathematics, 212. Springer-Verlag, New York, 2002.

[16] Ostrovskii M.I., Randrianantoanina B., *A new approach to low-distortion embeddings of finite metric spaces into non-superreflexive Banach spaces*. J. Funct. Anal. 2017, **273** (2), 598–651. doi:10.1016/j.jfa.2017.03.017

[17] Ostrovskii M.I. *Embeddability of locally finite metric spaces into Banach spaces is finitely determined*. Proc. Amer. Math. Soc. 2012, **140**, 2721–2730. doi:10.1090/S0002-9939-2011-11272-3

[18] Ostrovskii M.I. *Radon-Nikodým property and thick families of geodesics*. J. Math. Anal. Appl. 2014, **409** (2), 906–910. doi:10.1016/j.jmaa.2013.07.067

[19] Ostrovskii M.I. Metric Embeddings: Bilipschitz and Coarse Embeddings into Banach Spaces. In: de Gruyter Studies in Mathematics, **49**. Walter de Gruyter Co., Berlin, 2013. doi:10.1515/9783110264012

[20] Baudier F.P., Johnson W.B. *Metric embeddings: bilipschitz and coarse embeddings into Banach spaces [book review]*. Bull. Amer. Math. Soc. (N.S.) 2016, **53** (3), 495–506. doi:10.1090/bull/1523

[21] Dilworth S.J., Kutzarova D., Ostrovskii M.I. *Lipschitz-free spaces on finite metric spaces*. Canad. J. Math. 2020, **72**, 774–804. doi:10.4153/S0008414X19000087

[22] Khan S.S., Mim M., Ostrovskii M.I. Isometric copies of ℓ_∞^n and ℓ_1^n in transportation cost spaces on finite metric spaces. In: The Mathematical Legacy of Victor Lomonosov. Operator Theory, 189–203. De Gruyter, 2020. doi:10.1515/9783110656756

[23] Ostrovska S., Ostrovskii M.I. *On relations between transportation cost spaces and ℓ_1*. J. Math. Anal. Appl. 2020, **491** (2), 124338. doi:10.1016/j.jmaa.2020.124338

[24] Ostrovskii M.I., Rosenthal D. *Metric dimensions of minor excluded graphs and minor exclusion in groups*. Internat. J. Algebra Comput. 2015, **25** (4), 541–554.

[25] Catrina F., Ostrovskii M.I. *Images of nowhere differentiable Lipschitz maps of* $[0,1]$ *into* $L_1[0,1]$. Fund. Math. 2018, **243**, 75–83.

[26] Ostrovskii M.I. Weak* sequential closures in Banach space theory and their applications. In: Banakh T., Plichko A. (Eds.) General Topology in Banach Spaces, 21–34. New York, Nova Sci. Publishers, 2001.

[27] Ostrovska S., Ostrovskii M.I. *Generalized transportation cost spaces*. Mediterr. J. Math. 2019, **16** (6), Paper No. 157. doi:10.1007/s00009-019-1433-8

[28] Ostrovskii M.I., Plichko A.M. *On the Ukrainian translation of "Théorie des opérations linéaires" and Mazur's updates of the "Remarks" section*. Mat. Stud. 2009, **32** (1), 96–111.

[29] Ostrovskii M.I. *w^*-derivatives of transfinite order of the subspaces of a conjugate Banach space*. Dokl. Akad. Nauk Ukrain. SSR. Ser. A 1987, **10**, 9–12. (in Russian)

[30] Ostrovskii M.I. *Total subspaces in dual Banach spaces which are not norming over any infinite-dimensional subspace*. Studia Math. 1993, **105** (1), 37–49. doi:10.4064/sm-105-1-37-49

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