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Recall that a topological space $X$ is weakly semiregular, if $X$ has a base consisting of regular open sets, that is such sets $U$ that $U = \text{int} U$. In [1] is stated the following result.

**Lemma 3** ([1]). Let $(X, \tau)$ be a weakly semiregular space, $(Y, \sigma)$ be a space and $\pi: X \to Y$ be a continuous clopen surjection. Then $Y$ is a weakly semiregular space.

Unfortunately, lemma’s proof from [1] contains an error. Namely, the inclusion $\pi(\overline{U}) \subset \pi\pi^{-1}(V)$ fails, for instance, when $\pi$ is the identity map and $U = V$ is any regular open set such that $U \neq \overline{U}$.

Fortunately, in the paper [1] Lemma 3 is applied only once, namely in conjunction with Lemma 1 to prove Proposition 2. This application can be fixed because the map $\pi$ considered in Lemma 1 satisfies a condition $\pi^{-1}(\pi(U)) = U$ for every regular open subset of $X$. Adding this condition to Lemma 3, we can derive the required conclusion as follows.

Let $y \in Y$ be any point and $V \in \sigma$ be any open neighborhood of $y$. Pick a point $x \in \pi^{-1}(y)$. Since $\pi^{-1}(V)$ is an open neighborhood of $x$ and $X$ is weakly semiregular, there exists a regular open subset $U$ of $X$ such that $x \in U \subset \pi^{-1}(V)$. Since the mapping $\pi$ is continuous and clopen, we have $\pi(\overline{U}) = \pi(U)$ and $\pi(U)$ is open in $Y$. Since $U$ is open, the set $X \setminus \overline{U}$ is a regular open subset of $X$. Then $X \setminus \overline{U} = \pi^{-1}(\pi(X \setminus \overline{U}))$. Thus $\overline{U} = \pi^{-1}(\pi(\overline{U}))$.

Suppose that there exists a point $z \in \text{int}(\overline{\pi(U)}) \setminus V$. Then $\pi^{-1}(z) \subset X \setminus \pi^{-1}(V)$ and $\pi^{-1}(z) \subset \pi^{-1}(\text{int} \pi(U)) \subset \pi^{-1}(\pi(U)) = \overline{U}$. This contradicts with $\text{int} \overline{U} \subset \pi^{-1}(V)$. Thus $y \in \text{int} \pi(U) \subset V$, and hence $Y$ is a weakly semiregular space.

**References**


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