ISSN 2075-9827 **e-ISSN 2313-0210**
Carpathian Math. Publ. 2024, **16** (1), 246–258
doi:10.15330/cmp.16.1.246-258

https://journals.pnu.edu.ua/index.php/cmp
Карпатськi матем. публ. 2024, Т.16, №1, С.246–258

Conformal Ricci Soliton on 3-dimensional trans-Sasakian manifolds with respect to SVK connection

Akgün M.A., Acet B.E.

In this study, we adapt the Schouten-van Kampen (SVK) connection on trans-Sasakian 3-manifolds. Then we consider semi-symmetric conditions on trans-Sasakian 3-manifolds with respect to the SVK connection admitting conformal Ricci soliton.

Key words and phrases: Ricci soliton, conformal Ricci soliton, Schouten-van Kampen connection, trans-Sasakian 3-manifold.

Faculty of Arts and Sciences, Department of Mathematics, Adıyaman Univeristy, Adıyaman, Turkey
E-mail: muslumakgun@adiyaman.edu.tr (Akgün M.A.), eacet@adiyaman.edu.tr (Acet B.E.)

1 Introduction

In 1982, the concept of Ricci flow was introduced by R.S. Hamilton [12]. This concept was developed to answer Thurston's geometric conjecture, which says that each closed three manifold admits a geometric decomposition. Also, R.S. Hamilton classiffied all compact manifolds with positive curvature operator in dimension four [12]. The Ricci flow equation is given by

$$\frac{\partial g}{\partial t} = -2S,$$

on a compact Riemannian manifold M with Riemannian metric g.

A self-similar soliton to the Ricci flow [12, 28] is known a Ricci soliton [13], if it moves only by a one parameter family of diffeomorphism and scaling. The Ricci soliton equation is given by

$$\pounds_V g + 2S = 2\lambda g,$$

where $\pounds_V$ is the Lie derivative, S is Ricci tensor, g is a Riemannian metric, V is a vector field and λ is a scalar. The Ricci soliton is said to be shrinking if λ is positive, steady if λ is zero and expanding if λ is negative. Many studies on Ricci solitons have been reported by many geometers in different structure (see [3, 4, 7, 8, 14, 29, 30]).

The concept of conformal Ricci flow was studied by A.E. Fischer [10]. The conformal Ricci flow on M is defined by the equation

$$\frac{\partial g}{\partial t} + 2\left(S + \frac{g}{n}\right) = -pg$$

and $r(g) = -1$, where p is a scalar non-dynamical field (time dependent scalar field), $r(g)$ is the scalar curvature of the manifold and n is the dimension of manifold [10].

УДК 514.7, 514.764.226
2020 *Mathematics Subject Classification:* 53C15, 53A30, 53C25.

Also, the notion of conformal Ricci soliton equation is given by

$$\pounds_V g + 2S = \left[2\lambda - \left(p + \frac{2}{n}\right)\right] g, \tag{1}$$

where λ is constant [1]. The equation is the generalization of the Ricci soliton equation and it also satisfies the conformal Ricci flow equation. In [21], on a 3-dimensional trans-Sasakian manifold conformal Ricci soliton was studied.

Moreover, the Schouten-van Kampen (SVK) connection defined as adapted to a linear connection for studying non holonomic manifolds and it is one of the most natural connections on a differentiable manifold [2, 15, 22]. A.F. Solov'ev studied hyperdistributions in Riemannian manifolds using the SVK connection [23, 25–27]. Then Z. Olszak studied SVK connection to almost (para) contact metric structures [17]. In recent times, S.Y. Perktaş and A. Yıldız studied some symmetry conditions and some soliton types of quasi-Sasakian manifolds and f-Kenmotsu manifolds with respect to SVK connection [19, 20].

In this study, we consider some curvature conditions on a 3-dimensional trans-Sasakian manifolds with respect to the SVK connection admitting conformal Ricci soliton and give an example of a 3-dimensional trans-Sasakian manifold with respect to SVK connection.

2 Preliminaries

Let M be an almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g), where ϕ is a (1,1) tensor field, ξ is a vector field, η is a 1-form and g is the compatible Riemannian metric such that

$$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad \phi\xi = 0, \tag{2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{3}$$

$$g(X, \phi Y) = -g(\phi X, Y), \tag{4}$$

$$g(X, \xi) = \eta(X)$$

for all vector fields $X, Y \in \chi(M)$.

An almost contact metric structure (ϕ, ξ, η, g) on M is called a trans-Sasakian structure [18] if $(M \times R, J, G)$ belongs to the class W_4 [11], where J is the almost complex structure on $M \times R$ defined by $J\left(X, f\frac{d}{dt}\right) = \left(\phi X - f\xi, \eta(X)\frac{d}{dt}\right)$ for smooth functions f on $M \times R$.

It can be expressed [5] by the condition

$$(\nabla_X \phi) Y = \alpha\big(g(X, Y)\xi - \eta(Y)X\big) + \beta\big(g(\phi X, Y)\xi - \eta(Y)\phi X\big) \tag{5}$$

for some smooth functions α, β on M and we say that the trans-Sasakian structure is of type (α, β).

From the above expression, we have

$$\nabla_X \xi = -\alpha\phi X + \beta\big(X - \eta(X)\xi\big), \tag{6}$$

$$(\nabla_X \eta) Y = -\alpha g(\phi X, Y) + \beta g(\phi X, \phi Y). \tag{7}$$

For a 3-dimensional trans-Sasakian manifold the following relations hold [9]:

$$2\alpha\beta + \xi\alpha = 0,$$

$$S(X,Y) = \Big(\frac{r}{2} + \xi\beta - \big(\alpha^2 - \beta^2\big)\Big)g(X,Y) - \Big(\frac{r}{2} + \xi\beta - 3\big(\alpha^2 - \beta^2\big)\Big)\eta(X)\eta(Y)$$
$$- \big(Y\beta + (\phi Y)\alpha\big)\eta(X) - \big(X\beta + (\phi X)\alpha\big)\eta(Y),$$
$$S(X,\xi) = \Big(2\big(\alpha^2 - \beta^2\big) - \xi\beta\Big)\eta X - X\beta - (\phi X)\alpha,$$

where S denotes the Ricci tensor of type $(0,2)$, r is the scalar curvature of the manifold M.

For constant α, β, the following relations hold [9]:

$$S(X,Y) = \Big(\frac{r}{2} - \big(\alpha^2 - \beta^2\big)\Big)g(X,Y) - \Big(\frac{r}{2} - 3\big(\alpha^2 - \beta^2\big)\Big)\eta(X)\eta(Y),$$

$$\begin{aligned} S(X,\xi) &= 2\left(\alpha^2 - \beta^2\right)\eta(X), \\ R(X,Y)\xi &= \big(\alpha^2 - \beta^2\big)\left[\eta(Y)X - \eta(X)Y\right], \\ R(\xi,X)Y &= \big(\alpha^2 - \beta^2\big)\left[g(X,Y)\xi - \eta(Y)X\right], \\ \eta\big(R(X,Y)Z\big) &= \big(\alpha^2 - \beta^2\big)\left[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\right]. \end{aligned} \tag{8}$$

From (5) it follows that if α and β are constants, then the manifold is either α-Sasakian or β-Kenmotsu or cosymplectic, respectively.

Also, we have two naturally defined distributions in the tangent bundle TM of M as follows

$$\tilde{H} = \ker\eta, \qquad \tilde{V} = \operatorname{span}\xi,$$

which implies $TM = \tilde{H} \oplus \tilde{V}$, $\tilde{H} \cap \tilde{V} = 0$ and $\tilde{H} \perp \tilde{V}$. This decomposition allows one to define the Schouten-van Kampen connection $\tilde{\nabla}$ over an almost contact metric structure. The Schouten-van Kampen connection $\tilde{\nabla}$ on an almost contact metric manifold with respect to Levi-Civita connection ∇ is defined [24] by

$$\tilde{\nabla}_X Y = \nabla_X Y - \eta(Y)\nabla_X\xi + (\nabla_X\eta)(Y)\xi. \tag{9}$$

Moreover

$$\begin{aligned} \left(\pounds_\xi g\right)(X,Y) &= \left(\nabla_\xi g\right)(X,Y) - \alpha g(\phi X,Y) + 2\beta g(X,Y) - 2\beta\eta(X)\eta(Y) - \alpha g(\phi X,Y) \\ &= 2\beta g(X,Y) - 2\beta\eta(X)\eta(Y). \end{aligned}$$

Let M be a 3-dimensional trans-Sasakian manifold with constant α and β with respect to SVK connection. Then using (6) and (7) in (9), we get

$$\tilde{\nabla}_X Y = \nabla_X Y + \alpha\big\{\eta(Y)\phi X - g(\phi X,Y)\xi\big\} + \beta\big\{g(X,Y)\xi - \eta(Y)X\big\}. \tag{10}$$

Let R and $\tilde{R}$ be the curvature tensors of the Levi-Civita connection ∇ and SVK connection $\tilde{R}$ are given by

$$R(X,Y) = [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}, \quad \tilde{R}(X,Y) = \big[\tilde{\nabla}_X, \tilde{\nabla}_Y\big] - \tilde{\nabla}_{[X,Y]}.$$

Using (10), we obtain

$$\tilde{R}(X,Y)Z = R(X,Y)Z + \alpha^2 \left\{ \begin{array}{c} g(\phi Y,Z)\phi X - g(\phi X,Z)\phi Y + \eta(X)\eta(Z)Y \\ -\eta(Y)\eta(Z)X - g(Y,Z)\eta(X)\xi + g(X,Z)\eta(Y)\xi \end{array} \right\} + \beta^2\{g(Y,Z)X - g(X,Z)Y\}. \tag{11}$$

We will also consider the Riemannian curvature tensors $\tilde{R}$, R, the Ricci tensors $\tilde{S}$, S, the Ricci operators $\tilde{Q}$, Q and the scalar curvatures $\tilde{\tau}$, τ of the connections $\tilde{\nabla}$ and ∇ are given by

$$\tilde{R}(X,Y,W,Z) = R(X,Y,W,Z) + \alpha^2 \left\{ \begin{array}{c} g(\phi Y,W)g(\phi X,Z) - g(\phi X,W)g(\phi Y,Z) \\ +g(Y,Z)\eta(X)\eta(W) - g(X,Z)\eta(Y)\eta(W) \\ -g(Y,W)\eta(X)\eta(Z) + g(X,W)\eta(Y)\eta(Z) \end{array} \right\} + \beta^2\{g(Y,W)g(X,Z) - g(X,W)g(Y,Z)\},$$

$$\tilde{S}(X,Y) = S(X,Y) + 2\beta^2 g(X,Y) - 2\alpha^2\eta(X)\eta(Y), \tag{12}$$

$$\tilde{Q}X = QX + 2\beta^2 X - 2\alpha^2\eta(X)\xi,$$

$$\tilde{\tau} = \tau - 2\alpha^2 + 6\beta^2.$$

In a 3-dimensional trans-Sasakian manifold M endowed with respect to the SVK connection bearing a conformal Ricci soliton, we can write

$$\tilde{\pounds}_V g + 2\tilde{S} - \left\{2\lambda - \left(p + \frac{2}{3}\right)\right\}g = 0. \tag{13}$$

From (9) and (13), since $\tilde{\nabla}g = 0$ and $\tilde{T} \neq 0$, we have

$$\tilde{\pounds}_V g(X,Y) = g(\nabla_X V, Y) + g(X, \nabla_Y V) = (\pounds_V g)(X,Y),$$

that is

$$g(\nabla_X V, Y) + g(X, \nabla_Y V) + 2\tilde{S}(X,Y) - \left\{2\lambda - \left(p + \frac{2}{3}\right)\right\}g(X,Y) = 0. \tag{14}$$

Putting $V = \xi$ in (14), we obtain

$$g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi) + 2\tilde{S}(X,Y) - \left\{2\lambda - \left(p + \frac{2}{3}\right)\right\}g(X,Y) = 0. \tag{15}$$

Now using (6) with (4) in (15), we get

$$2\beta g(X,Y) - 2\beta\eta(X)\eta(Y) + 2\tilde{S}(X,Y) - \left\{2\lambda - \left(p + \frac{2}{3}\right)\right\}g(X,Y) = 0,$$

which yields

$$\tilde{S}(X,Y) = \left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right) - \beta\right\}g(X,Y) + \beta\eta(X)\eta(Y). \tag{16}$$

Thus M is an η-Einstein manifold with respect to SVK connection.

Also using (12) in (16), we have

$$S(X,Y) = \left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right) - \beta - 2\beta^2\right\}g(X,Y) + \left(\beta + 2\beta^2\right)\eta(X)\eta(Y). \tag{17}$$

Hence M is an η-Einstein manifold with respect to the Levi-Civita connection. Thus we have the following assertion.

Theorem 1. *Let M be a 3-dimensional trans-Sasakian manifold admitting a conformal Ricci soliton (g, ξ, λ) with respect to SVK connection. Then M is an η-Einstein manifold both with respect to the SVK connection and Levi-Civita connection.*

Putting $Y = \xi$ and using (12) in (16), we get

$$\tilde{S}(X, \xi) = \left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right\}\eta(X). \tag{18}$$

So we discuss the following conditions:

i) assume that $p > \frac{2}{3}$ and therefore the equation (18) shows that $\lambda > 0$, thus the Ricci soliton (g, ξ, λ) is expanding;

ii) assume that $p < \frac{2}{3}$ and therefore the equation (18) reveals that $\lambda < 0$, thus the Ricci soliton (g, ξ, λ) is shrinking;

iii) assume that $p = \frac{2}{3}$ and therefore the equation (18) allows that $\lambda = 0$, thus the Ricci soliton (g, ξ, λ) is steady.

Thus we state our results in the form of theorem as follows.

Theorem 2. *A conformal Ricci soliton (g, ξ, λ) on a 3-dimensional trans-Sasakian manifold with respect to SVK connection is said to be expanding, shrinking and steady if $p > \frac{2}{3}$, $p < \frac{2}{3}$ and $p = \frac{2}{3}$, respectively.*

3 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold satisfying $R(\xi, X).\tilde{B} = 0$

The C-Bochner curvature tensor $\tilde{B}$ in M is defined [16] (see also [6]) by

$$\begin{aligned}
\tilde{B}(X,Y)Z = \tilde{R}(X,Y)Z + \frac{1}{6}&\left\{\begin{array}{c}
g(X,Z)\tilde{Q}Y - g(Y,Z)\tilde{Q}X \\
-\tilde{S}(Y,Z)X + \tilde{S}(X,Z)Y \\
+g(\phi X,Z)\tilde{Q}\phi Y - g(\phi Y,Z)\tilde{Q}\phi X \\
-\tilde{S}(\phi Y,Z)\phi X + \tilde{S}(\phi X,Z)\phi Y \\
+2\tilde{S}(\phi X,Y)\phi Z + 2g(\phi X,Y)\tilde{Q}\phi Z \\
+\eta(Y)\eta(Z)\tilde{Q}X - \eta(X)\eta(Z)\tilde{Q}Y \\
-\eta(Y)\tilde{S}(X,Z)\xi + \eta(X)\tilde{S}(Y,Z)\xi
\end{array}\right\} \\
- \frac{D+2}{6}&\left\{\begin{array}{c}
g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X \\
+2g(\phi X,Y)\phi Z
\end{array}\right\} \\
+ \frac{D}{6}&\left\{\begin{array}{c}
\eta(Y)g(X,Z)\xi - \eta(Y)\eta(Z)X \\
+\eta(X)\eta(Z)Y - \eta(X)g(Y,Z)\xi
\end{array}\right\} \\
- \frac{D-4}{6}&\left\{g(X,Z)Y - g(Y,Z)X\right\},
\end{aligned}$$

where $D = \frac{\tilde{r}+2}{6}$.

Using (11) with (12) in the above equation, we get

$$\begin{aligned}
\tilde{B}(X,Y)Z = R(X,Y)Z + \alpha^2 &\left\{ \begin{array}{c} g(\phi Y,Z)\phi X - g(\phi X,Z)\phi Y \\ +\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ -g(Y,Z)\eta(X)\xi + g(X,Z)\eta(Y)\xi \end{array} \right\} \\
&+ \beta^2 \Big\{ g(Y,Z)X - g(X,Z)Y \Big\} \\
&+ \frac{1}{6} \left\{ \begin{array}{c} S(X,Z)Y - S(Y,Z)X \\ +g(X,Z)QY - g(Y,Z)QX \\ +g(\phi X,Z)Q\phi Y - g(\phi Y,Z)Q\phi X \\ +S(\phi X,Z)\phi Y - S(\phi Y,Z)\phi X \\ -S(X,Z)\eta(Y)\xi + S(Y,Z)\eta(X)\xi \\ +2S(\phi X,Y)\phi Z + 2g(\phi X,Y)Q\phi Z \\ +\eta(Y)\eta(Z)QX - \eta(X)\eta(Z)QY \\ +4\beta^2 g(X,Z)Y - 4\beta^2 g(Y,Z)X \\ +4\beta^2 g(\phi X,Z)\phi Y - 4\beta^2 g(\phi Y,Z)\phi X \\ -2\beta^2 g(X,Z)\eta(Y)\xi + 2\beta^2 g(Y,Z)\eta(X)\xi \\ +8\beta^2 g(\phi X,Y)\phi Z \\ +2\beta^2\eta(Y)\eta(Z)X - 2\beta^2\eta(X)\eta(Z)Y \\ +2\alpha^2 g(Y,Z)\eta(X)\xi - 2\alpha^2 g(X,Z)\eta(Y)\xi \\ +2\alpha^2\eta(Y)\eta(Z)X - 2\alpha^2\eta(X)\eta(Z)Y \end{array} \right\} \\
&- \frac{D+2}{6} \left\{ \begin{array}{c} g(\phi X,Z)\phi Y - g(\phi Y,Z)\phi X \\ +2g(\phi X,Y)\phi Z \end{array} \right\} \\
&+ \frac{D}{6} \left\{ \begin{array}{c} g(X,Z)\eta(Y)\xi - \eta(Y)\eta(Z)X \\ +\eta(X)\eta(Z)Y - g(Y,Z)\eta(X)\xi \end{array} \right\} \\
&- \frac{D-4}{6} \{ g(X,Z)Y - g(Y,Z)X \}.
\end{aligned} \tag{19}$$

Taking $Z = \xi$ in (19), we get

$$\tilde{B}(X,Y)\xi = \left\{ \frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} [\eta(X)Y - \eta(Y)X], \tag{20}$$

which gives

$$\eta\big(\tilde{B}(X,Y)Z\big) = \left\{ \frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3} \right) \right) + \frac{4}{6} \right\} \left[\begin{array}{c} g(X,Z)\eta(Y) \\ -g(Y,Z)\eta(X) \end{array} \right].$$

Now, we assume that the condition $R(\xi,X).\tilde{B} = 0$ is satisfied, then we get

$$R(\xi,X)\tilde{B}(Y,Z)W - \tilde{B}\big(R(\xi,X)Y,Z\big)W - \tilde{B}\big(Y,R(\xi,X)Z\big)W - \tilde{B}(Y,Z)R(\xi,X)W = 0. \tag{21}$$

Using (8) in (21), we obtain

$$\begin{aligned}
\eta\big(\tilde{B}(Y,Z)W,X\big) &- g\big(\tilde{B}(Y,Z)W,X\big)\xi + g(X,Y)\tilde{B}(\xi,Z)W - \eta(Y)\tilde{B}(X,Z)W \\
&+ g(X,Z)\tilde{B}(Y,\xi)W - \eta(Z)\tilde{B}(Y,X)W + g(X,W)\tilde{B}(Y,Z)\xi - \eta(W)\tilde{B}(Y,Z)X = 0.
\end{aligned}$$

By taking inner product with ξ and using (2), we have

$$\begin{aligned}\eta\big(\tilde{B}(Y,Z)W\big)\eta(X) - g\big(\tilde{B}(Y,Z)W,X\big)\xi + g(X,Y)\eta\big(\tilde{B}(\xi,Z)W\big) - \eta(Y)\eta\big(\tilde{B}(X,Z)W\big) \\ + g(X,Z)\eta\big(\tilde{B}(Y,\xi)W\big) - \eta(Z)\eta\big(\tilde{B}(Y,X)W\big) \\ + g(X,W)\eta\big(\tilde{B}(Y,Z)\xi\big) - \eta(W)\eta\big(\tilde{B}(Y,Z)X\big) = 0.\end{aligned} \tag{22}$$

By using (20) in (22), we arrive at

$$\left\{\frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6}\right\}\left[\begin{array}{c} g(Y,W)g(X,Z) \\ -g(Z,W)g(X,Y)\end{array}\right] - g(\tilde{B}(Y,Z)W,X) = 0. \tag{23}$$

Now using (19) in (23), we get

$$\begin{aligned}&\left\{\frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6}\right\}\left[\begin{array}{c} g(Y,W)g(X,Z) \\ -g(Z,W)g(X,Y)\end{array}\right] - g\big(R(Y,Z)W,X\big) \\
&- \alpha^2\left\{\begin{array}{c} g(\phi Z,W)g(\phi Y,X) - g(\phi Y,W)g(\phi Z,X) \\ +g(X,Z)\eta(Y)\eta(W) - g(X,Y)\eta(Z)\eta(W) \\ -g(Z,W)\eta(Y)\eta(X) + g(Y,W)\eta(Z)\eta(X)\end{array}\right\} \\
&- \beta^2\{g(Z,W)g(X,Y) - g(Y,W)g(X,Z)\} \\
&- \frac{1}{6}\left\{\begin{array}{c} S(X,Z)g(Y,W) - S(Z,W)g(Y,X) \\ +S(Y,W)g(X,Z) - S(Y,X)g(Z,W) \\ +S(\phi Z,X)g(\phi Y,W) - S(\phi Z,W)g(\phi Y,X) \\ -S(\phi Y,X)g(\phi Z,W) + S(\phi Y,W)g(\phi Z,X) \\ +2S(\phi Y,Z)g(\phi W,X) + 2S(\phi W,X)g(\phi Y,Z) \\ +S(Y,X)\eta(Z)\eta(W) - S(Y,W)\eta(Z)\eta(X) \\ +S(Z,W)\eta(Y)\eta(X) - S(Z,X)\eta(Y)\eta(W) \\ +4\beta^2 g(Y,W)g(X,Z) - 4\beta^2 g(Z,W)g(Y,X) \\ -2\alpha^2 g(Y,W)\eta(X)\eta(Z) + 2\alpha^2 g(X,Y)\eta(Z)\eta(W) \\ +2\alpha^2 g(Z,W)\eta(X)\eta(Y) - 2\alpha^2 g(X,Z)\eta(Y)\eta(W) \\ +4\beta^2 g(\phi Y,W)g(\phi Z,X) - 4\beta^2 g(\phi Z,W)g(\phi Y,X) \\ +4\beta^2 g(\phi Y,Z)g(\phi W,X) \\ +2\beta^2 g(Y,X)\eta(Z)\eta(W) - 2\beta^2 g(Y,W)\eta(Z)\eta(X) \\ +2\beta^2 g(Z,W)\eta(Y)\eta(X) - 2\beta^2 g(Z,X)\eta(Y)\eta(W)\end{array}\right\} \\
&+ \frac{D+2}{6}\left\{\begin{array}{c} g(\phi Y,W)g(\phi Z,X) - g(\phi Z,W)g(\phi Y,X) \\ +2g(\phi Y,Z)g(\phi W,X)\end{array}\right\} \\
&- \frac{D}{6}\left\{\begin{array}{c} g(Y,W)\eta(Z)\eta(X) - \eta(Z)\eta(W)g(X,Y) \\ -g(W,Z)\eta(X)\eta(Y) + \eta(Y)\eta(Z)g(X,Z)\end{array}\right\} \\
&+ \frac{D-4}{6}\{g(Y,W)g(Z,X) - g(Z,W)g(Y,X)\} = 0.\end{aligned}$$

Taking $Y = X = e_i$ and summing over $i = 1,2,3$, where e_i is an orthonormal basis of T_pM , we obtain

$$\begin{aligned}S(Z,W) = &\left\{\begin{array}{c} \frac{\tau}{6} - \beta + \beta^2 - \alpha^2 \\ +\frac{2}{3}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) - \frac{2D}{3} - 1\end{array}\right\}g(Z,W) \\ &+ \left\{\begin{array}{c} \frac{2D}{3} + 1 - \frac{\tau}{6} \\ -\frac{2}{3}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + 5\alpha^2 - 3\beta^2\end{array}\right\}\eta(Z)\eta(W).\end{aligned}$$

Taking $Z = W = \xi$ in the above equation with using (17), we arrive at

$$\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right) = 2\left(\alpha^2 - \beta^2\right).$$

Since $\alpha^2 \neq \beta^2$, in this case we have the following.

i) Assume that $\alpha^2 \geq \beta^2$, then $(\alpha - \beta)(\alpha + \beta) \geq 0$, which implies α greater than β. So, $\lambda > 0$ and Ricci soliton is shrinking.

ii) Assume that $\alpha^2 < \beta^2$ and $p + \frac{2}{3} > 4\left(\alpha^2 - \beta^2\right)$, then $(\alpha - \beta)(\alpha + \beta) < 0$, which gives α less than $-\beta$. So, $\lambda > 0$ and Ricci soliton is shrinking.

iii) Assume that $\alpha^2 < \beta^2$ and $p + \frac{2}{3} < 4\left(\alpha^2 - \beta^2\right)$, then $(\alpha - \beta)(\alpha + \beta) < 0$, which gives α less than $-\beta$. So, $\lambda < 0$ and Ricci soliton is expanding.

Theorem 3. *A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies $R(\xi, X).\tilde{B} = 0$ and admits conformal Ricci soliton, then*

i) for $\alpha > \beta$, the Ricci soliton is shrinking;

ii) for $\alpha < -\beta$ and $p + \frac{2}{3} > 4\left(\alpha^2 - \beta^2\right)$, the Ricci soliton becomes shrinking;

iii) for $\alpha < -\beta$ and $p + \frac{2}{3} < 4\left(\alpha^2 - \beta^2\right)$, the Ricci soliton becomes expanding.

4 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfying $\tilde{B}(\xi, X).S = 0$

The condition $\tilde{B}(\xi, X).S = 0$ implies that

$$S\big(\tilde{B}(\xi, X)Y, Z\big) + S\big(Y, \tilde{B}(\xi, X)Z\big) = 0. \tag{24}$$

Using (17) in (24), we get

$$\left\{ \begin{array}{c} \lambda - \frac{1}{2}\left(p + \frac{2}{3}\right) \\ -\beta - 2\beta^2 \end{array} \right\} \left[\begin{array}{c} g\big(\tilde{B}(\xi, X)Y, Z\big) \\ +g\big(Y, \tilde{B}(\xi, X)Z\big) \end{array} \right] + \left\{2\alpha^2 + \beta\right\} \left[\begin{array}{c} \eta\big(\tilde{B}(\xi, X)Y\big)\eta(Z) \\ +\eta\big(\tilde{B}(\xi, X)Z\big)\eta(Y) \end{array} \right] = 0.$$

By using (20) in the above equation, we obtain

$$\left\{2\alpha^2 + \beta\right\} \left\{ \begin{array}{c} \lambda - \frac{1}{2}\left(p + \frac{2}{3}\right) \\ -\beta - 2\beta^2 \end{array} \right\} \left[\begin{array}{c} 2\eta(X)\eta(Y)\eta(Z) \\ -g(X, Y)\eta(Z) - g(X, Z)\eta(Y) \end{array} \right] = 0.$$

Taking $Y = X = e_i$ and summing over $i = 1, 2, 3$, where e_i is an orthonormal basis of T_pM and taking condition $\eta(Z) \neq 0$, we obtain

$$\left\{2\alpha^2 + \beta\right\} \left\{ \frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6} \right\} = 0.$$

Thus we get the following result.

Theorem 4. *A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies $\tilde{B}(\xi, X).S = 0$ and admits conformal Ricci soliton with $2\alpha^2 \neq \beta$, then*

i) if $p = \frac{22}{3}$, then $\lambda = 0$ and Ricci soliton is steady;

ii) if $p < \frac{22}{3}$, then $\lambda < 0$ and Ricci soliton is expanding;

iii) if $p > \frac{22}{3}$, then $\lambda > 0$ and Ricci soliton is shrinking.

5 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfying $\tilde{S}(\xi, X).\tilde{R} = 0$

If we consider the condition $\tilde{S}(\xi, X).\tilde{R} = 0$, then we have

$$\begin{aligned}(\tilde{S}(\xi, X).\tilde{R})(U,V)W = {} & \tilde{S}(\xi, \tilde{R}(U,V)W)X - \tilde{S}(X, \tilde{R}(U,V)W)\xi + \tilde{S}(\xi,U)\tilde{R}(X,V)W \\ & - \tilde{S}(\xi,U)\tilde{R}(\xi,V)W + \tilde{S}(\xi,V)\tilde{R}(U,X)W - \tilde{S}(\xi,V)\tilde{R}(U,\xi)W \\ & + \tilde{S}(\xi,W)\tilde{R}(U,V)X - \tilde{S}(X,W)\tilde{R}(U,V)\xi.\end{aligned} \tag{25}$$

By using (16) in (25), we have

$$\begin{aligned}&\left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right\}\begin{bmatrix} \eta(\tilde{R}(U,V)W)X + \eta(U)\tilde{R}(X,V)W \\ +\eta(V)\tilde{R}(U,X)W + \eta(W)\tilde{R}(U,V)X \end{bmatrix} \\ &\qquad - \left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right) - \beta\right\}\begin{bmatrix} g(X, \tilde{R}(U,V)W)\xi + g(X,U)\tilde{R}(\xi,V)W \\ +g(X,V)\tilde{R}(U,\xi)W + g(X,W)\tilde{R}(U,V)\xi \end{bmatrix} \\ &\qquad - \beta\begin{bmatrix} \eta(X)\eta(\tilde{R}(U,V)W)\xi + \eta(X)\eta(U)\tilde{R}(\xi,V)W \\ +\eta(X)\eta(V)\tilde{R}(U,\xi)W + \eta(X)\eta(W)\tilde{R}(U,V)\xi \end{bmatrix} = 0.\end{aligned}$$

By taking inner product with ξ and using (11), we get

$$\left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right\}\begin{bmatrix} g(X, R(U,V)W) \\ +\alpha^2\begin{pmatrix} g(\phi V, W)g(\phi U, X) - g(\phi U, W)g(\phi V, X) \\ +g(X,V)\eta(U)\eta(V) - g(X,U)\eta(X)\eta(W) \\ -g(V,W)\eta(U)\eta(X) + g(U,W)\eta(V)\eta(X) \end{pmatrix} \\ +\beta^2(g(V,W)g(X,U) - g(U,W)g(X,V)) \end{bmatrix} = 0.$$

Taking $X = U = e_i$ and summing over $i = 1,2,3$, where e_i is an orthonormal basis of T_pM, we find

$$\left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right\}\begin{bmatrix} S(V,W) \\ +\alpha^2(g(V,W) - 4\eta(V)\eta(W)) \\ +2\beta^2 g(V,W) \end{bmatrix} = 0.$$

Taking $V = W = \xi$ in the above equation, we arrive at

$$\left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right\}\left\{\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right) - \alpha^2\right\} = 0,$$

which gives either $\lambda = \frac{1}{2}\left(p + \frac{2}{3}\right)$ or $\lambda = \alpha^2 + \frac{1}{2}\left(p + \frac{2}{3}\right)$.

So we can give the following result.

Theorem 5. *A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies $\tilde{S}(\xi, X).\tilde{R} = 0$ and admits conformal Ricci soliton, then*

Case I: *λ depends only on p*

- *i) if $p = -\frac{2}{3}$, then $\lambda = 0$ and Ricci soliton is steady;*
- *ii) if $p > -\frac{2}{3}$, then $\lambda > 0$ and Ricci soliton is shrinking;*
- *iii) if $p < -\frac{2}{3}$, then $\lambda < 0$ and Ricci soliton is expanding;*

Case II: *λ depends on both p and α*

- *i) if $p = -\frac{2}{3} - 2\alpha^2$, then $\lambda = 0$ and Ricci soliton is steady;*
- *ii) if $p > -\frac{2}{3} - 2\alpha^2$, then $\lambda > 0$ and Ricci soliton is shrinking;*
- *iii) if $p < -\frac{2}{3} - 2\alpha^2$, then $\lambda < 0$ and Ricci soliton is expanding.*

6 Conformal Ricci soliton on a 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfying $\tilde{B}.\phi = 0$

Now, we examine the curvature condition $\tilde{B}.\phi = 0$ on a 3-dimensional trans-Sasakian manifold with respect to SVK connection admitting a conformal Ricci soliton.

We know that if $\tilde{B}.\phi = 0$, then we have

$$(\tilde{B}.\phi)(X,Y)Z = 0,$$

which gives

$$\tilde{B}(X,Y)\phi Z - \phi\tilde{B}(X,Y)Z = 0. \tag{26}$$

If we take $Z = \xi$ in (26), we find

$$\phi\tilde{B}(X,Y)\xi = 0. \tag{27}$$

Using (20) in (27), we obtain

$$\left\{\frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6}\right\}\left[\eta(X)\phi Y - \eta(Y)\phi X\right] = 0. \tag{28}$$

Replacing X by ϕX in (28) and using (2), we get

$$\left\{\frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6}\right\}\left[\left(- X + \eta(X)\right)\eta(Y)\right] = 0. \tag{29}$$

Taking $Y = \xi$ and replacing X by ϕX in (29), we have

$$\left\{\frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6}\right\}\phi X = 0.$$

Finally, taking inner product with W, we obtain

$$\left\{\frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6}\right\}g(\phi X, W) = 0.$$

It follows that

$$\frac{1}{6}\left(\lambda - \frac{1}{2}\left(p + \frac{2}{3}\right)\right) + \frac{4}{6} = 0.$$

So we can state the following result.

Theorem 6. *A 3-dimensional trans-Sasakian manifold with respect to SVK connection satisfies $\tilde{B}.\phi = 0$ and admits conformal Ricci soliton, then*

i) if $p = \frac{22}{3}$, then $\lambda = 0$ and Ricci soliton is steady;

ii) if $p < \frac{22}{3}$, then $\lambda < 0$ and Ricci soliton is expanding;

iii) if $p > \frac{22}{3}$, then $\lambda > 0$ and Ricci soliton is shrinking.

Example 1. *We consider a 3-dimensional manifold* $= \left\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 \neq 0\right\}$, *where* (x_1, x_2, x_3) *are standard coordinates in* $\mathbb{R}^3$. *Let* $\{\omega_1, \omega_2, \omega_3\}$ *be a linearly independent global frame on* M *defined by*

$$\omega_1 = e^{-\omega_3}\left(\frac{\partial}{\partial\omega_1} + \frac{\partial}{\partial\omega_2}\right), \quad \omega_2 = e^{-\omega_3}\left(-\frac{\partial}{\partial\omega_1} + \frac{\partial}{\partial\omega_2}\right), \quad \omega_3 = \frac{\partial}{\partial\omega_3}.$$

Let g *be the Riemannian metric defined by*

$$g(\omega_1,\omega_1) = g(\omega_2,\omega_2) = g(\omega_3,\omega_3) = 1,$$
$$g(\omega_1,\omega_2) = g(\omega_1,\omega_3) = g(\omega_2,\omega_3) = 0.$$

If η *is the 1-form defined by* $\eta(W) = g(W, \omega_3)$ *and if* ϕ *is the* $(1,1)$*-tensor field defined by*

$$\phi(\omega_1) = \omega_2, \quad \phi(\omega_2) = -\omega_1, \quad \phi(\omega_3) = 0.$$

Also, we have

$$\eta(\omega_3) = 1,$$
$$\phi Y = -Y + \eta(Y)\xi,$$
$$g(\phi Y, \phi V) = g(Y, V) + \eta(Y)\eta(V).$$

Now, we get

$$[\omega_1,\omega_2] = 0, \quad [\omega_1,\omega_3] = 0, \quad [\omega_2,\omega_3] = 0.$$

From Koszul's formula, we obtain

$$\begin{array}{lll} \nabla_{\omega_1}\omega_1 = -\omega_3, & \nabla_{\omega_2}\omega_1 = 0, & \nabla_{\omega_3}\omega_1 = 0, \\ \nabla_{\omega_1}\omega_2 = 0, & \nabla_{\omega_2}\omega_2 = -\omega_3, & \nabla_{\omega_3}\omega_2 = 0, \\ \nabla_{\omega_1}\omega_3 = \omega_1, & \nabla_{\omega_2}\omega_3 = \omega_2, & \nabla_{\omega_3}\omega_3 = 0. \end{array} \tag{30}$$

In view of the above eqations, (ϕ, ξ, η, g) *satisfy (2) and (3) with* $\alpha = 0$ *and* $\beta = 1$. *So,* M *is a trans-Sasakian manifold [31]. In view of (30), we find*

$$\begin{array}{lll} R(\omega_1,\omega_2)\omega_3 = 0, & R(\omega_2,\omega_3)\omega_3 = -\omega_2, & R(\omega_1,\omega_3)\omega_3 = -\omega_1, \\ R(\omega_1,\omega_2)\omega_2 = -\omega_1, & R(\omega_2,\omega_3)\omega_2 = \omega_3, & R(\omega_1,\omega_3)\omega_2 = 0, \\ R(\omega_1,\omega_2)\omega_1 = \omega_2, & R(\omega_2,\omega_3)\omega_1 = 0, & R(\omega_1,\omega_3)\omega_1 = \omega_3. \end{array}$$

If we consider SVK connection to this equation from (10) with (30), we obtain

$$\begin{array}{lll} \tilde{\nabla}_{\omega_1}\omega_1 = (\beta-1)\omega_3, & \tilde{\nabla}_{\omega_2}\omega_1 = \alpha\omega_3, & \tilde{\nabla}_{\omega_3}\omega_1 = 0, \\ \tilde{\nabla}_{\omega_1}\omega_2 = -\alpha\omega_3, & \tilde{\nabla}_{\omega_2}\omega_2 = (\beta-1)\omega_3, & \tilde{\nabla}_{\omega_3}\omega_2 = 0, \\ \tilde{\nabla}_{\omega_1}\omega_3 = (1-\beta)\omega_1 + \alpha\omega_2, & \tilde{\nabla}_{\omega_2}\omega_3 = (1-\beta)\omega_2 + \alpha\omega_1, & \tilde{\nabla}_{\omega_3}\omega_3 = 0. \end{array} \tag{31}$$

We know that M *is a trans-Sasakian manifold with respect to SVK connection. In view of (31), we get*

$$\begin{array}{ll} \tilde{R}(\omega_1,\omega_2)\omega_3 = 0, & \tilde{R}(\omega_2,\omega_3)\omega_3 = \left(\beta^2-\alpha^2-1\right)\omega_2, \\ \tilde{R}(\omega_1,\omega_2)\omega_2 = \left(\beta^2-\alpha^2-1\right)\omega_1, & \tilde{R}(\omega_2,\omega_3)\omega_2 = \left(1+\alpha^2-\beta^2\right)\omega_3, \\ \tilde{R}(\omega_1,\omega_2)\omega_1 = \left(1+\alpha^2-\beta^2\right)\omega_2, & \tilde{R}(\omega_2,\omega_3)\omega_1 = 0, \end{array}$$

$$\begin{array}{l} \tilde{R}(\omega_1,\omega_3)\omega_1 = \omega_3, \\ \tilde{R}(\omega_1,\omega_3)\omega_2 = 0, \\ \tilde{R}(\omega_1,\omega_3)\omega_3 = \left(\beta^2-\alpha^2-1\right)\omega_1. \end{array}$$

From the above equation, we arrive at

$$\tilde{S}(\omega_1,\omega_1) = 2\left(\beta^2-1\right), \quad \tilde{S}(\omega_2,\omega_2) = 2\left(\beta^2-1\right), \quad \tilde{S}(\omega_3,\omega_3) = 2\left(\beta^2-\alpha^2-1\right).$$

Finally, in view of (1), *if* $\lambda = 2\beta^2 - \beta + 2 + \frac{p}{2} + \frac{1}{3}$ *and* $\beta = -2\alpha^2$, *then* M *admits a conformal Ricci soliton with respect to SVK connection.*

References

[1] Basu N., Bhattacharyya A. *Conformal Ricci soliton in Kenmotsu manifold*. Glob. J. Adv. Res. Class. Mod. Geom. 2015, **4** (1), 15–21.

[2] Bejancu A., Farran H. Foliations and geometric structures. In: Mathematics and Its Applications, 580. Springer, Dordrecht, 2006.

[3] Blaga A.M., Yüksel P.S., Acet B.E., Erdoğan F.E. *η-Ricci solitons in ϵ-almost paracontact metric manifolds*. Glas. Mat. 2018, **53** (1), 205–220. doi:10.3336/gm.53.1.14

[4] Blaga A.M., Yüksel P.S. *Remarks on almost η-Ricci solitons in ϵ-para Sasakian manifolds*. Comm. Fac. Sci. Uni. Ankara Series A1 Math. and Stat. 2019, **68** (2), 1621–1628. doi:10.31801/cfsuasmas.546595

[5] Blair D.E., Oubiña J.A. *Conformal and related changes of metric on the product of two almost contact metric manifolds*. Publ. Mat. 1990, **34** (1), 199–207.

[6] Bochner S. *Curvature and Betti numbers II*. Ann. of Math. (2) 1949, **50** (2), 77–93. doi:10.2307/1969353

[7] Chakrabotry D., Mishra V.N., Hui S.K. *Ricci soliton on three-dimensional β-Kenmotsu manifolds with respect to Schouten-van Kampen connection*. J. Ultra Scientist of Phys. Sci. 2018, **30** (1), 86–91. doi:10.22147/jusps-A/300110

[8] Chaubey S.K., De U.C. *Three-dimensional trans-Sasakian manifolds and solitons*. Arab. J. Math. Sci. 2022, **28** (1), 112–125. doi:10.1108/AJMS-12-2020-0127

[9] Dutta T., Basu N., Bhattacharyya A. *Almost conformal Ricci solitons on 3-dimensional trans-Sasakian manifold*. Hacet. J. Math. Stat. 2016, **45** (5), 1379–1392. doi:10.15672/HJMS.20164514287

[10] Fischer A.E. *An introduction to conformal Ricci flow*. Classical Quantum Gravity 2004, **21** (3), 171–218. doi:10.1088/0264-9381/21/3/011

[11] Gray A., Hervella M.L. *The sixteen classes of almost Hermitian manifolds and their linear invariants*. Ann. Mat. Pura Appl. (4) 1980, **123** (4), 35–58. doi:10.1007/BF01796539

[12] Hamilton R.S. *Three manifold with positive Ricci curvature*. J. Differential Geom. 1982, **17** (2), 255–306. doi:10.4310/jdg/1214436922

[13] Hamilton R.S. *The Ricci flow on surfaces*. Contemp. Math. 1988, **71**, 237–261. doi:10.1090/conm/071/954419

[14] Hui S.K., Chakrabotry D. *Ricci almost soliton on concircular Ricci pseudosymmetric β-Kenmotsu manifolds*. Hacet. J. Math. Stat. 2018, **47** (3), 579–587. doi:10.15672/HJMS.2017.471

[15] Ianus S. *Some almost product structures on manifolds with linear connection*. Ködai Math. Sem. Rep. 1971, **23**, 305–310.

[16] Kim J.S., Tripathi M.M., Choi J.-D. *On C-Bochner curvature tensor of a contact metric manifold*. Bull. Korean Math. Soc. 2005, **42** (4), 713–724. doi:10.4134/bkms.2005.42.4.713

[17] Olszak Z. *The Schouten-van Kampen affine connection adapted an almost (para) contact metric structure*. Publ. de l'Inst. Math. 2013, **94** (108), 31–42. doi:10.2298/PIM1308031O

[18] Oubina J.A. *New classes of almost contact metric structures*. Publ. Math. Debrecen 1985, **32** (4), 187–193.

[19] Perktaş S.Y., Yıldız A. *On quasi-Sasakian 3-manifolds with respect to the Schouten-van Kampen connection*. Int. Electron. J. Geom 2020, **13** (2), 62–74. doi:10.36890/iejg.742073

[20] Perktaş S.Y., Yıldız A. *On f-Kenmotsu 3-manifolds with respect to the Schouten-van Kampen connection*. Turkish J. Math. 2021, **45** (1), 387–409. doi:10.3906/mat-2003-121

[21] Roy S., Bhattacharyya A. *Conformal Ricci soliton on 3-dimensional trans-Sasakian manifold*. Jordan J. Math. Stat. 2020, **13** (1), 89–109.

[22] Schouten J., Van Kampen E. *Zur Einbettungs-und Krü mmungsthorie nichtholonomer Gebilde*. Math. Ann. 1930, **103**, 752–783.

[23] Solov'ev A.F. *On the curvature of the connection induced on a hyperdistribution in a Riemannian space*. Geom. Sb. 1982, **19**, 12–23. (in Russian)

[24] Solov'ev A.F. *On the curvature of the connection induced on a hyperdistribution in a Riemannian space*. Geom Sb. 1978, **19**, 12–23. (in Russian)

[25] Solov'ev A.F. *The bending of hyperdistributons*. Geom. Sb. 1979, **20**, 101–112. (in Russian)

[26] Solov'ev A.F. *Second fundamental form of a distribution*. Mat. Zametki 1982, **35**, 139–146.

[27] Solov'ev A.F. *Curvature of a distribution*. Mat. Zametki 1984, **35**, 111–124.

[28] Topping P. Lecture on the Ricci Flow. In: Süli E. (Eds.) London Mathematical Society Lecture Note Series, 325. Cambridge University Press, United Kingdom, 2006.

[29] Turan M., De U.C., Yıldız A. *Ricci solitons and gradient Ricci solitons in* 3*-dimensional trans-Sasakian manifolds*. Filomat 2012, **26** (2), 363–370. doi:10.2298/FIL1202363T

[30] Turan M., De U.C., Yıldız A., De A. *Ricci solitons and gradient Ricci solitons on* 3*-dimensional normal almost contact metric manifolds*. Publ. Math. Debrecen 2012, **80** (1-2), 127–142. doi:10.5486/PMD.2012.4947

[31] Vishnuvardhana S.V., Venkatesha V., Turgut V.A. *On* 3*-dimensional trans-Sasakian manifold admitting a semi-symmetric metric connection*. GU J Sci. 2019, **32** (1), 242–254.

Received 21.02.2022
Revised 30.05.2022

Акгюн М.А., Асет Б.Е. *Конформний солітон Річчі на тривимірних транс-Сасакянових многовидах відносно SVK зв'язності* // Карпатські матем. публ. — 2024. — Т.16, №1. — С. 246–258.

У цьому дослідженні ми адаптуємо зв'язність Схаутен-ван Кампена (SVK) до тривимірних транс-Сасакянових многовидів. Також, ми розглядаємо напівсиметричні умови на тривимірних транс-Сасакянових многовидах відносно SVK зв'язності, що допускає конформний солітон Річчі.

Ключові слова і фрази: солітон Річчі, конформний солітон Річчі, зв'язність Схаутен-ван Кампена, тривимірний транс-Сасакяновий многовид.