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Topological transitivity of translation operators in a non-separable Hilbert space

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We consider a Hilbert space of entire analytic functions on a non-separable Hilbert space, associated with a non-separable Fock space. We show that under some conditions operators, like the differentiation operators and translation operators, are topologically transitive in this space.

Key words and phrases: topologically transitive operator, hypercyclic operator, function space.

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Introduction

Linear dynamics is popular and new branch of mathematics. Linear dynamics is closely related to hypercyclic and topologically transitive operators [12]. We introduce the definition of topologically transitive operator.

Definition 1. *Let X be a Hausdorff locally convex space. A continuous operator $T\colon X \to X$ is called topologically transitive if for each pair U, V of non-empty open subsets of X there is some $n \in \mathbb{N}$ with $T^n(U) \cap V \neq \varnothing$.*

If the underlying space is a separable Baire space, transitivity is equivalent to hypercyclicity.

Definition 2. *A continuous linear operator $T\colon X \to X$ acting on a separable Fréchet space X is called hypercyclic if there is a vector $x \in X$ for which the orbit under T,*

$$\mathrm{Orb}(T, x) = \left\{x, Tx, T^2x, \dots\right\}$$

is dense in X.

Every such vector x is called a *hypercyclic vector* of T. Orbits can be quite complicated sets, even for very simple nonlinear mappings. However, there are some orbits which are especially simple and play a central role in the study of the dynamical system. We can suppose that $\mathrm{Orb}(T, x)$ is an at most countable set.

The assumption of separability for the existence of a hypercyclic operator is necessary since if X is not separable, then X has no countable dense subset and hence, $\mathrm{Orb}(T, x)$ cannot be dense in X in according to the operator T and vector x chosen. Non-separable spaces do not support hypercyclic operators.

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The weighted unilateral backward shift or also known as the Rolewicz operator was one of the first hypercyclic operators found in 1960's. It was shown that no linear operator is hypercyclic on any finite-dimensional space [21]. S. Rolewich posed the problem: *does an infinite-dimensional space support a hypercycic operator?* Results of S. Ansari [1] and L. Bernal-Gonzalez [5] state that every infinite-dimensional separable Banach space supports a hypercyclic operator.

An approach to obtain something similar to hypercyclicity in a non-separable Hilbert and Banach spaces was given by K. Chan [10] and A. Montes and C. Romero [17], respectively. Working on non-separable Banach spaces it is clear that the definition of hypercyclicity has to be replaced by the topological transitivity. Since every separable Banach space admits hypercyclic and hence topological transitive operators it is natural to ask whether every non-separable Banach space admits a topological transitive operator? However there exist several examples of Banach spaces which do not admit topologically transitive operator. For Hilbert spaces the latter question is true as shown in [4] by T. Bermudez and N.J. Kalton. Some non-separable spaces such as the space $L(\ell^2)$ of all bounded linear operators on ℓ_2, or the space ℓ^∞ do not support any topological transitive operator. This matter of fact leads to modifications which are for replacing the norm-topology by the weak star topology on ℓ^∞ and considering then hypercyclic operators on ℓ^∞, see [6].

A lot of results on hypercyclicity has been based on the Kitai-Gethner-Shapiro theorem (so-called Hypercyclicity Criterion). In [4] T. Bermudez and N.J. Kalton pointed that by the same way it is possible to prove the similar Criterion holds for topologically transitive operators. Here we present this theorem with a proof that is quite analogous to the proof of the hypercyclicity criterion (see, e.g., [3, p. 5]).

Theorem 1 (Topologically transitive criterion)**.** *Let T be a bounded linear operator on a complex Banach space X (not necessarily separable). Suppose that there exists a strictly increasing sequence $\{n_k\}$ of positive integers for which there are:*

(i) a dense subset $Z_0 \subset X$ such that $T^{n_k}(x) \to 0$ for every $x \in Z_0$ as $k \to \infty$;

(ii) a dense subset $Y_0 \subset X$ and a sequence of mappings (not necessary linear and not necessary continuous) $S_k \colon Y_0 \to X$ such that $S_k(y) \to 0$ for every $y \in Y_0$ and $T^{n_k} \circ S_k(y) \to y$ for every $y \in Y_0$ as $k \to \infty$.

Then T is topologically transitive.

Proof. Let U and V be open and nonempty subsets of X. Pick $x \in U \cap Z_0$, $y \in V \cap Y_0$ and $\varepsilon > 0$ so that $B(x, \varepsilon) \subset U$ and $B(y, 2\varepsilon) \subset V$. By *(i)* and *(ii)* there exists n_k, arbitrarily large, satisfying

$$T^{n_k}(x) \in B(0, \varepsilon),$$

(that is, the fact that T^{n_k} tends point-wise to zero on a dense subset of X insures that $T^{n_k}(x) \cap B(0, \varepsilon)$ is nonempty for all sufficiently large k), also $S_k(y) \in B(0, \varepsilon)$.

So $x + S_k(y) \in U$ and

$$T^{n_k}(x + S_k y) = T^{n_k}(x) + T^{n_k} S_k(y) \to 0 + y \in V.$$

Hence, $T^{n_k}(x + S_k y) \in V$ for sufficiently large k. This means $T^{n_k}(U) \cap V \neq \varnothing$. Thus, T is topologically transitive. $\square$

In 1929, G.D. Birkhoff proved that the translation operator $f(x) \mapsto f(x+a)$ is hypercyclic on the Fréchet space of entire functions on the complex plane $\mathbb{C}$ for any nonzero complex number a (see [7]). This result was generalized by many authors for composition operators in spaces of analytic functions in finite-dimensional spaces [8, 9, 13], and in infinite-dimensional Banach spaces [2, 11, 16, 18–20, 22].

In this paper, we generalize results of [18] for non-separable Hilbert spaces.

1 Non-separable function Hilbert space

In this section, we consider a general construction of a non-separable function Hilbert space as the dual space to the ℓ_2-sum of symmetric Hilbertian tensor products (see [14, 15, 18] for the separable case). Let E be a complex non-separable Hilbert space with an orthonormal basis $(e_i)_{i\in\mathcal{I}}$, where $\mathcal{I}$ is an uncountable set. E is endowed with the scalar product $(x|y)_E$ and the norm $\|x\|_E = (x \mid x)_E^{1/2}$, $x, y \in E$. Clearly, for every $n \in \mathbb{N}$ the nth tensor power $\otimes^n E$ is defined to be complex linear span of elements

$$\{x_1 \otimes \cdots \otimes x_n \colon \quad x_1, \ldots, x_n \in E\}.$$

It is well-known that it is possible to define a norm $\|\cdot\|_{\otimes_h^n E}$ on the vector space $\otimes^n E$ such that the corresponding completion $\otimes_h^n E$ is a Hilbert space. More exactly, the scalar product on $\otimes_h^n E$ is defined by the equality

$$\langle x_1 \otimes \cdots \otimes x_n \mid y_1 \otimes \cdots \otimes y_n \rangle_{\otimes_h^n E} = (x_1 \mid y_1)_E \cdots (x_n \mid y_n)_E$$

for all $x_i, y_i \in E$, $i = 1, \ldots, n$. Let $[i]$ denotes a multi-index $(i_1, \ldots, i_n)$, $i_k \in \mathcal{I}$. We denote by $\mathcal{M}$ the set of all multi-indexes $[i]$. Since the system

$$\left\{e_{i_1} \otimes \cdots \otimes e_{i_n} \in \otimes^n E : [i] \in \mathcal{M}\right\}$$

forms an orthonormal basis in $\otimes_h^n E$, every such vector $v \in \otimes_h^n E$ can be represented by the Fourier series expansion

$$v = \sum_{[i]\in\mathcal{M}} \alpha_{[i]} e_{i_1} \otimes \cdots \otimes e_{i_n},$$

and we put

$$\|v\|_{\otimes_h^n E} = \langle v \mid v \rangle_{\otimes_h^n E}^{1/2} = \left(\sum_{[i]\in\mathcal{M}} |\alpha_{[i]}|^2 \right)^{1/2}.$$

It is clear that the above norm, generated by the scalar product, is a cross-norm on $\otimes_h^n E$, that is,

$$\|x_1 \otimes \cdots \otimes x_n\|_{\otimes_h^n E} = \|x_1\|_E \cdots \|x_n\|_E.$$

We denote by $\otimes_s^n E$ the n-fold symmetric algebraic tensor product of space E. Every element from $\otimes_s^n E$ can be defined by formula

$$x_1 \otimes_s \cdots \otimes_s x_n := \frac{1}{n!} \sum_{\sigma\in S_n} x_{\sigma(1)} \otimes \cdots \otimes x_{\sigma(n)},$$

where $x_1, \ldots, x_n \in E$ and S_n is the group of permutations on the set $\{1, \ldots, n\}$.

We will use the following notation $e_i^{\otimes k} = \underbrace{e_i \otimes \cdots \otimes e_i}_{k}$ for any $k, i \in \mathbb{N}$. We denote by (k) an arbitrary multi-index $(k_1, \ldots, k_n) \in \mathbb{Z}_+^n$, and $\left|(k)\right| = \sum_i k_i$, $(k)! = \prod_i k_i!$.

The vectors

$$\left\{ e_{[i]}^{\otimes(k)} := e_{i_1}^{\otimes k_1} \otimes_s \cdots \otimes_s e_{i_n}^{\otimes k_n} : \quad [i] \in \mathcal{M}, \quad (k) \in \mathbb{Z}_+^n, \quad \left|(k)\right| = n \right\}$$

form an orthogonal basis in the closure $\otimes_{s,h}^n E$ of $\otimes_s^n E$ in $\otimes_h^n E$ and

$$\left\| e_{[i]}^{\otimes(k)} \right\|_{\otimes_h^n E} = \sqrt{\frac{(k)!}{n!}}, \qquad n = \left|(k)\right|.$$

Hermitian dual of a Hilbert space E we can define by the relation

$$E^* = \left\{ y^* := \langle \cdot | y \rangle_E : y \in E \right\}.$$

Note that the classical symmetric Fock space $\mathcal{F}$ is the Hilbert direct sum of $\otimes_{s,h}^n E$, $n = 0, 1, \ldots$, where $\otimes_{s,h}^0 E = \mathbb{C}$.

We say that a Hilbert space $\mathcal{F}_\eta$ with an arbitrary Hilbert norm $\| \cdot \|_\eta$ is a *(generalized) symmetric Fock space over a given Hilbert space E* if vectors

$$1,\ e_{[i]}^{(k)} = e_{i_1}^{\otimes k_1} \otimes_s \cdots \otimes_s e_{i_n}^{\otimes k_n}, \qquad k \in \mathbb{N}, \quad k_1 + \cdots + k_n = n, \quad i_1 < \ldots < i_n,$$

form an orthogonal basis in $\mathcal{F}_\eta$. Thus $\mathcal{F}_\eta$ can be represented by the Hilbert direct sum of symmetric tensor powers

$$\mathcal{F}_\eta = \mathbb{C} \oplus E \oplus \otimes_s^2 E \oplus \cdots \oplus \otimes_s^n E \oplus \cdots .$$

Evidently, the norm $\| \cdot \|_\eta$ is completely defined by its value on the basis vectors. Hence, setting $\left\| e_{[i]}^{\otimes(k)} \right\|_\eta$ by arbitrary positive numbers, we can get various symmetric Fock spaces over E. Let $\langle \cdot \mid \cdot \rangle_\eta$ be the scalar product in $\mathcal{F}_\eta$.

Put $c_{[i]}^{(k)} := \left\| e_{[i]}^{(k)} \right\|_\eta^{-2}$ and $c_0 = 1$. Let us consider a power series

$$\eta(x) = \sum_{k_1 + \cdots + k_n = 0}^{\infty} \sum_{i_1 < \cdots < i_n} c_{i_1 \ldots i_n}^{k_1 \ldots k_n} x_{i_1}^{k_1} \ldots x_{i_n}^{k_n} e_{i_1}^{\otimes k_1} \ldots e_{i_n}^{\otimes k_n} = \sum_{|(k)|=0}^{\infty} \sum_{[i] \in \mathcal{M}} c_{[i]}^{(k)} x_{[i]}^{(k)} e_{[i]}^{\otimes(k)} \tag{1}$$

for any $x = \sum_{i=1}^{\infty} x_i e_i \in E$.

It is known (see [18]), that there are constants $c > 0$ and $M > 0$ such that for all multi-indexes $[i] \in \mathcal{M}$, $(k) \in \mathbb{Z}_+^n$ and $n = k_1 + \cdots + k_n$ inequalities

$$0 < c_{[i]}^{(k)} = c_{i_1 \ldots i_n}^{k_1 \ldots k_n} \leq c M^{2n} \frac{(k_1 + \cdots + k_n)!}{k_1! \cdots k_n!} = c M^{2n} \frac{n!}{k_1! \cdots k_n!} \tag{2}$$

hold. Then there exists an open subset $U \subset E$, $U \ni 0$, such that for every $\varphi \in \mathcal{F}_\eta$ the map $f_\varphi(x) = \langle \eta(x) \mid \varphi \rangle_\eta$ is an analytic function on U. The function $\left\langle \eta(x) \middle| e_{[i]}^{\otimes(k)} \right\rangle_\eta$ is an n-homogeneous polynomial and

$$\left\langle \eta(x) \middle| e_{[i]}^{\otimes(k)} \right\rangle_\eta = x_{i_1}^{k_1} \cdots x_{i_n}^{k_n}.$$

Let us denote by $\mathcal{H}_\eta(E)$ the Hilbert space of analytic function $f_\phi = \langle \eta(\cdot) \mid \phi \rangle_\eta$ that is Hermitian dual to $\mathcal{F}_\eta(E)$.

2 Topologically transitive composition operator on Hilbert space

In this section, we consider the case when $\mathcal{H}_\eta(E) = \mathcal{F}^*_\eta(E)$ consists with entire functions on E.

Let us consider translation operator $T_a\colon \mathcal{H}_\eta(E) \to \mathcal{H}_\eta(E)$, defined as

$$T_a\colon f(x) \mapsto f(x+a), \quad x, a \in E, \quad a \neq 0.$$

Since $\mathcal{H}_\eta(E)$ is not separable space, T_a is not hypercyclic. Our purpose is to find conditions when T_a is topologically transitive.

Let us define a differentiation operator D_a on $\mathcal{H}_\eta(E)$ by

$$D_a\big(f(x)\big) = \sum_{i=1}^{\infty} a_i \frac{\partial}{\partial x_i} f(x),$$

where $a = (a_i) \in \ell_2$, $x \in E$. If we write

$$f = \sum_{n=0}^{\infty} f_n = \left(f^0, f^1, \dots, f^k, \dots\right),$$

then D_a can be considered as a weighted backward shift.

Let us denote a weight of the differentiation operator D_a by $\gamma_{k_j} = \dfrac{k_j\sqrt{c^{(k)}_{[i]}}}{\sqrt{c^{\left(k^{j'}\right)}_{[i]}}}$. Thus,

$$D_a f = \sum_{t,k_j} \sum_{(k),[i]} a_t \cdot \gamma_{k_j} \left\langle f \middle| \mathcal{E}^{(k)}_{[i]} \right\rangle_\eta \mathcal{E}^{\left(k^{j'}\right)}_{[i]},$$

where coefficients $c^{(k)}_{[i]}$ are defined in (1), $n \in \mathbb{N}$, $[i] \in \mathcal{M}$, $(k) \in \mathbb{Z}^n_+$, $n = \big|(k)\big|$, $\left(k^{j'}\right) = (k_1, k_2, \dots, k_j - 1, \dots, k_n)$, $j = 1, \dots, n$,

$$\mathcal{E}^{(k)}_{[i]}(x) = \frac{\left(x \middle| e^{\otimes(k)}_{[i]}\right)}{\left\| e^{\otimes(k)}_{[i]} \right\|_\eta} = \left(x \middle| e^{\otimes(k)}_{[i]}\right) \sqrt{c^{(k)}_{[i]}} = \sqrt{c^{(k)}_{[i]}}\, x^{(k)}_{[i]} = \sqrt{c^{k_1}_{i_1} \dots c^{k_n}_{i_n}}\, x^{k_1}_{i_1} \dots x^{k_n}_{i_n}.$$

Let us suppose, in sequel, that the set $\left\{ \dfrac{k_j\sqrt{c^{(k)}_{[i]}}}{\sqrt{c^{\left(k^{j'}\right)}_{[i]}}} \right\}$ is bounded. Then the operator D_a is continuous on $\mathcal{H}_\eta(E)$ [18]. Moreover, we will suppose that for every function of exponential type $\Psi = \sum_{n=0}^\infty c_n t^n$ and linear functional $\psi \in E^*$ the element $\Psi(\psi)$ belongs to $\mathcal{H}_\eta(E)$. From [18] it follows that this condition holds, for example, if $\eta = \sum_{n=0}^\infty \frac{x^{\otimes n}}{(n!)^2}$.

We denote

$$\widehat{\Psi}(D_a) f(x) = \sum_{n=0}^{\infty} c_n D_a^n f(x),$$

where D_a^n is the nth power of D_a and so $D_a^n f(x)$ is the nth Frèchet derivative of f at the point $x \in E$ towards a, $a \in E$, and c_n are coefficients of function of exponential type Ψ.

Theorem 2. *Let E be a Hilbert space and $\Psi = \sum_{n=0}^{\infty} c_n t^n$ be a nonconstant function of exponential type. If D_a is continuous, then the operator*

$$\widehat{\Psi}(D_a)\colon \mathcal{H}_\eta(E) \to \mathcal{H}_\eta(E),$$
$$f(x) \mapsto \sum_{n=0}^{\infty} c_n D_a^n f(x)$$

is topologically transitive.

The following lammas were proved in [18] for a separable Hilbert spaces E. However they are obviously true for the non-separable case as well.

Lemma 1. *$\mathcal{B} = \{e^{\psi}\colon \psi \in E^*, e^{\psi} \in \mathcal{H}_\eta\}$ is a linearly independent subset of $\mathcal{H}_\eta$.*

Lemma 2. *Let U be a non-empty open subset of a ball in E^* with radius δ and center at 0. Suppose that $e^{\psi} \in \mathcal{H}_\eta(E)$ for every $\psi \in U$. Then $S = \operatorname{span}\{e^{\psi} : \psi \in U\}$ is dense in $\mathcal{H}_\eta(E)$.*

Proof. Let a be fixed element in E. Consider the function $p\colon E^* \to \mathbb{C}$ defined by

$$p(\psi) = \sum_{n=1}^{\infty} c_n \psi^n(a),$$

where $n \geq 0$.

Since $\widehat{\Psi}$ is a function of exponential type, there exists $R > 0$ so that $|c_n| \leq \frac{R^n}{n!}$, $n \geq 1$.

Given $\psi \in E^*$ with $\|\psi\| \leq 1$, we have

$$|c_n| \cdot |\psi^n(a)| \leq \frac{R^n}{n!}\|\psi\|^n\|a\|^n \leq \left(\frac{e\|a\|R}{n}\right)^n,$$

and so

$$\Big(|c_n| \cdot \big|\psi^n(a)\big|\Big)^{\frac{1}{n}} \to 0, \quad n \to \infty.$$

It is clear that $p\colon E^* \to \mathbb{C}$ is a continuous and nonconstant function. So the sets

$$U\colon = \Big\{\psi \in E^*\colon \big\|p(\psi)\big\| < 1\Big\}, \quad V := \Big\{\psi \in E^*\colon \big\|p(\psi)\big\| > 1\Big\},$$

where $\big\|p(\psi)\big\| = \left|\sum_{n=1}^{\infty} c_n \psi^n(a)\right|$, are both open and non-empty. Hence, according to Lemma 2,

$$X_0 = \operatorname{span}\{e^{\psi}\colon \psi \in U\}, \quad Y_0 = \operatorname{span}\{e^{\psi}\colon \psi \in V\} \tag{3}$$

are both dense subspaces of $\mathcal{H}_\eta$. Next, notice that if $T = \widehat{\Psi}(D_a)$, given $\psi \in E^*$ we get

$$T(e^{\psi}) = \sum_{n=0}^{\infty} c_n D_a^n(e^{\psi}) = \sum_{n=0}^{\infty} c_n \psi^n(a) e^{\psi} = p(\psi)e^{\psi}.$$

By (3),

$$T^n \to 0 \text{ for } n \to \infty \text{ pointwise on } X_0.$$

Also, by Lemma 1 there exists a linear map $S\colon Y_0 \to Y_0$ determined by

$$S(e^{\psi}) = \big[p(\psi)\big]^{-1} e^{\psi}, \quad \psi \in E^*, \tag{4}$$

which by (3) and (4) satisfies

$$\begin{cases} S^n \to 0 & \text{for } n \to \infty \text{ pointwise on } Y_0, \\ TS = \mathrm{id}_{Y_0} & \text{on } Y_0. \end{cases}$$

By Theorem 1, $T = \widehat{\Psi}(D_a)$ is topologically transitive. $\square$

We note that $\widehat{\Psi}(D_a) = e^{D_a}$.

Corollary 1. *Let E be a Hilbert space and*

$$T_a \colon \mathcal{H}_\eta(E) \to \mathcal{H}_\eta(E),$$

where $\mathcal{H}_\eta(E)$ as in Theorem 2, and

$$T_a \colon f(x) \mapsto f(x+a).$$

Then

$$T_a = \sum_{n=0}^{\infty} \frac{1}{n!} D_a^n$$

and so T_a is topologically transitive.

References

[1] Ansari S.I. *Existence of hypercyclic operators on topological vector spaces*. J. Funct. Anal. 1997, **148** (2), 384–390. doi:10.1006/JFAN.1996.3093

[2] Aron R.M., Bès J. *Hypercyclic differentiation operators*. Contemporary Math. 1999, **232**, 39–46. doi:10.1090/conm/232/03382

[3] Bayart F., Matheron E. Dynamics of linear operators. In: Bertoin J. (Ed.) Cambridge Tracts in Mathematics, 179. Cambridge University Press, New York, 2009.

[4] Bermúdez T., Kalton N.J. *The range of operators on von Neumann algebras*. Proc. Amer. Math. Soc. 2002, **130** (5), 1447–1455.

[5] Bernal-González L. *On hypercyclic operators on Banach spaces*. Proc. Amer. Math. Soc. 1999, **127**, 1003–1010. doi:10.1090/S0002-9939-99-04657-2

[6] Bes J., Chan K.C., Sanders R. *Weak-Star Hypercyclicity and Supercyclicity of Shifts on l^∞*. Integral Equations Operator Theory 2006, **55**, 363–376.

[7] Birkhoff G.D. Démonstration d'un théorème élémentaire sur les fonctions entières. Gauthier-Villars, Paris 1929.

[8] Bourdon P.S., Shapiro J.H. *Cyclic phenomena for composition operators*. Mem. Amer. Math. Soc. 1997, **125** (596), 1–46. doi:10.1090/memo/0596

[9] Chan K.C., Shapiro J.H. *The cyclic behavior of translation operators on Hilbert spaces of entire functions*. Indiana Univ. Math. J. 1991, **40**, 1421–1449.

[10] Chan K.C. *Hypercyclicity of the operator algebra for a separable Hilbert space*. J. Operator Theory 1999, **42**, 231–244.

[11] Chernega I., Holubchak O., Novosad Z., Zagorodnyuk A. *Continuity and hypercyclicity of composition operators on algebras of symmetric analytic functions on Banach spaces*. Eur. J. Math. 2020, **6**, 153–163. doi:10.1007/s40879-019-00390-z

[12] Devaney L.R. An introduction to chaotic dynamical systems. Addison-Wesley, Reedwood City, 1989.

[13] Godefroy G., Shapiro H.J. *Operators with dense, invariant, cyclic vector manifolds*. J. Funct. Anal. 1991, **98** (2), 229–269. doi:10.1016/0022-1236(91)90078-J

[14] Lopushansky O., Zagorodnyuk A. *Hardy type spaces associated with compact unitary groups*. Nonlinear Anal. 2011, **74** (2), 556–572. doi:10.1016/j.na.2010.09.009

[15] Lopushansky O., Zagorodnyuk A. *Representing measures and infinite-dimensional holomorphy*. J. Math. Anal. Appl. 2007, **333** (2), 614–625.

[16] Manoussos A. *A Birkhoff type transitivity theorem for non-separable completely metrizable spaces with applications to linear dynamics*. J. Operator Theory 2013, **70** (1), 165–174. doi:10.7900/jot.2011may12.1971

[17] Montes-Rodríguez A., Romero-Moreno C. *Supercyclicity in the operator algebra*. Studia Math. 2002, **150**, 201–213. doi:10.4064/sm150-3-1

[18] Mozhyrovska Z.H., Zagorodnyuk A.V. *Hypercyclic behavior of translation operators on spaces of analytic functions on Hilbert spaces*. J. Funct. Spaces 2015, **2015**, article ID 139289. doi:10.1155/2015/139289

[19] Novosad Z., Zagorodnyuk A. *Analytic Automorphisms and Transitivity of Analytic Mappings*. Mathematics 2020, **8** (12), 2179. doi:10.3390/math8122179

[20] Novosad Z., Zagorodnyuk A. *Polynomial automorphisms and hypercyclic operators on spaces of analytic functions*. Arch. Math. 2007, **89**, 157–166. doi:10.1007/s00013-007-2043-4

[21] Rolewicz S. *On orbits of elements*. Studia Math. 1969, **32** (1), 17–22.

[22] Zagorodnyuk A., Novosad Z. *Topological Transitivity of Shift Similar Operators on Nonseparable Hilbert Spaces*. J. Funct. Spaces 2021, **2021**, article ID 9306342. doi:10.1155/2021/9306342

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У статті розглянуто гільбертів простір цілих аналітичних функцій на несепарабельному гільбертовому просторі, асоційованому з деяким несепарабельним простором Фока. Показано, що за деяких умов такі оператори, як оператор диференціювання та оператор зсуву, будуть топологічно транзитивними в цьому просторі.

Ключові слова і фрази: топологічно транзитивний оператор, гіперциклічний оператор, функційний простір.