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Majorization result for a certain class of meromorphic functions

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In this article, we investigate majorization results for a class of meromorphic univalent functions of complex order. By utilizing differential operators, various subclasses of analytic and meromorphic functions have been previously defined and studied. Our research focuses on majorization properties within this specific class. Because the q-calculus (quantum calculus) has applications in many mathematical fields, the present study aims to improve and generalize majorization results for the class of meromorphic functions combined by q-differential operator while presenting a valid form of previously reported results. Additionally, advances in the area under study were achieved, and various novel implications of the main conclusion presented as corollaries were provided. We also highlight some new or known consequences of our results in the form of corollaries. In particular, some of early known results are improved and refined.

Key words and phrases: univalent function, majorization, q-derivative operator, starlike and convex function, q-starlike function, subordination.

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1 Introduction and Preliminaries

Because the q-calculus (quantum calculus) has applications in many mathematical fields, its study has inspired and intrigued many researchers. The results of quantum calculus combined with the methods of the complex analysis are often used to investigate many structures of the function theory, while the q-calculus was utilized in many other areas of mathematics. Thus, new families of analytic functions have been recently defined and studied by a number of researchers, using the q-calculus operators (for example, see [18,19] and the references therein).

Definition 1 ([13])**.** *Let the functions f and g be analytic in an open subset G of the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$. We say that f is majorized by g in G and write $f(z) \ll g(z)$, $z \in G$, if there exists a function φ analytic in $\mathbb{D}$, such that $|\varphi(z)| \le 1$ for all $z \in \mathbb{D}$ and*

$$f(z) = \varphi(z) g(z), \quad z \in G.$$

If the functions f and g are analytic in $\mathbb{D}$, the function f is called to be *subordinate* to the function g, written $f(z) \prec g(z)$, if there exists a function w analytic in $\mathbb{D}$ with $|w(z)| < 1$, $z \in \mathbb{D}$, and $w(0) = 0$, such that $f = g \circ w$. According to the *Schwarz lemma* (see [2, Theorem 13]

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and [9, Lemma 1; Theorem 1]) it follows that $f(z) \prec g(z)$ implies $f(\mathbb{D}) \subset g(\mathbb{D})$, and if g is univalent in $\mathbb{D}$, then the following equivalence relationship

$$f(z) \prec g(z) \iff f(0) = g(0) \text{ and } f(\mathbb{D}) \subset g(\mathbb{D})$$

holds (see [16, Lemma 2.2]).

Let $\mathcal{M}$ denote the class of all functions of the form

$$f(z) = \frac{1}{z} + \sum_{k=0}^{\infty} a_k z^k, \tag{1}$$

that are analytic in the punctured open unit disk $\mathbb{D}^* := \{z \in \mathbb{C} : 0 < |z| < 1\} = \mathbb{D} \setminus \{0\}$, hence all the functions of the class $\mathcal{M}$ are meromorphic in $\mathbb{D}$ with the simple pole $z_0 = 0$.

In a subset $\mathbb{E} \subset \mathbb{C}$, for a function k and $q \in (0,1)$ the *q-derivative operator* is expressed (see [10, 11]) by

$$D_q k(z) := \begin{cases} \dfrac{k(qz) - k(z)}{(q-1)z}, & \text{if } z \neq 0, \\ k'(0), & \text{if } z = 0, \end{cases}$$

and if k is differentiable in $\mathbb{E}$, then $\lim_{q \to 1^-} D_q k(z) = k'(z)$, $z \in \mathbb{E}$. From the above definition, if $k \in \mathcal{M}$ has the form (1), then it is easy to check that

$$D_q k(z) = -\frac{1}{qz^2} + \sum_{k=1}^{\infty} [k]_q a_k z^{k-1}, \quad z \in \mathbb{D}^*,$$

where the *q-integer* $[n]_q$ defined by (see, for details, [10, 11] and also [4])

$$[n]_q := \begin{cases} \sum_{k=0}^{n-1} q^k, & \text{if } n \in \mathbb{N} := \{1, 2, \dots\}, \\ 0, & \text{if } n = 0, \\ \dfrac{1-q^n}{1-q}, & \text{if } n \in \mathbb{C}. \end{cases}$$

The class $\mathcal{S}^*(\alpha) \subset \mathcal{M}$ is called the subclass *meromorphic starlike functions of order* α in $\mathbb{D}^*$ if

$$\operatorname{Re}\left(-\frac{zf'(z)}{f(z)}\right) > \alpha, \quad z \in \mathbb{D}, \ 0 \leq \alpha < 1,$$

and we denote by $\mathcal{S}^*$ the class which consists of all *meromorphic starlike functions* of order $\alpha = 0$. C. Pommerenke [15], J. Clunie [7], W.C. Royster [17], and others have thoroughly studied this class and other related subclasses.

The families of meromorphic functions of complex order was introduced in [12] by using the q-derivative of a function $k \in \mathcal{A}$. For $\gamma = 1$ in [12, Definition 1] we get the following.

Definition 2. *Let χ be an analytic function in $\mathbb{D}$ with positive real part in $\mathbb{D}$, and $\chi(0) = 1$. A function $k \in \mathcal{M}$ is said to be in the subclass $\mathcal{MS}^q(\chi)$ of meromorphic functions if*

$$\mathcal{MS}^q(\chi) := \left\{k \in \mathcal{M} : -\frac{zqD_q k(z)}{k(z)} \prec \chi(z)\right\}.$$

T.H. MacGregor [13] has studied interesting problems connecting the normalized families of starlike functions and the majorization notion. Furthermore, majorization results for multivalent or univalent functions have been recently explored by many authors (see, for example, [3,6]).

In this paper, we will study majorization issue for the family $\mathcal{MS}^q(\chi)$ of meromorphic functions combined with q-differential operator. We also discuss new implications and corollaries arising from our results. In particular, the reported results in [12] are improved and refined.

2 Main results

The next lemma will be used in the proof of our first main result and then we will investigate majorization problem for the category $\mathcal{MS}^q(\chi)$ and represents the q-analogue form of Z. Nehari's result (see [14, p. 168]).

Lemma 1 ([1, Lemma 2.1]). *Let ω be analytic in $\mathbb{D}$, such that $|\omega(z)| < 1$, $z \in \mathbb{D}$. Then*

$$\left|D_q(\omega(z))\right| \leq \frac{\left|1-\overline{\omega(zq)}\omega(z)\right|}{1-|z|^2q}, \quad z \in \mathbb{D}, \ \ 0 < q < 1.$$

Taking $q \to 1^-$ in Lemma 1 we obtain the next well-known result of Z. Nehari.

Corollary 1 ([14, p. 168]). *If ω is analytic in $\mathbb{D}$, such that $|\omega(z)| < 1$, $z \in \mathbb{D}$, then*

$$|\omega'(z)| \leq \frac{1-|\omega(z)|^2}{1-|z|^2}, \quad z \in \mathbb{D}.$$

Now, we investigate majorization problem for the subclass $\mathcal{MS}^q(\chi)$.

Theorem 1. *Let l be analytic in $\mathbb{D}^*$ with $l \not\equiv 0$, and let $h \in \mathcal{MS}^q(\chi)$. If l is majorized by h in $\mathbb{D}^*$ such that $l \not\equiv c\,h$ with $|c| = 1$, and $q \in \left(\frac{(1-\eta)\rho}{(1-\eta)\rho+1+\eta}, 1\right)$, then*

$$\left|qzD_q l(z)\right| \leq \left|qzD_q h(z)\right|, \quad 0 < |z| = r \leq r^*,$$

where r^* is the positive root of the equation

$$(1-\eta)\rho q r^2 + (1+\eta)qr - (1-\eta)\rho = 0 \tag{2}$$

with $\eta = \eta(r,q) := \max\limits_{|\zeta|=qr} |\mu(\zeta)|$ and $\rho := \min\limits_{|z|=r} |\chi(z)|$. The function μ is that realize the majorization $l(z) \ll h(z)$ in $\mathbb{D}^*$ according to the Definition 1.

Proof. From the Definition 2, assuming that $h \in \mathcal{MS}^q(\chi)$, it follows that there exists a function λ analytic in $\mathbb{D}$ with $|\lambda(z)| < 1$, $z \in \mathbb{D}$ and $\lambda(0) = 0$, such that $-\dfrac{zqD_q h(z)}{h(z)} = \chi(\lambda(z))$, $z \in \mathbb{D}$, or

$$-\frac{h(z)}{D_q h(z)} = \frac{zq}{\chi(\lambda(z))}, \quad z \in \mathbb{D}. \tag{3}$$

Using the *Schwarz lemma*, we get $|\lambda(z)| \leq |z| = r < 1$, $z \in \mathbb{D}$, and because χ has positive real part in $\mathbb{D}$ we obtain $\chi(z) \neq 0$, $z \in \mathbb{D}$. Hence the *minimum modulus principle* (see [2, Theorem 12]) leads to the inequality

$$\min_{|z|=r} |\chi(z)| = \min_{|z|\leq r} |\chi(z)| \leq \min_{|\lambda(z)|\leq r} |\chi(\lambda(z))| = \min_{|\lambda(z)|=r} |\chi(\lambda(z))|.$$

Consequently, from (3) and using the above inequality, we obtain

$$\left|\frac{h(z)}{D_q h(z)}\right| = \frac{q|z|}{|\chi(\lambda(z))|} \leq \frac{qr}{\min\limits_{|z|=r}|\chi(z)|}, \quad 0<|z|\leq r<1. \tag{4}$$

From the assumption that l is majorized by h in $\mathbb{D}^*$, there exists an analytic function μ in $\mathbb{D}$ with $|\mu(z)| \leq 1$, $z \in \mathbb{D}$, satisfying $l(z) = \mu(z)h(z)$ for all $z \in \mathbb{D}^*$.

Using the q-derivative product rule for the previous relation (see [4]), we get

$$\begin{aligned} zqD_q l(z) &= zqD_q\mu(z)h(z) + zq\mu(qz)D_q h(z) \\ &= zqD_q h(z)\left(\mu(qz) + D_q\mu(z)\,\frac{h(z)}{D_q h(z)}\right), \quad z \in \mathbb{D}^*. \end{aligned} \tag{5}$$

From another side, using Lemma 1 for the function μ, we have

$$\left|D_q\mu(z)\right| \leq \frac{\left|1-\overline{\mu(zq)}\mu(z)\right|}{1-|z|^2 q}, \quad z\in\mathbb{D},\ 0<q<1. \tag{6}$$

According to the inequalities (4) and (6), from (5) we get

$$\begin{aligned} \left|zqD_q l(z)\right| &\leq \left|zqD_q h(z)\right|\left(|\mu(qz)| + \frac{rq}{\min\limits_{|z|=r}|\chi(z)|}\,\frac{\left|1-\overline{\mu(zq)}\mu(z)\right|}{1-|z|^2 q}\right) \\ &\leq \left|zqD_q h(z)\right|\left(|\mu(qz)| + \frac{rq}{\min\limits_{|z|=r}|\chi(z)|}\,\frac{1+|\mu(zq)|\,|\mu(z)|}{1-q|z|^2}\right), \quad 0<|z|\leq r<1. \end{aligned}$$

Let us denote

$$\begin{aligned} \eta = \eta(r,q) &:= \sup_{|z|<r}|\mu(qz)| = \max_{|\zeta|=qr}|\mu(\zeta)|, \\ \tau = \tau(r) &:= \sup_{|z|<r}|\mu(z)| = \max_{|\varsigma|=r}|\mu(\varsigma)|. \end{aligned} \tag{7}$$

We will prove that under the assumptions of the theorem, μ is a nonconstant function, hence $\eta, \tau \in (0,1)$ because of the following reasons.

(i) If $\eta\tau = 0$, then by using *the principle of the continuation of holomorphic functions* (see [5, Theorem 4.5.4]), we get $\mu \equiv 0$, that implies the trivial case $l \equiv 0$ excluded by the assumption of the theorem.

(ii) If $\eta = 1$ or $\tau = 1$, since $|\mu(z)| \leq 1$, $z \in \mathbb{D}$, it follows that $|\mu|$ attained its maximum in a point of $\mathbb{D}$ hence it is a constant of the form $\mu(z) = c$, $|c| = 1$. Consequently, we have $l(z) = c\,h(z)$, $z \in \mathbb{D}^*$, with $|c| = 1$, and this case was also excluded in our assumption.

From the above inequality we deduce

$$\left|zqD_q l(z)\right| \leq \left|zqD_q h(z)\right|\left(\eta + \frac{rq}{\min\limits_{|z|=r}|\chi(z)|}\,\frac{1+\eta\tau}{1-qr^2}\right), \quad 0<|z|\leq r<1,$$

where $\eta, \tau \in (0,1)$. If we define

$$m(r,\eta,\tau) := \eta + \frac{rq}{\min\limits_{|z|=r}|\chi(z)|}\,\frac{1+\eta\tau}{1-qr^2},$$

for determining r^* we must choose

$$r^* = \max\left\{r \in (0,1) : m(r,\eta,\tau) \leq 1,\ \eta,\tau \in (0,1)\right\}.$$

Since $m(r,\eta,\tau) \leq 1$ if and only if

$$0 \leq -(1+\eta\tau)rq + (1-\eta)\min_{|z|=r}|\chi(z)|\,(1-qr^2) =: n(r,\eta,\tau),$$

it follows that

$$\begin{aligned} n(r,\eta,\tau) &\geq \min\{n(r,\eta,\tau) : \eta,\tau \in [0,1]\} \\ &= \lim_{\tau\to 1^-} n(r,\eta,\tau) =: n(r,\eta) = -(1+\eta)rq + (1-\eta)\min_{|z|=r}|\chi(z)|\,(1-qr^2). \end{aligned}$$

Denoting $\rho = \rho(r) := \min\limits_{|z|=r}|\chi(z)|$, $0 < r < 1$, we have that $0 < \rho < 1$ because χ has positive real part in $\mathbb{D}$ and $\chi(0) = 1$. Since the value of η given by (7) is an arbitrary number with $\eta \in (0,1)$, we obtain

$$n(r) := -(1+\eta)qr + (1-\eta)\rho(1-qr^2), \quad r \in (0,1),$$

that is equivalent to the quadratic form

$$n(r) := -(1-\eta)\rho q r^2 - (1+\eta)qr + (1-\eta)\rho, \quad r \in (0,1), \tag{8}$$

where the parameters are $\eta \in (0,1)$, $\rho \in (0,1)$, $q \in (0,1)$. Therefore, the discriminant of this quadratic form is

$$\Delta = (1+\eta)^2 q^2 + 4(1-\eta)^2\rho^2 q > 0$$

for all the values of the above parameters, hence $n(r) = 0$ has two real and distinct zeroes r_1 and r_2. Using the *Viète's relations* we have

$$r_1\, r_2 = -\frac{1}{q} < 0,$$

hence $r_1 < 0$ and $r_2 > 0$. Since $\eta \in (0,1)$, $q \in \left(\frac{(1-\eta)\rho}{(1-\eta)\rho+1+\eta}, 1\right)$ and $\rho \in (0,1)$, we have

$$\lim_{r\to 0^+} n(r) = (1-\eta)\rho > 0, \qquad n(1) = (1-\eta)(1-q)\rho - (1+\eta)q < 0.$$

It follows that $n(r) = 0$ has only one positive root $r_2 \in (0,1)$. Therefore, $r^* := r_2$ and $n(r) \geq 0$ for all $r \in [0, r^*]$, where r^* is the positive zero of (8). □

Remark 1. *For the case $l \equiv 0$, the above theorem is trivial, and $r^* = 1$, i.e. the conclusion holds in $\mathbb{D}^*$.*

The other excepted case of Theorem 1, that is $l \equiv c\,h$ with $|c| = 1$, it is obtained for $\mu(z) = c$, $z \in \mathbb{D}$. Since $D_q\mu(z) = 0$, $z \in \mathbb{D}^*$, from (5) *we get*

$$zqD_q l(z) = zqD_q\mu(z)h(z) + zq\mu(qz)D_q h(z) = zqcD_q h(z), \quad z \in \mathbb{D}^*,$$

hence

$$\left|zqD_q l(z)\right| = \left|zqD_q h(z)\right|, \quad z \in \mathbb{D}^*.$$

Concluding, if h is analytic in $\mathbb{D}^*$ and $q \in (0,1)$, then $l(z) = ch(z) \ll h(z)$, $|c| = 1$, it implies

$$\left|zqD_q l(z)\right| = \left|zqD_q h(z)\right|, \quad z \in \mathbb{D}^*.$$

In the following corollaries we obtain a modified version of majorization problem for the different categories of $\mathcal{MS}^q(\chi)$ connected with the function χ of [12].

Corollary 2 ([12, Theorem 3])**.** *Let l be analytic in $\mathbb{D}^*$ with $l \not\equiv 0$, and let $h \in \mathcal{MS}^q_{\mathrm{SL}}(\gamma)$, where $\mathcal{MS}^q_{\mathrm{SL}}(\gamma) := \mathcal{MS}^q\big(1+\gamma(\sqrt{1+z}-1)\big)$ and $0<\gamma\le 1$.*

If l is majorized by h in $\mathbb{D}^*$ such that $l \not\equiv c\,h$ with $|c|=1$, and $q \in \left(\frac{(1-\eta)\rho}{(1-\eta)\rho+1+\eta}, 1\right)$, then

$$\left|qzD_q l(z)\right| \le \left|qzD_q h(z)\right|, \quad 0<|z|=r\le \widehat{r},$$

where $\widehat{r}$ is the positive root of the equation (2) *with $\eta=\eta(r,q):=\max_{|\zeta|=qr}|\mu(\zeta)|$ and $\rho := 1+\gamma\left(\sqrt{1-r}-1\right)$. Here the function μ is the one mentioned in Theorem 1.*

Proof. Let $M(z) := \sqrt{1+z}$, where $M(0)=1$, that is the square root function considered to the main branch. A simple computation shows that

$$\operatorname{Re}\left(1+\frac{zM''(z)}{M'(z)}\right) = \operatorname{Re}\frac{z+2}{2(z+1)} > \frac{3}{4} > 0, \quad z\in\mathbb{D},$$

hence M is a convex (univalent) function in $\mathbb{D}$. It follows that $\chi(z) := 1+\gamma\left(\sqrt{1+z}-1\right)$ is also a convex (univalent) function in $\mathbb{D}$, and because it has real positive part in $\mathbb{D}$ we could set this function in Theorem 1. Since $\overline{\chi(z)} = \chi(\overline{z})$, $z\in\mathbb{D}$, the domain $\chi(\mathbb{D})$ is symmetric with respect to the real axis, hence any disk $\{z\in\mathbb{C} : |z|<r<1\}$ will be mapped by the function χ onto a convex domain symmetric with respect to the real axis. Therefore,

$$\rho := \min_{|z|=r}|\chi(z)| = \chi(-r) = 1+\gamma\big(\sqrt{1-r}-1\big)$$

and our result follows from Theorem 1. □

Setting $\chi(z) := 1+\gamma\left(\dfrac{1+z}{1-z}-1\right)$, $0<\gamma\le 1$, from similar reasons as the above ones we get the following special case.

Corollary 3 ([12, Theorem 4])**.** *Let l be analytic in $\mathbb{D}^*$ with $l \not\equiv 0$, and let $h \in \mathcal{MS}^q_{\mathrm{S}}(\gamma)$, where $\mathcal{MS}^q_{\mathrm{S}}(\gamma) := \mathcal{MS}^q\left(1+\gamma\left(\frac{1+z}{1-z}-1\right)\right)$ with $0<\gamma\le 1$.*

If l is majorized by h in $\mathbb{D}^*$ such that $l \not\equiv c\,h$ with $|c|=1$, and $q \in \left(\frac{(1-\eta)\rho}{(1-\eta)\rho+1+\eta}, 1\right)$, then

$$\left|qzD_q l(z)\right| \le \left|qzD_q h(z)\right|, \quad 0<|z|=r\le \widetilde{r},$$

where $\widetilde{r}$ is the positive root of the equation (2) *with $\eta=\eta(r,q):=\max_{|\zeta|=qr}|\mu(\zeta)|$ and $\rho := 1+\gamma\left(\frac{1-r}{1+r}-1\right)$. Here the function μ is the one mentioned in Theorem 1.*

Corollary 4 ([12, Theorem 2])**.** *Let l be analytic in $\mathbb{D}^*$ with $l \not\equiv 0$, and let $h \in \mathcal{MS}^q_{\cos}(\gamma)$, where $\mathcal{MS}^q_{\cos}(\gamma) := \mathcal{MS}^q\big(1+\gamma(\cos z-1)\big)$ with $0<\gamma\le 1$.*

If l is majorized by h in $\mathbb{D}^*$ such that $l \not\equiv c\,h$ with $|c|=1$, and $q \in \left(\frac{(1-\eta)\rho}{(1-\eta)\rho+1+\eta}, 1\right)$, then

$$\left|qzD_q l(z)\right| \le \left|qzD_q h(z)\right|, \; 0<|z|=r\le r_{\cos}$$

where $r_{\cos}$ is the positive root of the equation (2) *with $\eta=\eta(r,q):=\max_{|\zeta|=qr}|\mu(\zeta)|$ and $\rho := 1+\gamma(\cos r-1)$. Here the function μ is the one mentioned in Theorem 1.*

Proof. Denote $N(z) := \cos z$ and set $\chi(z) := 1 + \gamma\,(N(z) - 1)$ in Theorem 1. If $z = x + iy \in \mathbb{D}$ is such that $x, y \in \mathbb{R}$, then we obtain $x^2 + y^2 < 1$. This implies $x, y \in [-1, 1]$, hence $\operatorname{Re} N(z) = \cos x \cosh y > 0$, $z \in \mathbb{D}$. Therefore, the function χ has real positive part in $\mathbb{D}$ for all $0 < \gamma \leq 1$, and because it is easy to check that $\overline{\chi(z)} = \chi\,(\overline{z})$, $z \in \mathbb{D}$, then the domain $\chi(\mathbb{D})$ is symmetric with respect to the real axis.

On the other hand, the inequality (3.1) from [8, Theorem 3.1] shows that

$$\left|\cos\left(re^{i\theta}\right)\right| \geq |\cos r|, \quad \theta \in [0, 2\pi),$$

and the equality holds if and only if $\theta \in \{0, \pi\}$, that implies $\min\limits_{|z|=r} |N(z)| = \cos r$, $0 \leq r < 1$. Therefore, we conclude that $\rho = \min\limits_{|z|=r} |\chi(z)| = \chi(r) = 1 + \gamma(\cos r - 1)$, and our result follows from Theorem 1. □

3 Conclusion

In the final section, we emphasize that we have presented a modified version of the majorization problem for the different categories of $\mathcal{MS}^q(\chi)$ connected with the function χ. Our study strength also lies in the originality of the results for this topic, which we provide through new approach. Furthermore, new developments in the field were made, and several innovative corollaries to the main conclusion were added. Additionally, the outcome provided in certain theorems can be applied in addition for the previously examined subclasses of meromorphic functions in the open unit disk.

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У цій статті досліджуються результати про мажорування для класу мероморфних однолистих функцій комплексного порядку. Використовуючи диференціальні оператори, раніше було визначено та вивчено різні підкласи аналітичних і мероморфних функцій. Наше дослідження зосереджене на властивостях мажорування в межах цього конкретного класу. Оскільки q-числення (квантове числення) має застосування в багатьох галузях математики, метою цієї роботи є покращення та узагальнення результатів про мажорування для класу мероморфних функцій, поєднаного з q-диференціальним оператором, а також подання коректної форми раніше опублікованих результатів. Крім того, було досягнуто нових результатів у досліджуваній галузі та наведено різноманітні нові наслідки основного результату у вигляді наслідків. Ми також подаємо деякі нові та відомі наслідки наших результатів. Зокрема, деякі з раніше відомих результатів були покращені та уточнені.

Ключові слова і фрази: однолиста функція, мажорування, оператор q-похідної, зіркоподібна та опукла функція, q-зіркоподібна функція, підпорядкування.