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Development of algorithms and software for studying the stability of complex power systems

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The mathematical model in this work describes both pendulum and power systems with multiple generators equally. As is well known, the industrial revolution led to an increase in energy consumption, the majority of which is now consumed in the form of electrical energy by modern society. Thus, this raises the issue of its transportation over long distances. The mathematical model of a modern power complex, consisting of turbo generators and complex interconnected energy blocks, represents a system of nonlinear ordinary differential equations. The task of optimizing the operation of these complexes, as well as developing algorithms for motion stability in such systems, continues to attract the attention of many researchers and remains highly relevant. The industrial development of modern society leads to a constant increase in electricity consumption. To meet the ever-growing demands power complexes are being created. When mathematically modeling such complexes, it is necessary to address a number of theoretical and practical issues. Ensuring the stability of motion is a critical issue at the design and operational stages of the systems under investigation. This work is devoted to the study of the asymptotic stability of the motion of phase systems.

Key words and phrases: Lyapunov functional, dynamic system, development of algorithms, control, robotics, electronics, technological progress.

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Introduction

Scientific, technological, and technical progress makes it necessary to investigate further the dynamics of multidimensional phase systems, which have important practical applications. A distinctive feature of phase systems is that they are described by differential equations, the right-hand sides of which are periodic with respect to angular coordinates. Phase systems are relevant to the study of the dynamics of pendulum systems, power systems, radio-navigation phase systems, and others. Ensuring the stability of motion, stabilizing motion, controllability,

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and optimality are crucial issues at the design and operational stages of the systems under investigation.

The stability of phase systems is a fundamental issue in the theory of dynamical systems and is important for various fields, including robotics, automatic control, biology and economics. The Lyapunov method is widely used to analyze the stability of systems. However, the development of effective algorithms and programs for the study of asymptotic Lyapunov stability of phase systems remains an urgent task. Asymptotic stability of systems is an important task in the field of dynamical systems and control research. The solution requires the development of effective algorithms and programs that allow us to study the properties of the system and obtain accurate predictions of its behavior. The Lyapunov asymptotic stability estimation makes it possible to determine whether the equilibrium of the system is maintained under small perturbations and what conditions must be met to ensure stability. This task has practical application in many fields, including automatic control, robotics, electronics and others. The development of algorithms and programs for the study of asymptotic Lyapunov stability of phase systems requires deep knowledge in the field of mathematical modeling, control theory and numerical methods. However, the development of effective tools for solving this problem can simplify and speed up the analysis of the system and help to obtain more accurate predictions of its behavior. This paper discusses the development of algorithms and programs for solving these problems, as well as their application in practical problems. Scientific, technological and technological progress makes it necessary to further study the dynamics of multidimensional phase systems with important practical applications. The mathematical model considered in this paper describes equally pendulum systems and electric power systems with many generators. The industrial revolution has led to an increase in energy consumption, the bulk of which is consumed by modern society in the form of electrical energy. Naturally, there is a question of transporting it over long distances [2]. The mathematical model of a modern electric power complex consisting of turbo generators and complex multi-connected power units is a system of nonlinear ordinary differential equations. The tasks of optimizing the functioning of these complexes, as well as the creation of motion stability algorithms for such systems, still attract the attention of many researchers and are relevant. The industrial development of modern society leads to a constant increase in electricity consumption. To meet these ever-growing needs, complex electric power complexes are being created. Mathematical modeling of such complexes requires solving a number of theoretical and practical issues. Ensuring the stability of movement is the most important problem at the design and operation stage of the systems under study.

1 Literature review

There are a number of works in the literature devoted to this topic, offering various methods and approaches to solving these problems.

The classical works devoted to the analysis of the stability of systems according to Lyapunov are [1,2]. In these papers, the authors describe in detail the principles and methods of stability analysis, including the use of Lyapunov functions and Lyapunov stability theorems. These works are important sources of information for understanding the basic principles and approaches to the analysis of the stability of systems. Due to the fact that the stability of the electrical energy systems depends very much on the accuracy of the representation of the parameters of the mechanisms, interval mathematics is applicable to guarantee the synchronicity of its operation. The papers [3, 4] present the properties of the introduced interval mathematics, which have found application in the study of the stability of various technical systems [5,6]. All of the above works and software tools represent valuable material and tools for the development of algorithms and programs for the study of asymptotic Lyapunov stability of phase systems, which allows researchers and engineers to more efficiently and accurately analyze the dynamics of systems and develop stable and optimal control strategies. These works are valuable resources for those interested in the application of numerical methods and software modeling for the analysis of dynamic systems. This direction has been studied by the authors for many years, the results of which can be seen in the works [7, 8]. One of the most common methods for studying stability and evaluating areas of attraction is the Lyapunov-Krasovsky method. In [9–11], the authors describe this method and propose algorithms and software implementations for its application. These works are important resources for researchers interested in the development of algorithms and programs for the study of stability and evaluation of areas of attraction. The study of determining the optimal parameters of optimal electric power systems is described in [12, 13]. Another important area in this field is the application of numerical methods and computer programs for stability analysis and evaluation of areas of attraction. In [14, 15], the authors propose methods for numerical analysis of systems of differential equations and software implementations for the study of asymptotic stability and evaluation of areas of attraction. Note that works [16, 17] are devoted to the study of the asymptotic stability of the motion of phase systems. These papers show the strategy of energy management, control systems and development of algorithms based on artificial intelligence for vehicles with hydrogen fuel cells. The paper [16] reviews modern methods for optimizing energy efficiency and improving the durability of fuel cells, focusing on the use of AI to improve their performance. A novel hybrid control algorithm with fixed and adaptive coefficients for power loss reduction in renewable energy conversion systems based on LLCL filter is presented in [17]. The book [18] is devoted to the analysis and control of hybrid systems that combine discrete and continuous dynamics. The main focus is on symbolic methods for verification and control, which allows to study complex systems with a large number of states and to guarantee their correct operation through rigorously mathematical methods.

2 Problem statement

Consider the system of the form

$$\frac{d\delta_i}{dt} = S_i, \qquad \frac{dS_i}{dt} = -D_i S_i - f_i(\delta_i) - \psi_i(\delta_i), \quad i = \overline{1, l}, \tag{1}$$

where δ_i is the angular coordinate, S_i is an angular velocity, f_i is the periodic function, $D_i > 0$ is a damping coefficient, function $\psi_i(\delta_i)$ is determined by the ratio

$$\psi_i(\delta_i) = \sum_{k=1,\, k\neq i}^{l} P_{i,k}(\delta_{i,k}), \quad \delta_{i,k} = \delta_i - \delta_k,$$

which defines the relationship between the subsystems and $P_{i,k}(\delta_{i,k})$ is a given continuously differentiable periodic function.

Let $f_i(\delta_i)$ be a continuously differentiable periodic function, which is the nonlinearity in the control object, satisfying the conditions:

$$f_i(\delta_i) = f_i(\delta_i + 2\pi), \quad \forall \delta_i \in \mathbb{R}_i^1, \quad \gamma_0 = \frac{1}{2\pi}\int_0^{2\pi} f_i(\delta_i)\, d\delta_i \leq 0, \quad f_i(0) = 0$$

and

$$\frac{df_i}{d\delta_i}(0) > 0, \quad f_i(\delta_{0,i}) = 0, \quad \frac{df_i}{d\delta_i}(\delta_{0,i}) < 0. \tag{2}$$

Due to the periodicity of the phase portrait of the system by the coordinates δ_i, it is sufficient to study it, for example, in the band $\bar{G}_{0,i}$, given by the inequalities

$$\delta_{-1,i} < \delta_i < \delta_{0,i}, \quad S_i \in \mathbb{R}_i^1, \quad i = \overline{1,l}.$$

The set of singular points of system (1) is located inside the band $\bar{G}_{0,i}$, is defined by the set

$$\begin{aligned}\bar{\Delta} &= \left\{ (\delta, S):\ S_i = 0,\ f_i(\delta_i) + \sum_{r=1,\, k\neq i}^{l} P_{i,k}(\delta_{i,k}) = 0,\ (\delta_i, S_i) \in \bar{G}_{0,i},\ i = \overline{1,l} \right\} \\ &= \{T_0, T_1, \ldots, T_N\}.\end{aligned}$$

Note that the point $T_0 = \{\delta_i = 0,\ S_i = 0,\ i = \overline{1,l}\}$ is also an element of the stationary set $\bar{\Delta}$. Composing the characteristic equation of the first approximation system, it is possible to establish the nature of the stability of the singular points. As is known, in order for the singular point of the system to be unstable, the negativity of at least one coefficient of the characteristic equation of the first approximation system in the vicinity of this singular point is sufficient.

Consider the function

$$V(\delta, S) = \sum_{i=1}^{l} v_{0,i}(\delta_i, S_i) + \sum_{j=2}^{l}\sum_{i=1}^{j-1}\int_0^{\delta_{i,j}} P_{i,j}(\lambda)\, d\lambda, \tag{3}$$

where $\nu_{0,i}(\delta_i, S_i)$ is defined by

$$\begin{aligned}\nu_{0,i}(\delta_i, S_i) &= \frac{1}{2}(S_i + \alpha_i D_i \delta_i)^2 + \frac{1}{2}\alpha_i D_i^2 (1-\alpha_i)\delta_i^2 + F_i(\delta_i) + 2D_i\sqrt{\alpha_i(1-\alpha_i)}\bar{F}_i(\delta_i) \\ &= \frac{1}{2}(S_i + \alpha_i D_i \delta_i)^2 + \int_0^{\delta_i} N_i(\delta)\, d\delta,\end{aligned}$$

where

$$0 < \alpha_i < 1, \quad F_i(\delta_i) = \int_0^{\delta_i} f_i(\delta_i)\, d\delta_i, \quad \widetilde{F}_i(\delta_i) = \int_0^{\delta_i} \sqrt{\delta_i f_i(\delta_i)}\, d\delta_i,$$

and

$$N(\delta_i) = \alpha_i D_i^2 (1-\alpha_i)\delta_i + f_i(\delta_i) + 2D_i\sqrt{\alpha_i(1-\alpha_i)}\sqrt{\delta_i f_i(\delta_i)}.$$

Functions $F_i(\delta_i)$, $\tilde{F}_i(\delta_i)$ are continuous in the band $\bar{G}_{0,i}$.

Theorem 1. *Let the following conditions be fulfilled:*

1) *function $f_i(\delta_i)$ satisfies the condition* (2)*,*

2) *function $P_{i,j}(\lambda)$ satisfies the conditions*

$$-P_{i,j}(\lambda) = P_{ji}(\lambda), \quad P_{i,j}(\lambda) = -P_{i,j}(-\lambda), \quad P_{i,j}(\delta_{i,j})\delta_{i,j} \geq 0,$$

3) *constants α_i, $D_i > 0$ are such that*

   a) $\alpha_i = K/D_i, 0 < K < \min\{D_1, \ldots, D_l\}, i = \overline{1, l},$

   b) $f_i(0) = \alpha_i D_i^2(1 - \alpha_i), i = \overline{1, l}.$

Then the zero equilibrium position T_0 is asymptotically stable according to Lyapunov, and the internal estimate of the region of attraction of the singular point T_0 is determined by the region, bounded by the surface $V(\delta, S) = T$ with $T = \min_{1 \leq i \leq N} V(T_i)$, where T_i, $i = \overline{1, N}$, are unstable singular points of the system (1).

Proof. Under condition of the theorem, the functions $v_{0,i}(\delta_i, S_i)$, $i = \overline{1, l}$, are definitely positive in the band $\bar{G}_{0,i}$, the total time derivative, by virtue of system (1), is sign-negative and the set $\dot{v}_{0,i} = 0$ does not contain integer trajectories of system (1), except of a singular point T_0. Function $V(\delta, S)$ in the band $\bar{G}_{0,i}$ is also definitely positive, by virtue of (3).

Under condition (3) and item *3 a)* of the theorem, the following equalities are valid

$$\sum_{i=1}^{l} S_i \sum_{j=1,\, j\neq i}^{l} P_{i,j}(\delta_{i,j}) = \sum_{j=1}^{l}\sum_{i=1}^{j-1} P_{i,j}(\delta_{i,j})(S_i - S_j),$$

$$\sum_{i=1}^{l} \alpha_i D_i \delta_i \sum_{j=1,\, j\neq i}^{l} P_{i,j}(\delta_{i,j}) = K \sum_{i=1}^{l}\sum_{j=1,\, j\neq i}^{l} P_{i,j}(\delta_{i,j}) = K \sum_{j=2}^{l}\sum_{i=1}^{j-1} P_{i,j}(\delta_{i,j})\delta_{i,j}. \tag{4}$$

Taking into account (4), the total time derivative of the function $V(\delta, S)$, by virtue of system (1), takes the form

$$\dot{V}(\delta, S) = \sum_{i=1}^{l} D_i \left[\sqrt{\alpha_i\,(1-\alpha_i)}S_i - \sqrt{\alpha_i \delta_i f_i(\delta_i)}\right]^2 - \sum_{i=1}^{l} \alpha_i D_i \delta_i \sum_{j=1,\, j\neq i}^{l} P_{i,j}(\delta_{i,j})$$

$$= -\sum_{i=1}^{l} D_i \left[\sqrt{\alpha_i(1-\alpha_i)}S_i \sqrt{\alpha_i \delta_i f_i(\delta_i)}\right]^2 - K \sum_{j=2}^{l}\sum_{i=1}^{j-1} P_{i,j}(\delta_{i,j})\delta_{i,j}.$$

The above expression is sign-negative and the set $\dot{V}(\delta, S) = 0$ does not contain integer trajectories of system (1), except of the singular point T_0. Then the regions of attraction of the singular point T_0 can be determined using a bounded surface $V(\delta, S) = T$ with $T = \min_{1 \leq i \leq N} V(T_i)$, where T_i are unstable singular points of the system (1). The theorem is proved. □

Note that the regions of attraction of the singular point T_0 can be found by the method from [1].

Let us now consider the function

$$v_{0,i}(\delta_i, S_i) = \frac{S_i^2}{2} + \alpha_i S_i \delta_i + \frac{\alpha_i D_i}{2}\delta_i^2 + F_i(\delta_i) + 2\beta_i(\delta_i)\widetilde{F}_i(\delta_i)$$

and function $V(\delta, S)$ from (3).

Theorem 2. *Let the items 1), 2) and 3) of the Theorem 1 hold and there are constants* α_i*,* $D_i > 0$ *such that:*

a) $\alpha_i = K \in (0, D_i)$*,* $i = \overline{1,l}$*,*

b)

$$\sqrt{\frac{df_i}{d\delta_i}}(0) \neq -\,sign\,\beta_i'\sqrt{(D_i-\alpha_i)\alpha_i}, \quad \sqrt{\frac{df_i}{d\delta_i}}(0) \neq +\,sign\,\beta_i''\sqrt{(D_i-\alpha_i)\alpha_i}.$$

Then the zero equilibrium position T_0 *is asymptotically stable according to Lyapunov, and the internal estimate of the region of attraction of the singular point* T_0 *is determined by the region, bounded by the surface* $V(\delta, S) = T$ *with* $T = \min_{1\le i\le N} V(T_i)$*, where* T_i*,* $i = \overline{1,N}$*, are unstable singular points of the system* (1)*.*

Proof. Under the condition of the theorem and according to Theorem 1, the functions $v_{0,i}(\delta_i, S_i)$, $i = \overline{1,l}$, $V(\delta, S)$ definitely positive in the band $\bar{G}_{0,i}$. Full time derivative of function $v_{0,i}(\delta_i, S_i)$, by virtue of the system (1), is sign-negative, and the set $v_{0,i} = 0$ does not contain entire trajectories, except of a singular point T_0. Taking into account the equalities (4), the time derivative of the function $V(\delta, S)$, by virtue of the system (1), takes the form

$$\begin{aligned}\dot{V}(\delta,S) &= \sum_{i=1}^{l}\dot{v}_{0,i}(\delta_i,S_i) - \sum_{i=1}^{l}\alpha_i\delta_i\sum_{j=1,\,j\neq i}^{l}P_{i,j}(\delta_{i,j}) \\ &= \sum_{i=1}^{l}\dot{v}_{0,i}(\delta_i,S_i) - K\sum_{j=2}^{l}\sum_{i=1}^{j-1}P_{i,j}(\delta_{i,j})\delta_{i,j},\end{aligned} \tag{5}$$

where the right side of equality is sign-negative and the set $\dot{V}(\delta, S) = 0$ does not contain entire trajectories, except of a singular point T_0. Hence, as in Theorem 1, it is easy to obtain statements of the theorem. □

Let us now consider the system

$$\frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = w_i - D_iS_i - f_i(\delta_i) - \psi_i(\delta_i), \qquad w_i = C_i^* x_i, \tag{6}$$

$$\frac{dx_i}{dt} = A_ix_i + q_iS_i + B_iu_i, \quad i = \overline{1,l}. \tag{7}$$

Due to the periodicity of the phase portrait of the system by coordinates δ_i, it is enough to study it in the strip $G_{0,i}$, defined by inequalities

$$\delta_{-1,i} < \delta_i < \delta_{0,i}, \quad S_i \in \mathbb{R}_i^1, \quad x_i \in \mathbb{R}_i^{n_i}, \quad i = \overline{1,l}.$$

Consider the set of singular points of the system (6)–(7) located inside the band $G_{0,i}$. To do this, we introduce a set of stationary points

$$\begin{aligned}\bar{\Lambda} &= \Big\{(\delta,S,x):\ S_i = 0,\ f_i(\delta_i) + \sum_{k=1,\,k\neq i}^{l}P_{i,k}(\delta_{i,k}) = 0,\ x_i = 0,\ (\delta_i,S_i,x_i)\in G_{0,i},\ i=\overline{1,l}\Big\} \\ &= \big\{\bar{T}_0, \bar{T}_1, \ldots, \bar{T}_N\big\},\end{aligned}$$

where

$$\bar{T}_0 = \{\delta_i = 0,\ S_i = 0,\ x_i = 0, i = \overline{1,l}\} \in \bar{\Lambda}.$$

Consider the function

$$\bar{V}(\delta, S, x) = \sum_{i=1}^{l} \widetilde{v}_{0,i}(\delta_i, S_i, x_i) + \sum_{j=2}^{l}\sum_{i=1}^{j-1}\int_0^{\delta_{i,j}} P_{i,j}(\lambda)\, d\lambda, \tag{8}$$

where the function $\widetilde{v}_{0,i}(\delta_i, S_i, x_i)$ is defined as follows

$$\widetilde{v}_{0,i}(\delta_i, S_i, x_i) = \frac{1}{2}x_i^* H_i x_i + \frac{\varepsilon_{1i}}{2}S_i^2 + v_{0,i}(\delta_i, S_i), \quad \varepsilon_{1i} > 0. \tag{9}$$

It is defined in the band $G_{0,i}$ in the space $R_i^{n_i+2}$.

Then combining the results of Theorems 1 and 2, it is not difficult to prove the following assertion.

Theorem 3. *Let the conditions of Theorems 1 and 2 be satisfied. Then with the control $u_i = a_i^* x_i + \alpha_{1i}S_i + \alpha_{2i}\delta_i + \alpha_{3i}f_i(\delta_i)$ the internal estimate of the attraction region of the origin of the special point T_0 is determined by the region, bounded by the surface $\hat{V}(\delta, S, x) = \hat{T}$ with $\bar{T} = \min_{1\le i\le N} \bar{V}(T_i)$, where T_i, $i = \overline{1,N}$, are unstable special points of the system* (6)–(7).

Again considering the function (8), where $\widetilde{v}_{0,i}(\delta_i, S_i, x_i)$ is defined by the relation (9) in the band $G_{0,i}$, we have the following result.

Theorem 4. *Let there be a vector a_i, scalars $\alpha_{1i}, \alpha_{2i}, \alpha_{3i}, \varepsilon_{1i} > 0$, $\alpha_i \in (0,1)$, $D_i > 0$ such that*

1) A_i is a Hurwitz matrix,

2) the pair (A_i, B_i) is fully controllable,

3) the pair (A_i, q_i^*) is completely observable,

4) $P_{i,j}(\omega) > 0\ \forall\, \omega \in (-\infty, +\infty)$,

5) $f_i'(0) \ne \alpha_i D_i^2(1-\alpha_i)$.

Then, with the control $u_i = a_i^* x_i + \alpha_{1i}S_i + \alpha_{2i}\delta_i + \alpha_{3i}f_i(\delta_i)$, the attraction region of the origin in the band $G_{0,i}$, which can be estimated using the Lyapunov function (9)*, is determined by the inequality $\widetilde{v}_{0,i}(\delta_i, S_i, x_i) < \overline{\overline{v}}_{0,i}$, where the critical value $\overline{\overline{v}}_{0,i} = \min(\widetilde{v}'_{0,i}, \widetilde{v}''_{0,i})$ can be found from the condition of maximizing the estimated region*

$$\dot{\overline{\overline{v}}}_{0,i} = \min\{\overline{\overline{\rho}}_{0,i}, \overline{\overline{\rho}}_{-1,i}\}, \quad \overline{\overline{\rho}}_{0,i} = \widetilde{v}_{0,i}(\delta_{0,i}, 0, 0), \quad \overline{\overline{\rho}}_{-1,i} = \widetilde{v}_{0,i}(\delta_{-1,i}, 0, 0).$$

Using Theorems 1 and 2, it is easy to prove the following assertion.

Theorem 5. *Let the conditions of Theorems 1 and 2 are satisfied. Then, with the control $u_i = a_i^* x_i + \alpha_{1i}S_i + \alpha_{2i}\delta_i + \alpha_{3i}f_i(\delta_i)$, the internal estimate of the attraction region of the critical point $\bar{T}_0$ is determined by the region, bounded by the surface $\bar{V}(\delta, S, x) = \bar{T}$ with $\bar{T} = \min_{1\le i\le N} \bar{T}_i$, where $\bar{T}_i$, $i = \overline{1,N}$, are the unstable critical points of the system* (6)–(7).

Remark 1. *In Theorems 1 and 5, finding all the critical points T_i and $\bar{T}_i$, $i = \overline{1,N}$, is a labor-intensive task. Therefore, there are special methods for finding the critical saddle point $\overline{T}$.*

3 Numerical example

Consider a system

$$\frac{d\delta_1}{dt} = S_1, \quad \frac{d\delta_2}{dt} = S_2, \quad \frac{dS_1}{dt} = -D_1S_1 - f_1(\delta_1) - \psi_1(\delta_1), \quad \frac{dS_2}{dt} = -D_2S_2 - f_2(\delta_2) - \psi_2(\delta_2), \tag{10}$$

where $f_1(\delta_1) = f_{0,1}\,[\sin(\delta_1 + \theta_{0,1}) - \sin\theta_{0,1}]$, $f_2(\delta_2) = f_{0,2}\,[\sin(\delta_2 + \theta_{0,2}) - \sin\theta_{0,2}]$, and $\psi_1(\delta_1) = \frac{P_{1,2}}{T_1}\,[\sin(\delta_{1,2} + \theta_{0,1}) - \sin\theta_{0,1}]$, $\psi_2(\delta_2) = \frac{P_{2,1}}{T_2}\,[\sin(\delta_{2,1} + \theta_{0,2}) - \sin\theta_{0,2}]$.

Let the numerical data of the system (10) be as follows: $D_i = 50.5 \times 10^{-4}$, $T_1 = 2135$, $T_2 = 1256$, $P_{1,2} = 0.85$, $P_{2,1} = 0.69$, $\theta_{0,i} = 0.3562$, $f_{0,i} = 1.513 \times 10^{-4}$ with the initial conditions $\delta_1(0) = 0.18$, $\delta_2(0) = 0.1$, $S_1(0) = 0.001$, $S_2(0) = 0.005$.

To study stability, we consider the function (3), which is defined in the band $\bar{G}_{0,i}$. To numerically solve the problem under consideration, a software module was created, written in the C# programming language using the Windows Forms application creation interface. The program uses the two-step Adams-Bashfort method for numerical integration of the system (10). The formula for this method is the following one

$$y_{i+1} = y_i + h\Big(\frac{3}{2}f(x_i, y_i) - \frac{1}{2}f(x_{i-1}, y_{i-1})\Big).$$

Using the above formula, we rewrite the system (10) in the form

$$\delta_{i+1} = \delta_i + h\Big(\frac{3}{2}S_i - \frac{1}{2}S_{i-1}\Big)$$

and

$$S_{i+1} = S_i + h\Big(\frac{3}{2}\big(-DS_i - f(\delta_i) - \psi_i(\delta_i)\big) - \frac{1}{2}\big(-DS_{i-1} - f(\delta_{i-1}) - \psi_{i-1}(\delta_{i-1})\big)\Big).$$

Figures 1–4 show graphs of variable changes δ_1, δ_2, S_1, and S_2, respectively. Finally, in Figures 5 and 6 we can see the difference variable changes δ and S, respectively. In all figures, N is a number of iterations.

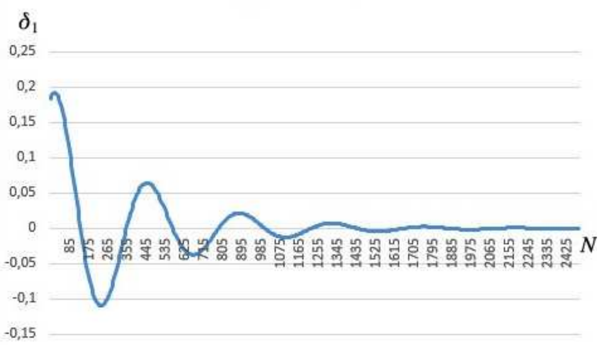

Figure 1. Graph of variable change δ_1

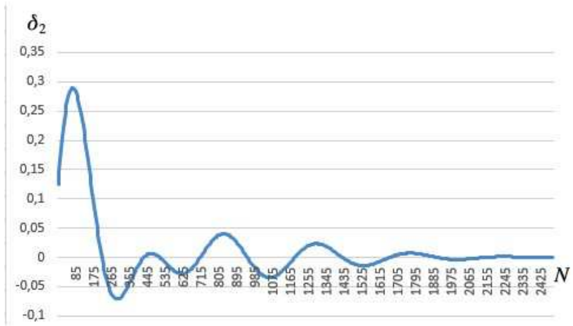

Figure 2. Graph of variable change δ_2

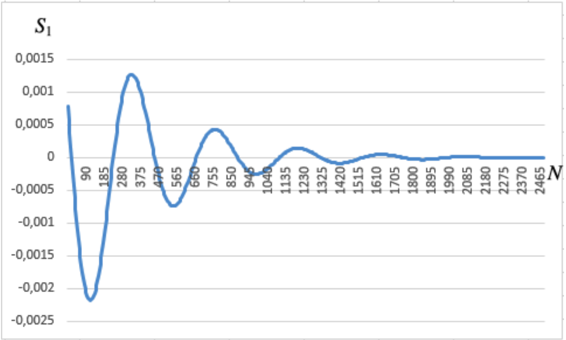

Figure 3. Graph of variable changes S_1

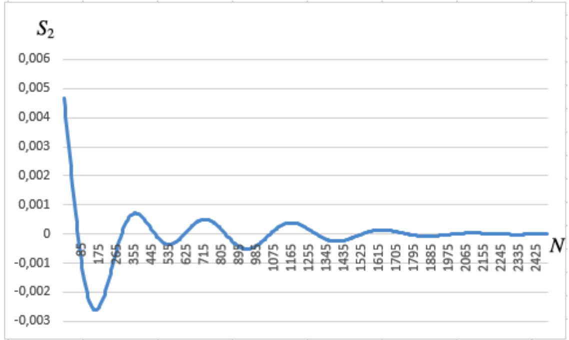

Figure 4. Graph of variable changes S_2

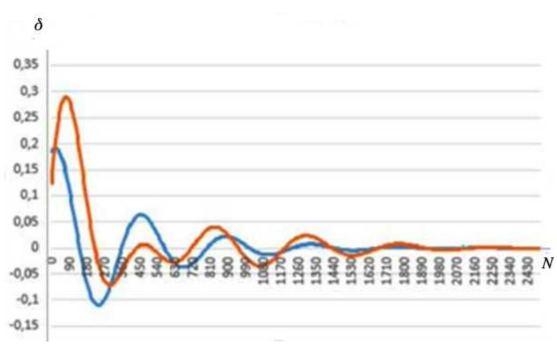

Figure 5. Graph of variable changes δ_1 and δ_2

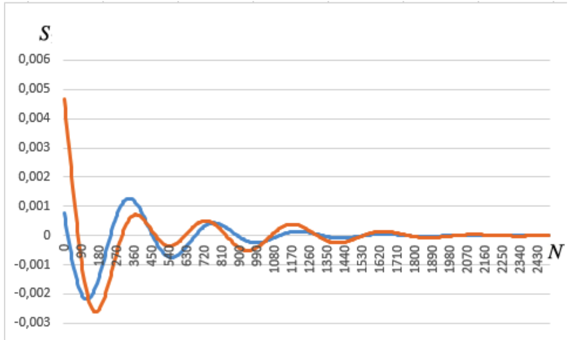

Figure 6. Graphs of variable changes S_1 and S_2

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Жолдангарова Г.І., Калімолдаєв М.Н., Зіятбекова Г.З., Ахметжанов М.А., Аршидинова М.Т. *Розробка алгоритмів і програмного забезпечення для дослідження стійкості складних енергосистем* // Карпатські матем. публ. — 2025. — Т.17, №2. — С. 376–385.

У цій роботі математична модель однаково описує як маятникові системи, так і енергетичні системи з багатьма генераторами. Як відомо, промислова революція призвела до зростання споживання енергії, більшість якої в сучасному суспільстві споживається у вигляді електричної енергії. Це, своєю чергою, піднімає питання її транспортування на великі відстані. Математична модель сучасного енергетичного комплексу, що складається з турбогенераторів і складних взаємопов'язаних енергетичних блоків, є системою нелінійних звичайних диференціальних рівнянь. Завдання оптимізації роботи таких комплексів, а також розробка алгоритмів забезпечення стійкості руху в подібних системах продовжують привертати увагу багатьох дослідників і залишаються надзвичайно актуальними. Промисловий розвиток сучасного суспільства зумовлює постійне зростання споживання електроенергії. Для задоволення постійно зростаючих потреб створюються енергетичні комплекси. Під час математичного моделювання таких комплексів необхідно вирішувати низку теоретичних і практичних питань. Забезпечення стійкості руху є критично важливим як на етапі проєктування, так і на етапі експлуатації досліджуваних систем. Ця робота присвячена дослідженню асимптотичної стійкості руху фазових систем.

Ключові слова і фрази: функціонал Ляпунова, динамічна система, розробка алгоритмів, керування, робототехніка, електроніка, технологічний прогрес.