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Automorphisms of the endomorphism semigroup of a free monogenic strict n-tuple semigroup

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Free objects are fundamental in algebra and play a central role in B. Plotkin's universal algebraic geometry. A key approach, initiated by B. Plotkin and developed with collaborators, involves studying automorphisms of the category of finitely generated free algebras. This problem is related to the study of automorphisms of endomorphism semigroups of free finitely generated algebras. Algebras of dimension one play a special role in examining properties of higher-dimensional algebras. Free monogenic algebras form a natural class from which the study of automorphisms of endomorphism semigroups of free algebras of arbitrary rank can naturally begin.

The concept of a strict n-tuple semigroup and a free strict n-tuple semigroup naturally arise in several frameworks, including trialgebra and trioid theory, dialgebra and dimonoid theory, strong doppelsemigroup theory, and n-tuple semigroup theory. The n-tuple semigroups are, in turn, closely related to the notion of an n-tuple algebra of associative type which was introduced to provide an analogue of the Chevalley construction for modular Lie algebras of Cartan type. In every strict n-tuple semigroup, any two semigroups are $\mathcal{P}$-related, and free strict n-tuple semigroups are determined by their endomorphism semigroups.

In this paper, we construct a semigroup isomorphic to the endomorphism semigroup of a free monogenic strict n-tuple semigroup and establish that the automorphism group of the endomorphism semigroup of the free monogenic strict n-tuple semigroup is isomorphic to the direct product of two symmetric groups.

Key words and phrases: strict n-tuple semigroup, free monogenic strict n-tuple semigroup, endomorphism semigroup, automorphism group.

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1 Introduction and preliminaries

Free objects are among the most fundamental concepts in algebra. Every variety contains free objects, and understanding their structure is essential for addressing classical problems, such as the word problem, as well as for studying the variety itself. Free objects provide a natural and powerful framework for exploring algebraic operations, morphisms, and identities. Free objects in varieties also play a central role in B. Plotkin's universal algebraic geometry. A central question in universal algebraic geometry is when the geometries defined by different algebras in a given variety coincide. A far-reaching approach, first proposed by B. Plotkin

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and further developed with his collaborators (see, e.g., [6, 7, 10, 11]), involves the study of the structure of automorphisms of the category of finitely generated free algebras. This question is related to the study of automorphisms of endomorphism semigroups of free finitely generated algebras. The problem of investigating the automorphism group of the endomorphism semigroup of a free algebra in a variety is highly nontrivial, quite interesting by itself, and has been considered in many papers. For free (commutative) dimonoids and free commutative g-dimonoids the automorphism groups of their endomorphism semigroups have been characterized in [18–20], respectively. Automorphisms of endomorphism semigroups of free groups and free semigroups have been considered in [1] and [8], respectively. Algebras of dimension one play a special role in studying various properties of algebras of arbitrary dimension. For example, free trioids of rank 1 were described in [5] and used for constructing free trialgebras, while semigroups of cohomological dimension one appear naturally in algebraic topology [9]. Therefore, a natural starting point in the investigation of automorphisms of endomorphism semigroups of free algebras is the case of free monogenic algebras; even in this setting, a description remains unknown, for example, for free monogenic trialgebras, dialgebras, and other related structures.

The concept of a strict n-tuple semigroup (see definition below) and the construction of the free strict n-tuple semigroup were introduced in [12]. Every semigroup can be regarded as a particular case of a strict n-tuple semigroup. However, there exist many natural examples of strict n-tuple semigroups that are not semigroups. The motivation for strict n-tuple semigroups arises from the observation that these algebras naturally appear in several algebraic frameworks such as trialgebra and trioid theory [5, 17], dialgebra and dimonoid theory [4, 15], strong doppelsemigroup theory [14], and n-tuple semigroup theory [16]. The n-tuple semigroups are, in turn, closely related to the notion of an n-tuple algebra of associative type which was introduced in [3] to provide an analogue of the Chevalley construction for modular Lie algebras of Cartan type. Moreover, in every strict n-tuple semigroup, any two semigroups are known to be $\mathcal{P}$-related in the sense of Hewitt and Zuckerman [2]. In [13], it was established that free strict n-tuple semigroups are determined by their endomorphism semigroups. More general information on strict n-tuple semigroups can be found in [12].

In this paper, we construct a semigroup which is isomorphic to the endomorphism semigroup of a free monogenic strict n-tuple semigroup and prove that the automorphism group of the endomorphism semigroup of the free monogenic strict n-tuple semigroup is isomorphic to the direct product of two symmetric groups.

Let $\mathbb{N}$ denote the set of all positive integers, and let $\overline{n}$ stand for the set $\{1, 2, \ldots, n\}$ of the first n positive integers. Following [12], a nonempty set G equipped with n binary operations denoted by $\boxed{1}, \boxed{2}, \ldots, \boxed{n}$ is called a *strict n-tuple semigroup* if it satisfies the axioms

$$\left(x \boxed{r} y\right) \boxed{s} z = x \boxed{i} \left(y \boxed{j} z\right) \quad \text{for all} \quad x, y, z \in G \quad \text{and} \quad r, s, i, j \in \overline{n}.$$

The class of all strict n-tuple semigroups forms a variety. A strict n-tuple semigroup which is free in the variety of strict n-tuple semigroups is called a *free strict n-tuple semigroup*.

Now we construct the singly generated free object in the variety of strict n-tuple semigroups. For this, we add $n-1$ $(n > 1)$ arbitrary elements $x \notin \mathbb{N}$ to $\mathbb{N}$. Each added i-th element is conveniently denoted by 2_i and imagined as a copy of the number 2. Define n

binary operations $\boxed{1}, \boxed{2}, \ldots, \boxed{n}$ on $\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$ by

$$m * s = \begin{cases} 2_i, & \text{if } * = \boxed{i},\ m = s = 1 \neq i, \\ m + s, & \text{otherwise,} \end{cases}$$

$m * 2_i = 2_i * m = m + 2$, $2_i * 2_j = 4$ for all $m, s \in \mathbb{N}$, $i, j \in \{2, \ldots, n\}$ and $* \in \left\{\boxed{1}, \boxed{2}, \ldots, \boxed{n}\right\}$. The algebra

$$\left(\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \boxed{1}, \boxed{2}, \ldots, \boxed{n}\right)$$

is denoted by $\mathbb{N}^{\sharp}(n)$.

Corollary 1 ([12, Corollary 4]). *For every $n > 1$, $\mathbb{N}^{\sharp}(n)$ is the free strict n-tuple semigroup of rank 1.*

Let $\mathrm{End}(A)$, $\mathrm{Aut}(A)$, and P denote, respectively, the endomorphism semigroup, the automorphism group of an algebraic system A, and the set of all prime numbers. Let further X be a nonempty set. The symmetric group on X is denoted by $S(X)$. For any mapping $f : B \to B'$ and any nonempty subset $C \subseteq B$, we write $f\big|_C$ for the restriction of f to C.

2 The automorphism group of $\mathrm{End}(\mathbb{N}^{\sharp}(n)), n > 1$

In this section, we present a semigroup which is isomorphic to the endomorphism semigroup of a free monogenic strict n-tuple semigroup and prove that the automorphism group of the endomorphism semigroup of the free monogenic strict n-tuple semigroup is isomorphic to the direct product of two symmetric groups.

Define a binary operation $\perp$ on $\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$ by

$$m \perp s = ms, \quad 2_i \perp 2_j = 4, \quad m \perp 2_i = 2_i \perp m = \begin{cases} 2_i, & \text{if } m = 1, \\ 2m, & \text{if } m \neq 1 \end{cases}$$

for all $m, s \in \mathbb{N}$ and $i, j \in \{2, \ldots, n\}$.

Lemma 1. *$\left(\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp\right)$ is a commutative monoid.*

Proof. The proof amounts to a routine verification, and we omit it. □

Theorem 1. *Let $\mathbb{N}^{\sharp}(n)$, $n > 1$, be the free strict n-tuple semigroup of rank 1. Then*

(*i*) $\mathrm{End}(\mathbb{N}^{\sharp}(n)) \cong (\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp)$;

(*ii*) $\mathrm{Aut}(\mathrm{End}(\mathbb{N}^{\sharp}(n))) \cong S(P) \times S(\overline{n})$.

Proof. (*i*) For $n > 1$, note that $\mathbb{N}^{\sharp}(n)$ is generated by the singleton $\{1\}$. Let ϕ be an endomorphism of $\mathbb{N}^{\sharp}(n)$ and suppose that $1\phi = m$ for some $m \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$. For any $a \in \mathbb{N}$ and $i \in \{2, \ldots, n\}$, we have

$$a\phi = (\underbrace{1 \boxed{1} 1 \boxed{1} \ldots \boxed{1} 1}_{a})\phi = \underbrace{m \boxed{1} m \boxed{1} \ldots \boxed{1} m}_{a} = a \perp m,$$

$$2_i\phi = (1 \boxed{i} 1)\phi = 1\phi \boxed{i} 1\phi = m \boxed{i} m = 2_i \perp m.$$

Conversely, for each $m \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$, the mapping ϕ_m defined by $a\varphi_m = a\perp m$ for all $a \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$ is an endomorphism, that is, $(a\boxed{r}b)\phi_m = a\phi_m\boxed{r}b\phi_m$ for all $a, b \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$ and $r \in \overline{n}$. Indeed, we have the following seven distinguished cases.

Case 1: $a, b, m \in \mathbb{N}$.

$$
\begin{aligned}
(a\boxed{r}b)\phi_m = (a\boxed{r}b)\perp m &= \begin{cases} 2_r\perp m, & \text{if } a = b = 1 \neq r, \\ (a+b)\perp m, & \text{otherwise} \end{cases} \\
&= \begin{cases} 2_r, & \text{if } a = b = 1 \neq r,\, m = 1, \\ 2m, & \text{if } a = b = 1 \neq r,\, m \neq 1, \\ (a+b)m, & \text{otherwise} \end{cases} \\
&= (am)\boxed{r}(bm) = (a\perp m)\boxed{r}(b\perp m) = a\phi_m\boxed{r}b\phi_m.
\end{aligned}
$$

Case 2: $a, b \in \mathbb{N}$, $m = 2_j$.

$$
\begin{aligned}
(a\boxed{r}b)\phi_{2_j} = (a\boxed{r}b)\perp 2_j &= \begin{cases} 2_r\perp 2_j, & \text{if } a = b = 1 \neq r, \\ (a+b)\perp 2_j, & \text{otherwise} \end{cases} \\
&= \begin{cases} 4, & \text{if } a = b = 1 \neq r, \\ 2(a+b), & \text{otherwise} \end{cases} \\
&= \begin{cases} 4, & \text{if } a = b = 1 \neq r, \\ 2(b+1), & \text{if } a = 1, b \neq 1, \\ 2(a+1), & \text{if } a \neq 1, b = 1, \\ 2(a+b), & \text{if } a \neq 1, b \neq 1 \end{cases} \\
&= \begin{cases} 2_j\boxed{r}2_j, & \text{if } a = b = 1 \neq r, \\ 2_j\boxed{r}(2b), & \text{if } a = 1, b \neq 1, \\ (2a)\boxed{r}2_j, & \text{if } a \neq 1, b = 1, \\ (2a)\boxed{r}(2b), & \text{if } a \neq 1, b \neq 1 \end{cases} \\
&= (a\perp 2_j)\boxed{r}(b\perp 2_j) = a\phi_{2_j}\boxed{r}b\phi_{2_j}.
\end{aligned}
$$

Case 3: $a \in \mathbb{N}, b = 2_k, m \in \mathbb{N}$.

$$
\begin{aligned}
(a\boxed{r}2_k)\phi_m = (a\boxed{r}2_k)\perp m = (a+2)\perp m &= \begin{cases} a+2, & \text{if } m = 1, \\ am + 2m, & \text{if } m \neq 1 \end{cases} \\
&= \begin{cases} a\boxed{r}2_k, & \text{if } m = 1, \\ (am)\boxed{r}(2m), & \text{if } m \neq 1 \end{cases} \\
&= (am)\boxed{r}(2_k\perp m) = (a\perp m)\boxed{r}(2_k\perp m) = a\phi_m\boxed{r}2_k\phi_m.
\end{aligned}
$$

Case 4: $a \in \mathbb{N}, b = 2_k, m = 2_j$.

$$
\begin{aligned}
(a\boxed{r}2_k)\phi_{2_j} = (a\boxed{r}2_k)\perp 2_j = (a+2)\perp 2_j &= 2(a+2) = 2a+4 \\
&= \begin{cases} 6, & \text{if } a = 1, \\ 2a+4, & \text{if } a \neq 1 \end{cases} = \begin{cases} 2_j\boxed{r}4, & \text{if } a = 1, \\ (2a)\boxed{r}4, & \text{if } a \neq 1 \end{cases} \\
&= (a\perp 2_j)\boxed{r}4 = (a\perp 2_j)\boxed{r}(2_k\perp 2_j) = a\phi_{2_j}\boxed{r}2_k\phi_{2_j}.
\end{aligned}
$$

Case 5: $a = 2_k, b \in \mathbb{N}, m \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$.

Using the commutativity of the operation $\boxed{r}$, $r \in \overline{n}$, and calculations in Cases 3 and 4, we get

$$(2_k \boxed{r} b)\phi_m = (b \boxed{r} 2_k)\phi_m = b\phi_m \boxed{r} 2_k\phi_m = 2_k\phi_m \boxed{r} b\phi_m.$$

Case 6: $a = 2_k, b = 2_j, m \in \mathbb{N}$.

$$\begin{aligned}(2_k \boxed{r} 2_j)\phi_m = (2_k \boxed{r} 2_j)\perp m = 4\perp m &= \begin{cases} 4, & \text{if } m = 1, \\ 4m, & \text{if } m \neq 1 \end{cases} \\ &= \begin{cases} 2_k \boxed{r} 2_j, & \text{if } m = 1, \\ (2m)\boxed{r}(2m), & \text{if } m \neq 1 \end{cases} \\ &= (2_k \perp m)\boxed{r}(2_j \perp m) = 2_k\phi_m \boxed{r} 2_j\phi_m.\end{aligned}$$

Case 7: $a = 2_k, b = 2_\ell, m = 2_j$.

$$(2_k \boxed{r} 2_\ell)\phi_{2_j} = (2_k \boxed{r} 2_\ell)\perp 2_j = 4\perp 2_j = 8 = 4 + 4 = (2_k \perp 2_j)\boxed{r}(2_\ell \perp 2_j) = 2_k\phi_{2_j} \boxed{r} 2_\ell\phi_{2_j}.$$

Hence,

$$\operatorname{End}(\mathbb{N}^\sharp(n)) = \{\phi_m : m \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})\}.$$

Define a mapping Φ from $\operatorname{End}(\mathbb{N}^\sharp(n))$ into $\big(\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp\big)$ by $\phi_m\Phi = m$ for all $\phi_m \in \operatorname{End}(\mathbb{N}^\sharp(n))$. We now verify that Φ is an isomorphism. For any $\phi_m, \phi_{m'} \in \operatorname{End}(\mathbb{N}^\sharp(n))$ we get

$$(\phi_m\phi_{m'})\Phi = (\phi_{m\perp m'})\Phi = m\perp m' = \phi_m\Phi \perp \phi_{m'}\Phi,$$

since for all $a \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$, we have

$$a\phi_m\phi_{m'} = (a\perp m)\perp m' = a\perp(m\perp m') = a\phi_{m\perp m'},$$

by Lemma 1. It is easy to see that Φ is a bijection: different endomorphisms ϕ_m correspond to different elements m, and for each $m \in \mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\})$, there exists an endomorphism ϕ_m. Therefore, Φ is an isomorphism of semigroups.

(ii) Let $\widetilde{P} = P \cup (\cup_{i=2}^{n}\{2_i\})$ and $P^* = P \setminus \{2\}$. The subset $(\mathbb{N} \setminus \{1\}) \cup (\cup_{i=2}^{n}\{2_i\})$ is a subsemigroup of the semigroup $\big(\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp\big)$. This subsemigroup is generated by $\widetilde{P}$, and for $p, q \in (\mathbb{N} \setminus \{1\}) \cup (\cup_{i=2}^{n}\{2_i\})$, we have

$$p\perp p = q\perp q = p\perp q \quad \text{if and only if} \quad p, q \in \{2\} \cup (\cup_{i=2}^{n}\{2_i\}).$$

This implies that for every automorphism γ of $\big(\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp\big)$, we get

$$\{2, 2_2, \ldots, 2_n\}\gamma = \{2, 2_2, \ldots, 2_n\} \quad \text{and} \quad P^*\gamma = P^*.$$

On the other hand, every permutation $f : \widetilde{P} \to \widetilde{P}$ such that

$$f\big|_{\{2,2_2,\ldots,2_n\}} \in S\big(\{2, 2_2, \ldots, 2_n\}\big)$$

uniquely determines an automorphism of $(\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp)$.

An immediate check shows that the mapping

$$\xi : \operatorname{Aut}\big((\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp)\big) \to S(P^*) \times S(\{2, 2_2, \ldots, 2_n\})$$

defined as

$$g\xi = \left(g|_{P^*},\, g|_{\{2,2_2,\ldots,2_n\}}\right)$$

for all $g \in \mathrm{Aut}\big((\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp)\big)$ is an isomorphism. It is clear that $S(P^*) \cong S(P)$ and $S(\{2, 2_2, \ldots, 2_n\}) \cong S(\overline{n})$. By (i), $\mathrm{End}(\mathbb{N}^\sharp(n)) \cong (\mathbb{N} \cup (\cup_{i=2}^{n}\{2_i\}), \perp)$, and therefore

$$\mathrm{Aut}(\mathrm{End}(\mathbb{N}^\sharp(n))) \cong S(P) \times S(\overline{n}).$$

□

Corollary 2.

(i) $\mathrm{End}(\mathbb{N}^\sharp(n))$, $n > 1$, *is a commutative monoid.*

(ii) *The operation* $\perp$ *distributes over each operation* $\boxed{r}$, *where* $r \in \overline{n}$.

Proof. Part (i) follows directly from Lemma 1 and the isomorphism Φ of semigroups established above. Part (ii) is a consequence of the calculations in Cases 1–7 of the proof of Theorem 1 combined with the commutativity of the operation $\perp$. □

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Жучок А.В. *Автоморфізми напівгрупи ендоморфізмів вільної моногенної строгої n-кратної напівгрупи* // Карпатські матем. публ. — 2026. — Т.18, №1. — С. 29–35.

Вільні об'єкти мають фундаментальне значення в алгебрі та відіграють центральну роль в універсальній алгебраїчній геометрії Б. Плоткіна. Ключовий підхід, започаткований Б. Плоткіним і розвинений у співавторстві з іншими дослідниками, полягає у вивченні автоморфізмів категорії скінченно породжених вільних алгебр. Ця задача пов'язана з дослідженням автоморфізмів напівгруп ендоморфізмів вільних скінченно породжених алгебр. Алгебри розмірності один відіграють особливу роль у вивченні властивостей алгебр вищих розмірностей. Вільні моногенні алгебри формують природний клас, з якого доцільно починати дослідження автоморфізмів напівгруп ендоморфізмів вільних алгебр довільного рангу.

Поняття строгої n-кратної напівгрупи та вільної строгої n-кратної напівгрупи природно виникають у кількох теоретичних контекстах, зокрема у теорії триалгебр і тріоїдів, теорії діалгебр і дімоноїдів, теорії сильних допельнапівгруп та теорії n-кратних напівгруп. У свою чергу, n-кратні напігрупи тісно пов'язані з поняттям n-кратної алгебри асоціативного типу, яке було введено для отримання аналога конструкції Шевалле для модулярних алгебр Лі типу Картана. У кожній строгій n-кратній напівгрупі будь-які дві напівгрупи є $\mathcal{P}$-зв'язаними, а вільні строгі n-кратні напівгрупи визначаються своїми напівгрупами ендоморфізмів.

У цій статті побудовано напівгрупу, ізоморфну напівгрупі ендоморфізмів вільної моногенної строгої n-кратної напівгрупи, і доведено, що група автоморфізмів напівгрупи ендоморфізмів вільної моногенної строгої n-кратної напівгрупи є ізоморфною прямому добутку двох симетричних груп.

Ключові слова і фрази: строга n-кратна напівгрупа, вільна моногенна строга n-кратна напівгрупа, напівгрупа ендоморфізмів, група автоморфізмів.