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INNOVATIVE TECHNOLOGIES FOR ENHANCED PHYSICAL AND MATHEMATICAL LEARNING IN CENTRAL EUROPE: CONTEMPORARY ISSUES AND TRENDS OF FRACTAL STRUCTURING OF INFORMATION

*MYKHAYLO MAR'YAN**, *NATALIYA YURKOVYCH*, *VLADIMIR SEBEN*, *NIKOLA TESĽUKOVÁ*

*Corresponding author: mykhaylo.maryan@uzhnu.edu.ua

Abstract. The implementation of the functioning of fractals and fractality for open and holistic systems of physical and mathematical education is substantiated. It is shown that it is thanks to fractality that the synergy of the nature of the functioning of education, business, ecology, and human life as open systems is realized. In this article, the main emphasis is placed on considering the fractality of the functioning of physical and mathematical education in the countries of Central Europe on the example of cooperation between two universities – the University of Presov (Slovakia) and Uzhhorod National University (Ukraine). The relationship between fractality and information is analyzed. Iterations of the integrated fractal structure are presented using the example of studying physics sections, "Geometric optics" and "Wave optic" with information use of features geometric, algebraic, and stochastic fractals. It is shown that the principles of object-oriented programming, namely, encapsulation, inheritance, and polymorphism, are also characteristic of fractals. The application of the fractal approach to the structuring of information in physical and mathematical education is substantiated. It is shown that each iteration is characterized by synergy - the addition of a new iteration provides a high-quality and in-depth perception of new information. It manifests also the formation of an integrated fractal structure and corresponding iterations that reflect the integrity and spontaneity of the fractal perception of information. Based on the proposed approach to the use of interdisciplinary areas of knowledge and computer modeling, the fractal structure is formed at an intuitive level for students. The proposed approach provides the opportunity to most adequately implement the integration of education and information technologies to achieve the goals of holistic and continuous development adopted by the UN General Assembly.

Keywords*:* physical and mathematical education, fractal approach, information technology, synergetics, self-organized systems and wholeness of education, object-oriented modeling.

1. INTRODUCTION

Fractals and fractality, their manifestation and implementation in various fields of science, information technology, information processing and structuring, and education are extremely promising and relevant (Falconer, 2003; Mandelbrot, 2010; Mar'yan & Yurkovych, 2022). At the modern stage of the development of science education, business, and information technology, their integration, complementarities, and implementation are extremely important (Mar'yan et al., 2020). Despite a significant number of scientific publications and scientific conferences, the issues of considering fractals as open systems, the influence of self-organization processes, and the relationship of fractality with information are at the initial stage of research. However, it is the openness and integrity of the system that determines its structuring and development paths, and the formation of new types of ordering and integration (Packard, 2019; Stanley, 2019; Mar'yan & Yurkovych, 2022). Thus, the search for methods and principles of teaching natural sciences that holistically implement the principles of self-organization, fractality, and computer modeling corresponds to the immediate tasks of today. Innovative teaching of the synergy of natural sciences and computer modeling of natural phenomena, as well as the application of these methods by teachers, is the focus of special attention in the scientific literature (Hodson, 2014; Kafyulilo et al.2015; Kopylchak et al., 2024). However, special training of future teachers on numerical modeling of natural phenomena, bibliographic data in the pedagogical literature, as well as in educational practice is encountered less (Yurkovych et al., 2023; Oladele & Ndlovu, 2024). For example, the curriculum of training future teachers of physics in all five Slovakian universities does not contain this subject (Sladek et al., 2011; Seben et al., 2024). At the same time, this direction is well developed at the Uzhhorod National University at the physical and engineering-technical faculties, the faculties of information technologies, and mathematics. Within the framework of the proposed fractal approach, this gap can be compensated for by using computer modeling and the possibility of providing appropriate resources of Uzhhorod National University to colleagues at the University of Presov.

In addition, features of the fractal approach in teaching natural sciences reproduce the unique functioning of fractals in the natural environment. Namely, it is the implementation of the "simple" algorithm, self-sufficiency, and self-similarity for various areas, disciplines, and practical applications (Yurkovych et al., 2017; Maŕyan et al., 2019). It points also to the need for information perception in higher educational establishments at an intuitive level using visualization means and modern advances in programming and information technology – object-oriented modeling (Yurkovych et al., 2017; Yurkovych et al., 2023). The direction presented in the article, in addition to scientific research, is connected with modern innovative methods of synergy, self-organization processes, and fractality and is directly related not only to the sections of physics, mathematics, and engineering but also has an interdisciplinary architecture. This is extremely relevant at the current stage of development of higher educational institutions (Tsependa & Budnyk, 2021), the selection of educational materials and disciplines, the use of digital technologies, the synergy of the interaction between students and teachers, and their mobility (Budnyk et al., 2021). The issue of integrating education and digital technologies is currently receiving significant attention at the international level. This is evidenced, in particular, by the Sustainable Development Goals (SDGs) adopted by the UN General Assembly to 2030 (Blyznyuk, 2024).

2. THEORETICAL BACKGROUND

Learning physical and mathematical disciplines, like education in general, is related to the processing and distribution of significant volumes of information, the use of modern information technologies and artificial intelligence (Windschitl, 2004; Kuo et al., 2013; Mar'yan et al., 2020; Lopes et al., 2024). These questions are actively discussed and relevant in scientific and pedagogical literature (Luft, 2001; Lotter et al., 2007; Özcan, 2015). It is the flow of information that forms unique types of ordering of nature (Mar'yan & Yurkovych, 2022). And it is primarily characteristic of fractal structures (Yurkovych et al., 2017; Seben et al., 2024). Indeed, fractals as self-organized information structures are formed in open systems (Fig. 1), which correspond to the minimum of dissipation energy (Fig. 1, (a), (b)). That is achieved through self-organization and self-sufficiency of processes (Fig. 1, (a)), defining and initiating this is the information component of the structure itself (Fig. 1, (c)), which ensures the structuring and branching of the system with a minimum of energy dissipation (Fig. 1, (d)).

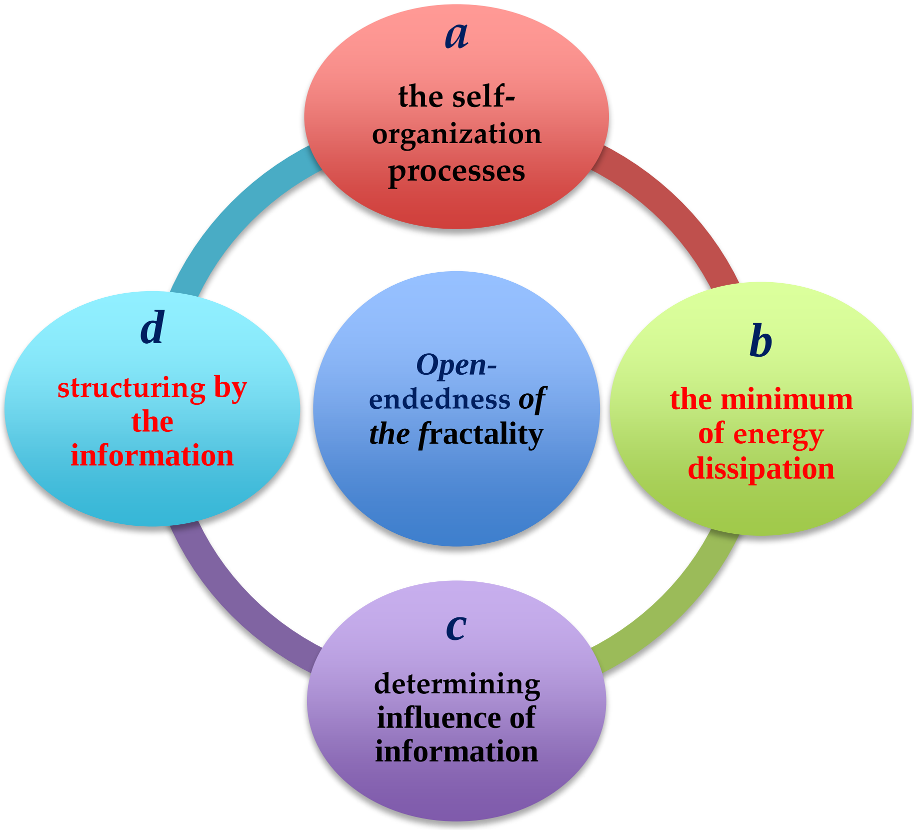

Fig. 1. Fractality, integrity, and complexity of the information in the open-endedness system (Seben et al., 2024)

Source: author's development

The sequence of steps presented in Figure 1 is an algorithm for forming a self-organized structure (fractal) and its structuring. There are well-known examples of self-organized structures in physical and chemical systems (Bénard cells, Belousov-Zhabotinsky reaction, Conway flows (Haken & Mikhailov, 2012; Mar'yan et al., 2019; Mar'yan et al., 2020). Natural fractals (correlation of cotton price changes and the Mandelbrot set (Mandelbrot, 2010), fractality of stock markets and Williams' indicators (Gregory-Williams & Williams, 2008), stochastic fractals of natural landscapes, cloud flows, plasma (Falconer, 2003; Mar'yan et al., 2018)) are implemented in open systems and are also self-organized structures.

The nature is unique in the formation of self-organized structures and fractals (Mar'yan et al., 2019). It is fractality and self-organization that most adequately define and implement the SDGs. From this point of view, fractality, and information technologies form algorithms and methods of perception, development, and creation of the surrounding information environment – science, education, economy, ecology, and improvement of the architecture of the person himself (intellectual, emotional, mental, spiritual development).

The use of object-oriented modeling, which is de facto built on the principles of synergetics, is essential in the implementation of the fractal approach in physical and mathematical education (Yurkovych et al., 2017; Seben et al., 2024). The principles of object-oriented modeling (encapsulation, inheritance, and polymorphism) can also be applied to create smart environments and respond SDG's. The basis of self-organized structures and fractality is information (Mar'yan & Yurkovych, 2022). Modern information principles of programming, the development of artificial intelligence (AI), namely encapsulation, inheritance, and polymorphism are also inherent in fractals (Fig. 2).

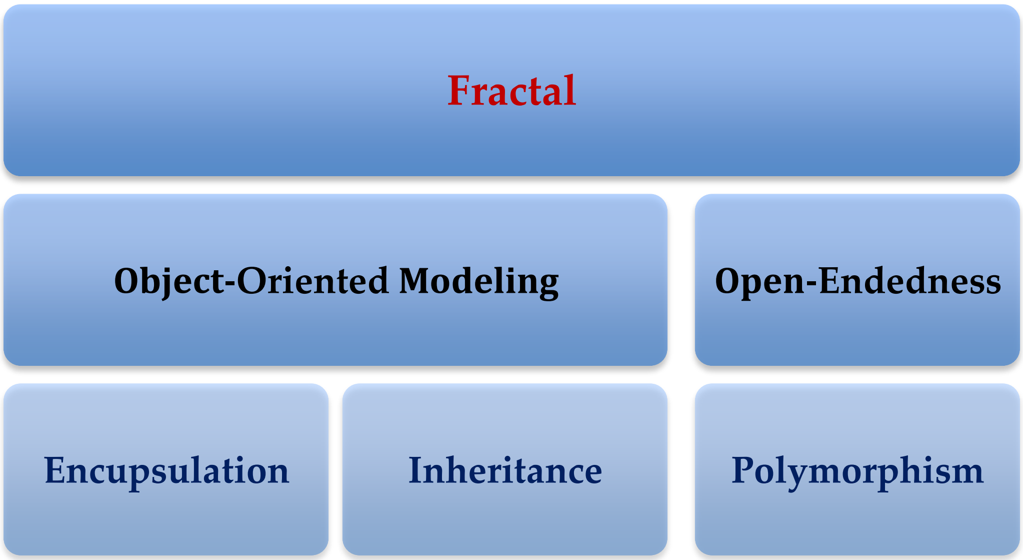

Fig. 2. Fractality and object-oriented modeling

Source: author's development

It applies the environment of visual programming, Delphi, and the algorithmic programming language, Object Pascal, which is built on the principles of object-oriented programming and the latest information technologies (Yurkovych et al., 2017). (At computer modeling, along with the use of Object Pascal other object-oriented languages such as C++, Java, and Python, can be used.) The educational experiment involved students of two universities: Uzhhorod National University (Uzhhorod, Ukraine) and the University of Presov (Presov, Slovakia) (Seben et al., 2024). Control groups consisted of 10-18 students. Upon the completion of the student's studies, the test control was conducted, which aimed to find out changes in the knowledge and skills of students after the implementation of the offered approach. A part of the test questions were aimed at ascertaining the interest of students towards the further study of computer modeling of natural phenomena (Yurkovych et al., 2023; Yurkovych et al., 2024).

3. RESEARCH OBJECTIVE

The objective of this study is to investigate the fractality structuring of information in physical and mathematical education. This area of education is associated with the processing and dissemination of significant amounts of information, the use of modern information technologies and artificial intelligence, interaction with a dynamic and self-renewing information environment, and synergy between the teacher and students. Therefore, the purpose of the present investigation is the implementation of the educational experiment based on the positive impact of the applied fractal measures and computer simulations of natural phenomena using the example of cooperation between the Uzhhorod National University and the University of Presov in Presov, as well as interaction with other higher educational institutions of Central Europe.

4. METHODOLOGY AND DATA

The application of the principles of object-oriented programming to the formation of geometric, algebraic, and stochastic fractals is analyzed (Marýan et al., 2022). Geometric fractals are determined by iterations and the self-similarity of simple geometric objects (Falconer, 2003). For example, consider the formation of Cantor's fractal (Fig. 3).

The initial geometric object is a line segment. Iterative propagation is based on a simple, elementary algorithm that repeats spontaneously to infinity. Namely, we divide the segment into three parts and remove the central part. This is an encapsulation – a combination of parameters, description, and action (algorithm) (Fig. 3). So – the 1st iteration: added, synchronized algorithm, which extends to the spatial and temporal scales. This algorithm is stored for subsequent iterations. That is, an action is added to the description, and properties, and a new information object is formed. This is no longer just a line, namely

$$\{ \textit{algorithm \& line} \rightarrow \textit{new information object} \} \qquad (1)$$

and access to another spatial and temporal scale of fractality. The usual spatial scale changes

$$d=1 \rightarrow \{ d=1, d=0 \} \rightarrow d_f \ . \qquad (2)$$

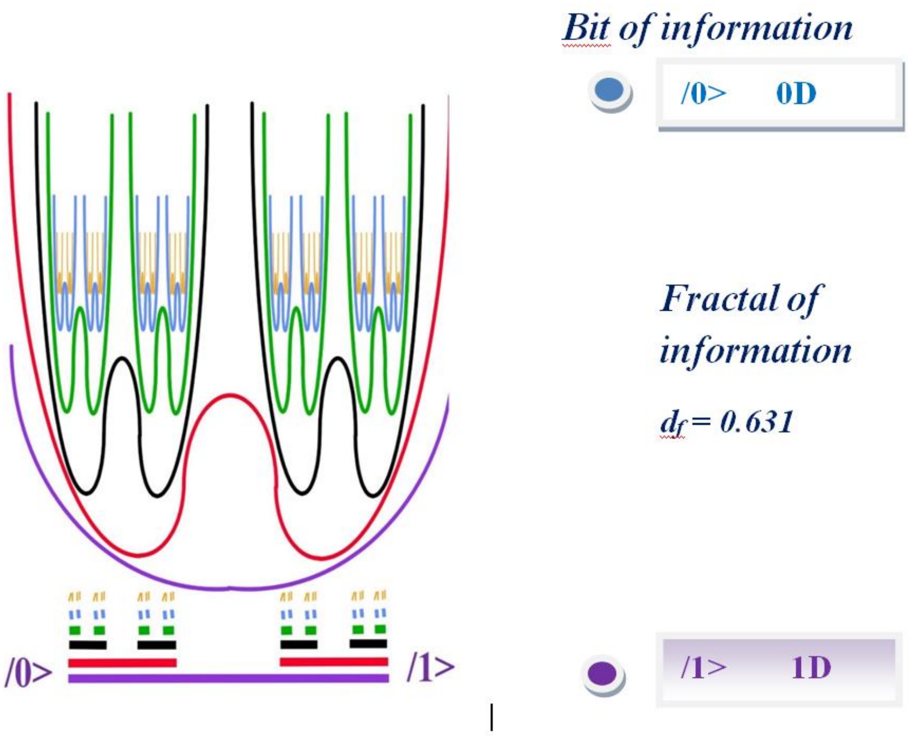

Fig. 3. Interpretation of bit and fractal of information: Cantor's fractal
Source: author's development

And space becomes completely different – this is the space of change and development of the fractal and structuring of information. The main attribute of encapsulation is information recovery. In this case, because of fractality (Fig. 4, Tab. 1). These are self-sufficiency, self-similarity, and self-organization.

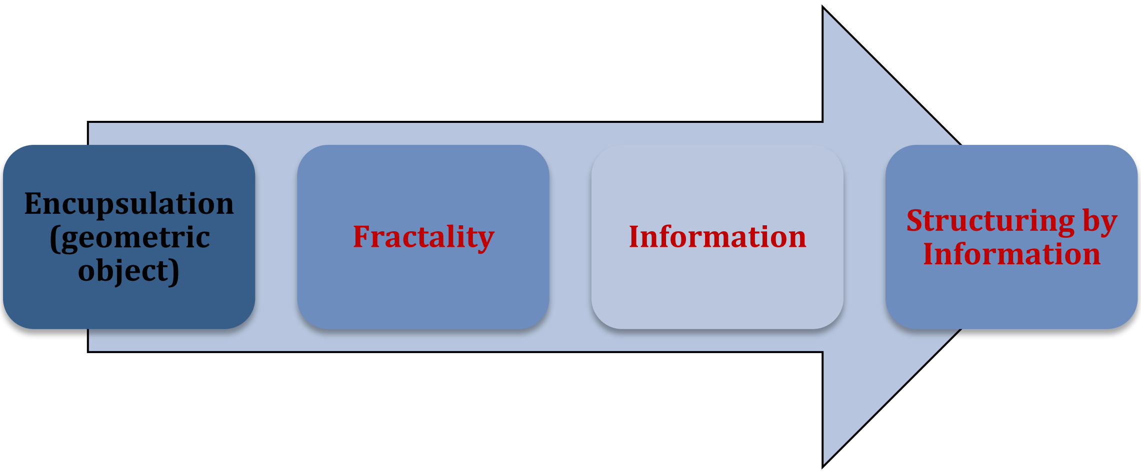

Fig. 4. Creating and presence information in the Fractality
Source: author's development

Tab. 1

Formation of fractals in open-endedness systems according to the principles of object-oriented modeling (OOM) and implementation of SDGs

Principles of OOM and AI	Types of fractality
Encapsulation	Geometric fractals are primarily an encapsulation that transforms the ordinary geometry of an object and its perception into information by combining the description and properties of parameters with an action (algorithm). Thanks to this, the geometric fractal becomes a holistic piece of information.
Inheritance	For algebraic fractals, the principle of inheritance is spontaneously and self-organized. All information properties and algorithms acquired for geometric fractals are inherited, and a completely new aspect of self-organization is born - this is the creation of levels of abstraction, new information spaces of fractality. Algebraic fractals are the development, new embodiment, and manifestation of abstract levels of geometric fractals and their implementation.
Polymorphism	Stochastic fractals develop the principles of encapsulation and inheritance, that is, implemented on the integrity of information. In addition, a new principle is spontaneously introduced and developed – is polymorphism: the implementation of properties and methods (algorithms) in various new interfaces, and environments, which are dynamically self-created for them.

Analogous structures and corresponding algorithms of functioning can be implemented in physical and mathematical education (Tab. 1). The principles of structuring information while preserving its integrity are decisive here.

5. RESULTS AND DISCUSSION

The iterations of the fractal structure on the example of studying physics sections, "Geometric optics" and "Wave optic" with integrated use of features geometric, algebraic, and stochastic fractals we have elaborated (Yurkovych et al., 2023; Seben et al., 2024).

The algorithm contains and implements the features of computer modeling of the process of light propagation at the interface of two media and is further used for subsequent phenomena (steps, iterations), forming a more complex, but internally self-sufficient and self-organized fractal structure. During the lecture, students examine in detail the basic laws of geometric optics and types of light reflection: mirror (parallel light rays remain parallel after reflection (smooth even surfaces) and diffuse (parallel rays after reflection are scattered in all directions (rough uneven surfaces) with the immediate transition to computer modeling (Yurkovych et al., 2017). After consideration of these laws in the environment of visual programming Delphi, the students create the interface modeling. Students have the opportunity to directly modify the parameters of the optical system (angle of incidence, refractive index, the factors of reflectivity and diffuseness), means of visualization of the rays in the Delphi environment (colors of the incident and reflected rays, types of lines) and become active self-sufficient participants in conducting computer experiment (Sherin, 2006; Strogatz, 2014; Soroko et al., 2023). It is important to develop the algorithm of information perception by students on the intuitive level that will be used and developed further in later iterations (lectures). Important attributes are the presence of game elements, the spontaneity of creating visual interfaces, and the students' sense of the significance of the direct experience. This approach provides opportunities to draw on knowledge from other sections of physics and forms ways of their practical application.

Each iteration of the integrated fractal structure is characterized by synergy - the addition of a new iteration ensures a high-quality and updated perception of information (without mechanical external introduction of division by topics), the formation of integrity, of which the student becomes an active participant. This synergy creates a unique fractal structure, capable of development and functioning. The iteration discussed above can be complemented and developed, in particular by the involvement of learning using the testing tools, and the exchange of information using the Internet, that is, the process generates and allows an infinite number of steps, which is essential for fractal structures (Mar'yan et al., 2020; Seben et al., 2024).

Based on the presented fractal approach of joint synergistic use of physics lectures and computer modeling at the AI level, a fractal structure is formed at the intuitive level. The functioning of this structure is manifested in the transition to the presence of self-sufficiency of students, which involves the use of a creative approach and the desire to apply the information received in fundamentally new areas of physics, mathematics, biology, and art (Strogatz, 2014; Mar'yan et al., 2018; Yurkovych et al., 2023).

Thus, a fractal structure in teaching one of the sections of physics, "geometrical optics," is formed (it can be easily spread to other branches of physics). The advantages of this approach are obvious: the corresponding physics section is perceived as a single unit without the mechanical separation into its parts; and the possibility of forming branched structures according to a single algorithm that can be extended to other branches of physics, while maintaining the integrity (fractality) at the level of several sections. It should be noted that unlike the classical approach, which is based on the assimilation of a certain amount of material, the fractal connections reflect the internal structure of the sections that are assigned spontaneously (Yurkovych et al., 2023).

The offered fractal approach was tested at Uzhhorod National University (Ukraine) for students at the Faculty of Physics and the University of Preshov (Slovakia) for students of humanities and natural sciences at the Department of Physics, Mathematics, and Technology. The research was conducted in the 2018/2019–2023/2024 academic years.

The study carried out the evaluation of the quality of training in the discipline "Programming and mathematical modeling" for students according to the specializations "physics teacher", and "applied physics" (Fig. 5).

The conducted rating control indicates the students' interest in computer simulation used in teaching physics in 88-93% of future physics teachers and the increase in their level of mastering material by 9-11% (Fig. 6). During these classes, the activation of students, and in-depth perception of the material have been noted.

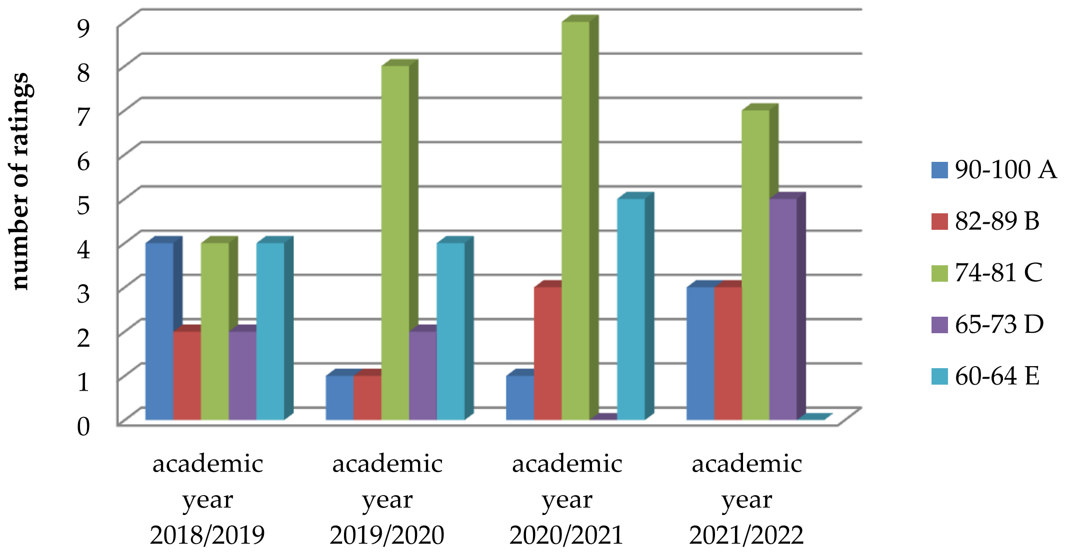

Fig. 5. The diagram of academic performance in the study of the discipline "Programming and mathematical modeling" for 2018/2019 - 2021/2022 academic years

Source: author's development

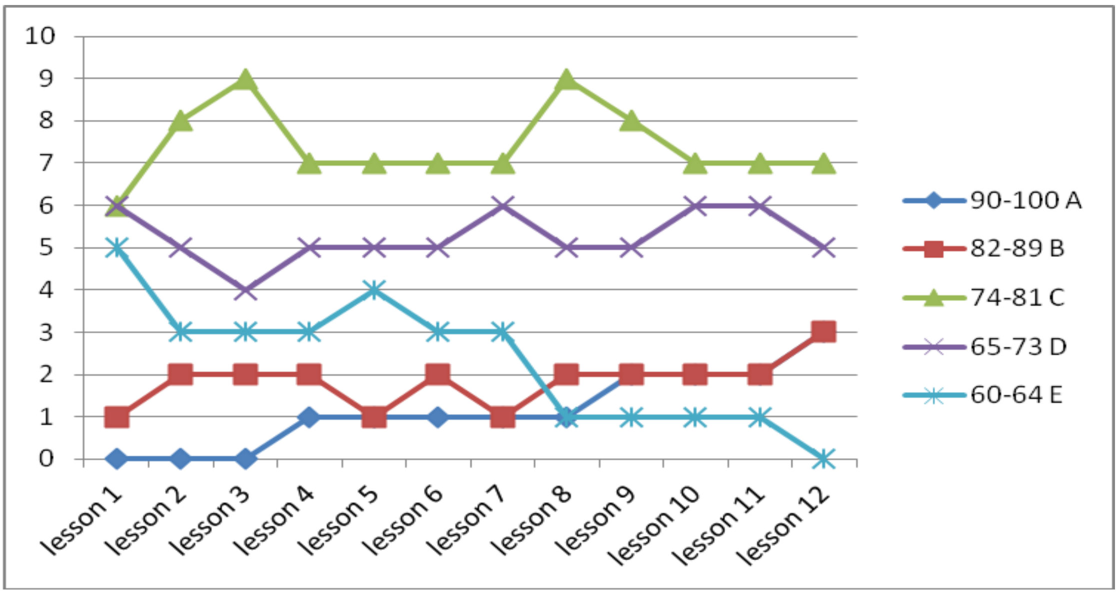

Fig. 6. The diagram of the dependence of assessment ratings of the students in the 2023/2024 academic year

Source: author's development

In addition, the fractal approach provides a harmonious and natural opportunity for the synchronized development of natural science teaching using modern information technologies and achieving the goals of sustainable holistic development. We also intend to deepen and implement this in the proposed approach further thanks to the cooperation between Uzhhorod National University and the University of Presov in Presov, as well as interaction with other higher educational institutions in Central Europe.

6. CONCLUSIONS

The implementation of information and its structuring in the formation of fractals and the possibility of application in physical and mathematical education are discussed. The fractal approach to the teaching of physical and mathematical disciplines using object-oriented computer modeling is substantiated. The formation of a fractal structure is established and iterations are defined that reflect the integrity and spontaneity of information presentation. The involvement of students of related specialties from two universities – the University of Presov (Faculty of Humanities and Natural Sciences) and Uzhhorod National University (Faculty of Physics) – demonstrates the feasibility of using the fractal approach in training future teachers and scientists of natural sciences in higher educational institutions. Studies of self-organized fractal structures in the context of physical and mathematical education are effective and will be continued in the future within the framework of the concept of sustainable development goals for international cooperation.

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Mykhaylo Mar'yan, Doctor of Sciences (Physics & Mathematics), Professor in the Department Solid-State Electronics & Information Security Faculty of Physics Uzhhorod National University, Uzhhorod, Ukraine;

ORCID ID: 0000-0002-1667-4945

Nataliya Yurkovych, PhD (Physics & Mathematics), Ass. Prof. in the Department of Solid-State Electronics & Information Security Faculty of Physics Uzhhorod National University, Uzhhorod, Ukraine;

ORCID ID: 0000-0001-7686-9667

Vladimir Seben, PhD in Pedagogical Sciences, Ass. Prof. in the Department of Physics, Mathematics & Technics Faculty of Humanities and Natural Science University of Presov, Presov, Slovakia;

ORCID ID: 0000-0002-7836-4125

Nikola Tesľuková, postgraduate student (Pedagogy), Department of Physics, Mathematics & Technics Faculty of Humanities and Natural Science University of Presov, Presov, Slovakia;

Address: Mykhaylo Mar'yan, Nataliya Yurkovych, Uzhhorod National University, 54 Voloshina Str., Uzhhorod, 88000, Ukraine.

Vladimir Seben, Nikola Tesľuková, University of Presov in Presov, 17 Novembra str., 1, Presov, 08116, Slovakia.

E-mail: mykhaylo.maryan@uzhnu.edu.ua, nataliya.yurkovych@uzhnu.edu.ua, vladimir.seben@unipo.sk, nikola.teslukova@smail.unipo.sk

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Михайло Мар'ян, Наталія Юркович, Володимир Шебень, Нікола Теслюкова. Інноваційні технології для поглибленого фізико-математичного навчання в Центральній Європі: сучасні проблеми та тенденції фрактального структурування інформації. *Журнал Прикарпатського університету імені Василя Стефаника*, **12** (1) (2025), 69-79.

Обґрунтовано реалізацію функціонування фракталів та фрактальності для відкритих і цілісних систем фізико-математичної освіти. Показано, що саме завдяки фрактальності реалізується синергія природи функціонування освіти, бізнесу, екології та життя людини як відкритих систем. У цій статті основний акцент зроблено на розгляді фрактальності функціонування фізико-математичної освіти в країнах Центральної Європи на прикладі співпраці двох університетів – Пряшівського університету (Словаччина) та Ужгородського національного університету (Україна). Проаналізовано зв'язок між фрактальністю та інформацією. Розроблено ітерації фрактальної структури на прикладі вивчення розділів фізики "Геометрична оптика" і "Хвильова оптика" з комплексним використанням ознак геометричних, алгебраїчних та стохастичних фракталів. Показано, що принципи об'єктно-орієнтованого програмування, а саме інкапсуляція, успадкування та поліморфізм, також характерні для фракталів. Обґрунтовано застосування фрактального підходу до структурування інформації у фізико-математичній освіті. Показано, що кожна ітерація характеризується синергією – додавання нової ітерації забезпечує якісне та поглиблене сприйняття нової інформації. Проявляється також формування цілісної фрактальної структури та відповідних ітерацій, які відображають цілісність і спонтанність фрактального сприйняття інформації. На основі запропонованого підходу до використання міждисциплінарних галузей знань та комп'ютерного моделювання формується фрактальна структура на інтуїтивному рівні студентів. Запропонований підхід надає можливість найбільш адекватно реалізувати інтеграцію освіти та інформаційних технологій для досягнення прийнятих Генеральною Асамблеєю ООН цілей цілісного та безперервного розвитку.

Ключові слова: фізико-математична освіта, фрактальний підхід, інформаційні технології, синергетика, самоорганізовані системи та цілісність освіти, об'єктно-орієнтоване моделювання.