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Aharonov-Bohm effect in lens-shaped quantum dots

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This study theoretically investigates the Aharonov–Bohm (AB) effect in InAs/GaAs/AlAs lens-shaped quantum dots under constant axial magnetic and transverse electric fields. Electron states are calculated by solving the Schrödinger equation using the finite element method implemented in COMSOL Multiphysics. It is shown that a transverse electric field, applied perpendicular to the dot base, significantly modifies the topology of the ground-state electron wave function, inducing a quasi-ring-like spatial distribution. The formation of this electrically induced state enables the emergence of AB oscillations in an external magnetic field. Increasing the electric field strength modulates the AB oscillation period due to an increase in the effective radius of the quasi-ring state and a reduction of the critical magnetic field. These effects demonstrate the feasibility of electrical control of the AB effect in semiconductor nanostructures.

Keywords: nanostructure, semiconductor, quantum dot, quantum nanoring, electric field.

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Introduction

Quantum dots (QDs) and other low-dimensional systems are subjects of intense theoretical and experimental research owing to their discrete energy levels, which allow for controlling the optical, electronic, and magnetic properties of nanostructures. A particular area of interest involves quantum phenomena related to the topology of nanostructures, notably the fundamental Aharonov-Bohm (AB) effect. The Aharonov–Bohm effect provides direct evidence of the physical significance of the electromagnetic vector potential in quantum mechanics [1]. When a magnetic flux threads a multiply connected geometry, the wave function of a charged particle acquires a flux-dependent phase shift even though the particle propagates in a region of vanishing magnetic induction and experiences no Lorentz force. This phase shift, proportional to the magnetic flux enclosed by the particle's closed trajectory, results in observable interference phenomena in nanoscale systems. In nanostructures, the Aharonov–Bohm effect was originally predicted and experimentally confirmed in ring-shaped systems, such as quantum rings. In these structures, the AB effect manifests as periodic oscillations of energy levels and changes in the symmetry of the ground state, depending on the magnetic flux [2]. Studies have shown that in multilayered nanosystems featuring an internal potential barrier (like inverted core-shell QDs), the AB effect causes the electron ground state to be successively formed by states with the magnetic quantum number $m = 0, -1, -2, \dots$ as the magnetic field induction increases [3-6]. However, the combined influence of electric and magnetic fields has proven critical for manipulating these quantum phenomena. For instance, in some spherical core-shell QDs, applying an electric field parallel to the magnetic field can suppress the AB effect, fixing the ground state to $m=0$ across the magnetic field range [7-9].

Recently, particular attention has been directed toward quantum dots exhibiting complex geometries, such as lens-shaped quantum dots (LQDs) [10-12]. It has been demonstrated that applying a transverse electric field (perpendicular to the base) can significantly alter the topology of the electron's ground-state wave function in LQDs (e.g., InAs/GaAs/AlAs). As this electric field increases, the electron density becomes more compressed toward the base of the quantum dot, leading to an expansion of the region near the axis where the wave function approaches zero. Consequently, the electron density distribution adopts a ring-shaped configuration.

This artificially induced ring-shaped electron distribution within the lens-shaped quantum dot means that the system will exhibit the Aharonov-Bohm oscillations when subjected to an external magnetic field. The aim of this study is, therefore, to investigate the influence of a transverse electric field on the manifestations of the Aharonov-Bohm effect in lens-shaped quantum dots grown on a substrate and covered by a layer that forms a potential well for the electron. This research seeks to quantitatively examine how the electric field controls the formation of the ring-shaped electron density distribution and, consequently, modulates the period of the AB effect oscillations.

I. Theory

To investigate Aharonov–Bohm oscillations induced by external fields in nanoscale systems, we consider a multilayer lens-shaped InAs/GaAs/AlAs quantum dot consisting of a GaAs core in the form of a spherical segment with height h and base radius r_s, grown on an AlAs substrate and capped by an InAs shell of thickness Δ. The geometric structure of the quantum dot are presented in Fig. 1.

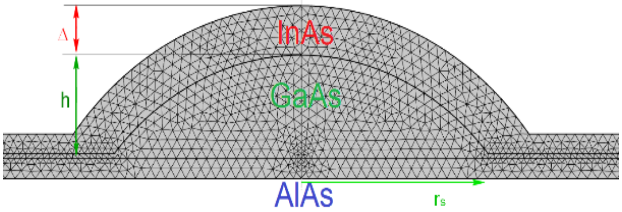

Fig. 1. The geometric scheme of the lens-shaped InAs/GaAs/AlAs quantum dot.

The quantum state of an electron in this system is described by the Schrödinger equation:

$$\left[\left(\vec{p}-\frac{e}{c}\vec{A}\right)\frac{1}{2\mu(\vec{r})}\left(\vec{p}-\frac{e}{c}\vec{A}\right)+e\vec{F}\cdot\vec{r}+U(\vec{r})\right]\psi_{jm}(\vec{r})=E_{nm}\psi_{jm}(\vec{r}), \tag{1}$$

where $\vec{A}$ is a vector potential, $\vec{F}$ is a magnitude of external electric field strength. Effective mass and confining potential have the forms

$$\mu(\vec{r})=\begin{cases} m_1, & \vec{r}\in \text{region of InAs} \\ m_2, & \vec{r}\in \text{region of GaAs QD} \\ m_3, & \vec{r}\in \text{region of AlAs substrate}\end{cases}, \tag{2}$$

$$U(\vec{r})=\begin{cases} 0, & \vec{r}\in \text{InAs region} \\ V_2, & \vec{r}\in \text{GaAs region} \\ V_3, & \vec{r}\in \text{AlAs region}\end{cases}, \tag{3}$$

The Schrödinger equation (1) for the electron states in the multilayer quantum dot is solved using the finite element method (FEM) implemented in COMSOL Multiphysics. Due to the cylindrical symmetry of the structure, the original three-dimensional eigenvalue problem is reduced to a two-dimensional formulation in the radial and axial coordinates (ρ, z), which significantly reduces which significantly reduces the computational time while increasing the accuracy of the solution.

At the outer boundary of the simulation domain, Dirichlet boundary conditions are imposed by setting the wave function to zero, ψ=0, which reflects the vanishing probability of finding the electron outside the heterostructure region. The computational domain is chosen sufficiently large to ensure that the boundary does not introduce artificial confinement. At the InAs/GaAs/AlAs interfaces, the wave function remains continuous, while the normal component of the probability current is conserved, which correctly accounts for the discontinuity of the effective mass. The magnetic field enters the Hamiltonian through the vector potential, and the resulting azimuthal phase factor is preserved along closed electron trajectories within the quantum dot. The vector potential $\boldsymbol{A}$ and the magnetic field $\boldsymbol{B}$ are related by the gauge symmetry relation $\boldsymbol{B}=\boldsymbol{\nabla}\times\boldsymbol{A}$; for a uniform magnetic field along the z-axis, the symmetric gauge in cylindrical coordinates is used, $\boldsymbol{A}=A_\varphi\,\boldsymbol{n}_\varphi$, where $A_\varphi=\frac{1}{2}\cdot B\cdot\rho$, which is consistent with the cylindrical symmetry of the system. In this gauge, the circulation of the vector potential along a closed azimuthal path gives the magnetic flux through the enclosed area, $\oint \boldsymbol{A}\cdot d\boldsymbol{l}=\Phi$, which directly determines the Aharonov–Bohm phase shift $\Delta\varphi=e\Phi/\hbar$.

In a nanostructure where the electron density exhibits a ring-like spatial distribution, the Aharonov–Bohm phase directly modifies the quantization condition for the azimuthal motion. For an electron circulating around a closed trajectory that encloses a magnetic flux Φ, the wave function acquires a phase factor $\psi(\varphi+2\pi)=\psi(\varphi)exp(\frac{ie}{\hbar}\Phi)$, which is equivalent to a shift of the magnetic quantum number. In the absence of a magnetic field, the eigenfunctions of the azimuthal motion take the form $\psi(\varphi)\sim e^{im\varphi}$, where m is an integer. When a magnetic flux is present, the Aharonov–Bohm phase modifies this condition as $m\rightarrow m+\Phi/\Phi_0$, where $\Phi_0=h/e$ is the magnetic flux quantum.

As a result, the eigenenergy of a state characterized by the magnetic quantum number m becomes a periodic function of the magnetic flux. For a narrow ring of radius R, where the electron motion is effectively one-dimensional, the energy spectrum is expressed as

$$E_m(\Phi) = \frac{\hbar^2}{2m^*R^2}\left(m+\frac{\Phi}{\Phi_0}\right)^2 \qquad (4)$$

with m^* denoting the electron effective mass. This expression shows that the presence of the magnetic flux removes the degeneracy between the states with quantum numbers $+m$ and $-m$, leading to a characteristic parabolic dependence of the energy levels on Φ and resulting in oscillatory behavior with period Φ_0.

The dependence of $E_m(\Phi)$ on the enclosed flux may be interpreted as a direct consequence of the topological nature of the Aharonov–Bohm phase shift. Rather than arising from the local action of the magnetic field on the electron density, the energy shift reflects the modification of the boundary conditions for the wave function over a closed path. In this sense, the magnetic flux acts as an effective parameter that continuously tunes the angular momentum quantization in the ring-like system.

Moreover, in realistic multilayer nanostructures the confinement potential and effective mass distribution can lead to the formation of quasi-ring states, in which the electron probability density is strongly concentrated at a certain radius. In such cases, the structure of the energy spectrum retains the periodic dependence on the magnetic flux, providing a clear signature of Aharonov–Bohm oscillations even in systems with non-ideal geometry.

II. Results of numerical calculations and their discussion

The following parameters of lense-shaped QD InAs/GaAs/AlAs were used for numerical calculations. The electron effective mass: $m_1 = 0.04m_e$, $m_2 = 0.067m_e$ $m_3 = 0.15m_e$, potential energies: $V_2 = 0.5\ eV$, $V_3 = 1.66\ eV$, geometric sizes: $\Delta = 5\ nm$, $h = 10\ nm$, $r_s = 23{,}4\ nm$.

In the lens-like multilayer InAs/GaAs/AlAs quantum dot considered in this work, the emergence of a ring-shaped electron density distribution is induced by an external transverse electric field. The electric field directed along the axial axis shifts the electron wave function to the base of the quantum dot. As a result, the electron density is concentrated at a finite radius. This leads to the formation of a quasi-ring state in which the electron motion acquires an azimuthal character, similar to the motion of a narrow quantum ring (Figure 3). With increasing electric field strength, the effective radius of the ring-shaped state increases. Since the Aharonov-Bohm phase shift depends on the magnetic flux $\Phi \propto B\,R^2$, penetrating the ring, an increase in the effective radius changes the period of oscillations of the ground state energy of the electron in the quantum dot with increasing magnetic field induction (Figure 3a,b,c). Therefore, the Aharonov-Bohm effect can be controlled by the electric field.

In the case of the opposite direction of the electric field ($F < 0$), the electron density is concentrated near the axial axis at the top of the quantum dot and the effect of the magnetic field on the energy states is reduced (Figure 3e,f).

With increasing electric field strength ($F > 0$), the magnetic field induction B_1 required for the first change of the ground-state configuration decreases, indicating an electric-field-controlled modulation of the Aharonov–Bohm oscillation period. For $F = 20\ kV/cm$, the first ground-state transition (crossing of the $m = 0$ and $m = -1$ levels) occurs at $B_1 \approx 6\ T$, whereas increasing the field to $F = 100\ kV/cm$ and $F = 200\ kV/cm$ reduces the critical magnetic field to approximately 3.7 T and 2.9 T, respectively (Figure 3a,b,c).

Similar electric-field-induced changes in the Aharonov–Bohm oscillation period have been observed in spherical multilayer quantum dots, where the effect arises from variations in the donor impurity position, leading to a redistribution of the electron density and an effective modification of the enclosed magnetic flux [7-8].

These shifts in the electron energy spectrum caused by coupled electric and magnetic fields directly influence the selection rules and energies of quantum transitions, enabling the development of novel magneto-optical semiconductor nanodevices and applications in quantum

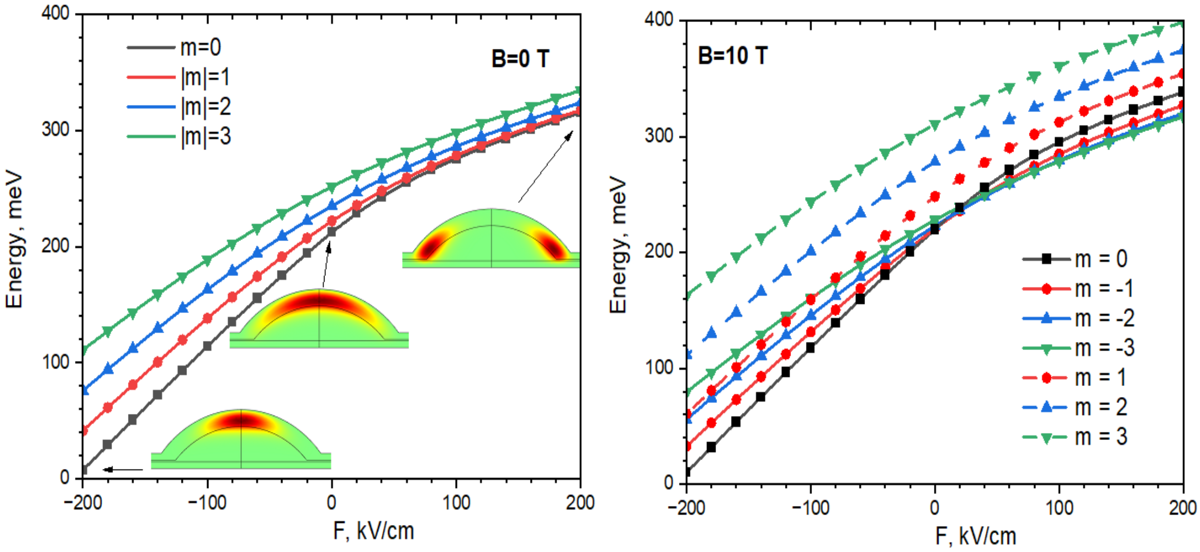

Fig. 2. Dependence of electron energies on electric field strength at $B = 0$ and $B = 10\ T$.

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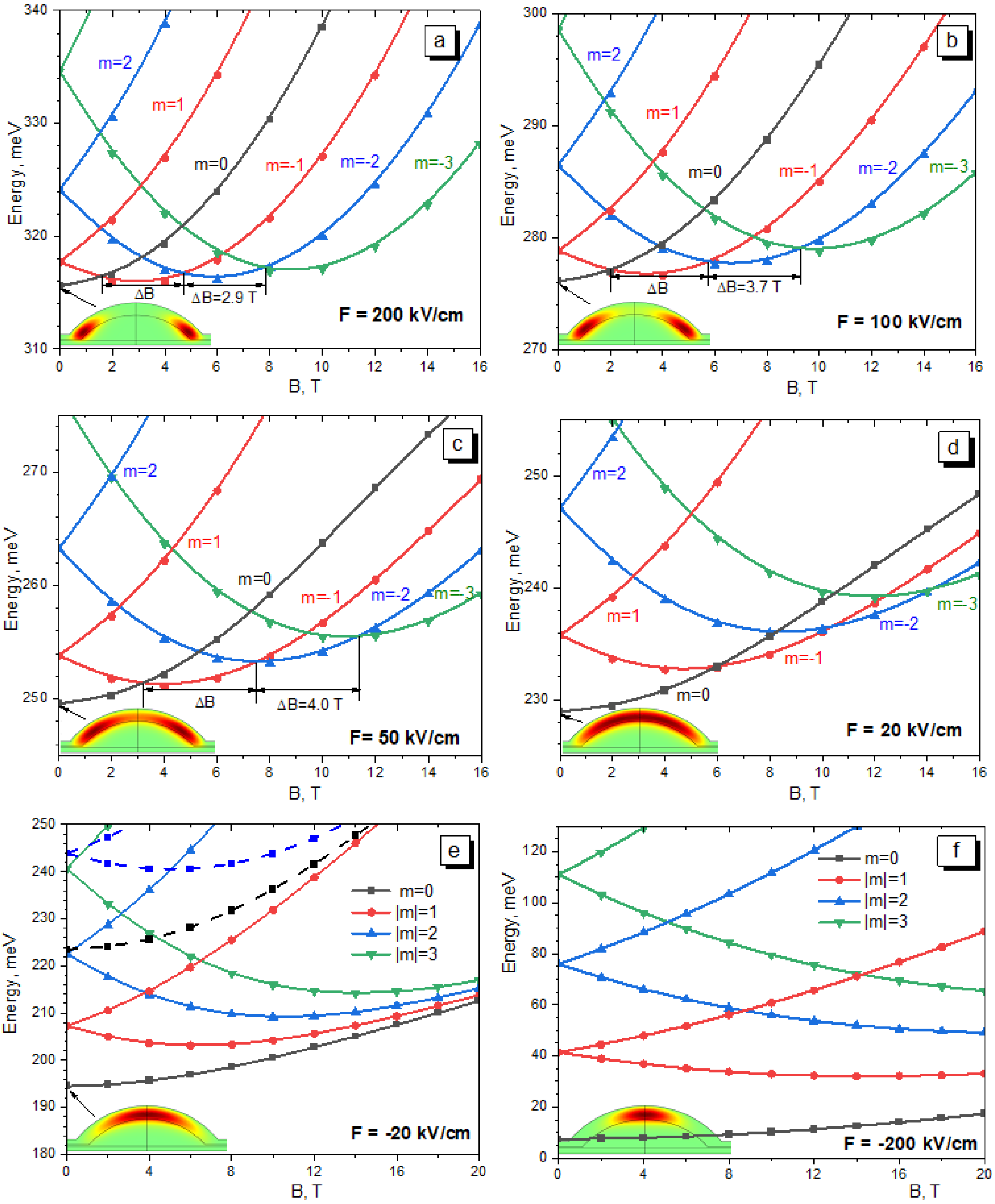

Fig. 3. Dependence of electron energies on magnetic field induction at $F = 200\ kV/cm$ (**a**), $F = 100\ kV/cm$ (**b**), $F = 50\ kV/cm$ (**c**), $F = 20\ kV/cm$ (**d**), $F = -20\ kV/cm$ (**e**), $F = -200\ kV/cm$ (**f**).

Conclusion

The influence of a transverse electric field on the Aharonov-Bohm effect in lens-shaped quantum dots (LQDs) was theoretically studied by solving the Schrödinger equation using the finite element method. We established that the electric field induces a ring-shaped electron distribution, which modulates the AB oscillation period. This occurs because the field increases the effective radius of the electron state, consequently lowering the critical magnetic field required for ground state transitions. These findings confirm the possibility of electrically controlling the AB effect, facilitating the development of novel magneto-optical semiconductor nanodevices and applications in quantum information.

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Ефект Ааронова-Бома в квантових точках у формі лінзи

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Це дослідження теоретично досліджує ефект Ааронова–Бома (АВ) у квантових точках у формі лінзи InAs/GaAs/AlAs у постійних аксіальних магнітних і поперечних електричних полях. Стани електронів розраховуються шляхом розв’язання рівняння Шредінгера за допомогою методу скінченних елементів, реалізованого в COMSOL Multiphysics. Показано, що поперечне електричне поле, прикладене перпендикулярно до основи точки, суттєво змінює топологію основної хвильової функції електрона, індукуючи квазікільцеподібний просторовий розподіл. Утворення цього електроіндукованого стану забезпечує виникнення коливань АВ у зовнішньому магнітному полі. Збільшення напруженості електричного поля модулює період коливань АВ за рахунок збільшення ефективного радіуса квазікільцевого стану та зменшення критичного магнітного поля. Ці ефекти демонструють можливість електричного контролю АВ-ефекту в напівпровідникових наноструктурах.

Ключові слова: наноструктура, напівпровідник, квантова точка, квантове нанокільце, електричне поле.