The Phenomenon of Electroohmic Transformation

Peculiarities of electric current distribution in anisotropic electrically conductive medium are considered and dependences of its longitudinal and transverse components on geometrical factors are established.

In the case of a rectangular plate of length a, height b and width c, the selected crystallographic axes are located in the plane of the side face \((a \times b)\), and one of these axes is oriented at an angle \(\alpha\) to the edge \(a\). Application to the upper and lower end faces of the plate of some potential difference leads to the appearance of longitudinal and transverse components of the flowing electric current. This leads to the possibility of transforming the electric current magnitude. The methods of optimizing the transformation coefficient magnitude which is determined by both the magnitude of the anisotropy of the electrical conductivity of the plate material and the coefficient of its shape \(k = a/b\). The design variants of anisotropic electrically conductive transformers are proposed, one of which is of the spiral shape is characterized by a high value of the transformation coefficient.

Information on existing monocrystalline and artificial anisotropic materials is given. The dependence of the transformation coefficient on the magnitude of the anisotropy \(k\) of the transforming element material is presented.

The perspective materials for the real creation of anisotropic electrically conductive transformers with the necessary functional characteristics. Silicon can be used in this case.

The use of this transformation effect makes it possible to expand the practical use of electroohmic phenomena. This principle of transformation will expand the areas of its use in metrology and measurement technology.

**Key words:** anisotropy, electrically conductive medium, tensor, vector, electric current components, electric current, transformer, transforming element.

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I. Equation of transformation of electric current in anisotropic electrically conductive media

In the general case, the electrical conductivity tensor \(\sigma\) of an anisotropic electrically conducting medium in the case when its main crystallographic axes \(\sigma_{11}, \sigma_{22}\) and \(\sigma_{33}\) coincide, respectively, with the axes \(OX, OY\) and \(OZ\) of the selected laboratory coordinate system \(OXYZ\), has the following form [6]:

\[
\sigma = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}.
\]  

(1)

If the vector of the external electric field \(\vec{E}\), which is applied to such a medium, oriented along one of its main crystallographic axes, for example, \(\sigma_{22}\), coincides with the \(OY\) axis, then an electric current \(\sigma_{22}\) arises in the volume of the medium only in this direction:

\[
\vec{j}_{yy} = \vec{E}_y \cdot \sigma_{22}.
\]  

(2)

Electric current is not observed in other crystallographic directions of the medium.

Another situation arises when the electric field vector \(\vec{E}\) is located in the plane created by the crystallographic axes \(\sigma_{11}\) and \(\sigma_{22}\), one of which, for example, \(\sigma_{11}\), is oriented at an angle \(\alpha\) to the \(OX\) axis of the laboratory coordinate system (Fig. 1a). This arrangement of the axes makes it possible to represent the tensor \(\sigma\) in the following form:
\( \dot{\hat{\sigma}} = \begin{bmatrix} \sigma_{11}\cos^2\alpha + \sigma_{22}\sin^2\alpha & (\sigma_{11} - \sigma_{22})\sin\alpha\cdot\cos\alpha & 0 \\ (\sigma_{11} - \sigma_{22})\sin\alpha\cdot\cos\alpha & \sigma_{11}\sin^2\alpha + \sigma_{22}\cos^2\alpha & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} \)  

and is characterized by the presence of both longitudinal \( \sigma_{ll} \) and transverse \( \sigma_{tt} \) components of the electrical conductivity tensor \( \dot{\hat{\sigma}} \).

The creation of an electric field voltage \( \vec{E}_f \) in the direction \( OT \) by means of external electrodes leads to the appearance of both longitudinal \( \dot{j}_{yy} \) and transverse \( \dot{j}_{xx} \) components of the electric vectors current, which have the following form:

\[
\dot{j}_{yy} = \vec{E}_f(\sigma_{11}\cos^2\alpha + \sigma_{22}\sin^2\alpha), \\
\dot{j}_{xx} = \vec{E}_f(\sigma_{11} - \sigma_{22})\sin\alpha\cdot\cos\alpha.
\]

Optimization of values (4) and (5) by angle \( \alpha - \left( \frac{\partial j}{\partial \alpha} = 0, \frac{\partial^2 j}{\partial \alpha^2} < 0 \right) \) shows that their maximum value is observed at \( \alpha_{opt} = 45^\circ \):

\[
\dot{j}_{yy} = 0.5 \cdot \vec{E}_f(\sigma_{11} + \sigma_{22}), \\
\dot{j}_{xx} = 0.5 \cdot \vec{E}_f(\sigma_{11} - \sigma_{22}).
\]

Consider a rectangular plate with length \( a \), height \( b \), and width \( c \) (Fig. 1a), which is made of a material with certain characteristics of the anisotropy of electrical conductivity \( \dot{\hat{\sigma}} \). Namely, the crystallographic axes \( \sigma_{11} \) and \( \sigma_{22} \) are located in the plane of its lateral face \( (a \times b) \), while one of these axes, for example, \( \sigma_{11} \), is oriented at an angle \( \alpha_{opt} = 45^\circ \) to the \( X \) axis. Applying to the upper and lower faces \( (a \times c) \) a certain potential difference \( U_f \) causes the flow of longitudinal \( I_y \) and transverse \( I_x \) electric currents:

\[
I_y = \frac{U_f}{\sigma_{11} \sigma_{22}} \cdot \frac{b}{ac}, \\
I_x = \frac{U_f}{\sigma_{11} \sigma_{22}} \cdot \frac{a}{bc}.
\]

and the transformation ratio \( n \) of such a device is expressed by the formula:

\[
n = \frac{l_x}{l_y} = \frac{(\sigma_{11} - \sigma_{22})}{(\sigma_{11} + \sigma_{22})} \cdot \frac{b^2}{a^2}.
\]

It should be noted, that in the case under consideration, transformation of both direct and alternating electric currents is possible, in which the value of \( n \) must be determined in the short circuit mode. In this case, the lines of the transformed current \( I_y \) are placed at an angle \( \beta \) (Fig. 1a), which are determined by the following formula:

\[
\beta = \arctg \frac{\sigma_{11} - \sigma_{22}}{\sigma_{11} + \sigma_{22}},
\]

where \( m = \frac{\sigma_{11} - \sigma_{22}}{\sigma_{11} + \sigma_{22}} \) is the transformation coefficient of the material of the anisotropic transforming plate, \( p = \frac{a}{c} \) is the coefficient of its shape.

Fig. 2 shows the dependence of the conversion coefficient \( m \) on the anisotropy value \( k \) of the plate material, from which it follows that with increasing \( k \) the value of \( m \) monotonically increases, reaching saturation at \( k = 50 \) (\( m = 89.1 \% \)).

Thus, the application of Ohm’s law in anisotropic electrically conductive media under certain conditions allows to propose a new approach to the transformation of electricity. This process can be called an electroohmic transformation method [5].
II. Design features of anisotropic electrically conductive transformers (AECT) of alternating current

In general, the choice of a specific transformer design based on an anisotropic electrically conductive material is determined by both the features of the physical phenomena occurring in it and the conditions of its operation [7]. One of the possible design options for a device designed to transform an alternating electric current is shown in Fig. 3.

The basis of such a device is a transforming element in the form of a rectangular plate 1 – length \(a\), height \(b\) and width \(c\) (Fig. 1a) from an anisotropic electrically conductive material. The selected crystallographic axes \(\sigma_{11}\) of the aforementioned plate and \(\sigma_{22}\) are located in the plane of its lateral face \((a\times b)\) (Fig. 3), while the \(\sigma_{11}\) axis is located at an angle \(\alpha_{opt} = 45^\circ\). The left and right sides \((b\times c)\) of this plate contain insulating layers 2 with a dielectric constant \(\epsilon\). The outer sides of these layers, in turn, contain electrically conductive layers 3, to which output electrical leads 6 and 7 are attached. Input electrical leads 4 and 5 are located on the upper and lower end \((b\times c)\) faces of the plate 1.

This design of the AECT provides a uniform distribution of the transforming electric current in the volume of the plate 1 and serves as its protector against electrical shunting by conductive layers 3.

An analysis of the distribution of the density of the transformed current in the volume of plate 1 showed that the orientation of the crystallographic axis \(\sigma_{11}\) at an angle \(\alpha_{opt} = 45^\circ\) leads to some distortion of its equipotentiality, and, accordingly, to a decrease in the value of the transformation coefficient.

Fig. 2. Dependence of the coefficient \(m\) on the value of anisotropy \(k\) of the material of an anisotropic electrically conductive plate: 1 – for monocrystalline materials, 2 – for artificial anisotropic materials.

Fig. 3. Schematic design of AECT alternating electric current: 1 – transforming element; 2 – electrical insulating layers; 3 – electrically conductive layers; 4, 5 – input and 6, 7 – output electrical contacts.

To eliminate this phenomenon, in some cases, the orientation of the \(\sigma_{11}\) axis must be carried out at an angle \(\gamma = \alpha_{opt} - \beta\) (Fig. 1b). In this case, the lines of current densities, transformed are not bent, and the value of the transformation ratio \(n_1\) will be determined by the
following expression:

\[ n_1 = \frac{(\sigma_{11} - \sigma_{22}) \sin \gamma \cos \gamma}{\sigma_{11} \cos^2 \gamma + \sigma_{22} \sin^2 \gamma} \cdot \frac{a}{b}. \]  

(12)

This constructive approach makes it possible to eliminate the curvature of the distribution of electric currents in the volume of plate 1.

The electrical circuit of such a device relative to the input electrical terminals 4 and 5 is an active resistance \( R_1 \), whose value is equal to:

\[ R_1 = \frac{\sigma_{11} + \sigma_{22}}{\sigma_{11} \sigma_{22}} \cdot \frac{b}{ac}. \]  

(13)

and its output impedance with respect to terminals 6, 7 is of the active-capacitive, the value of \( Z \) is determined by the following expression:

\[ Z = \sqrt{R_2^2 + \frac{1}{\omega^2 c^2}}, \]  

(14)

where:

\[ R_2 = \frac{\sigma_{11} - \sigma_{22}}{\sigma_{11} \sigma_{22}} \cdot \frac{a}{bc}. \]  

(15)

\[ \varepsilon = \varepsilon_1 \cdot \varepsilon_2 \cdot \frac{bc}{\Delta^2}. \]  

(16)

where \( \Delta \) is the thickness of the insulating layer, \( \omega = 2\pi f \) is the frequency of the transforming current.

Thus, the considered device has an active input and active-capacitive output resistance and can be used to transform only alternating electric current.

If necessary, AECT with a large value of the transformation ratio \( n_3 \), transforming element 1 (Fig. 3), which is its basis, is characterized by large linear dimensions. This feature leads to some limitation of the possibilities of its practical application. This limitation is eliminated by the following construction of AECT shown in Fig. 4.

Such a device consists of a transforming element 1 in the form of a plate with length \( a \), height \( b \) and width \( c \) based on anisotropic electrically conductive material. This plate is rolled into a spiral and is a disk of height \( b \) with outer \( r_1 \) and inner \( r_2 \) radii, respectively.

The upper and lower faces of this disk with area \( S = \pi (r_1^2 - r_2^2) \) contain dielectric layers with thickness \( \Delta_1 \), on the outer sides of which in In turn, electrically conductive layers with a thickness of \( \Delta_2 \) are placed. Input electrical contacts 4, 5 are located, respectively, on the inner and outer end faces \((b \times c)\) of the disk. The output terminals of such a transformer are located on the outer sides of the electrically conductive layers 3. One of the side faces \((a \times b)\) of the transforming spiral element 1 contains an electrically insulating layer 8 with a thickness of \( \Delta_3 \) \( c \) of a dielectric material.

In this case, the transformation ratio \( n_3 \), the length of the transforming element \( a \) and the number of turns \( N \) of the spiral are related to each other by the following relationships:

\[ a = \frac{\pi (r_1^2 - r_2^2)}{c + \Delta_3}, \]  

(17)

\[ N = \frac{\pi (r_1^2 - r_2^2)}{c + \Delta_3}, \]  

(18)

\[ n_3 = \frac{\sigma_{11} - \sigma_{22}}{\sigma_{11} + \sigma_{22}} \cdot \frac{\pi (r_1^2 - r_2^2)}{b(c + \Delta_3)}. \]  

(19)

Comparison of the geometric dimensions of these structures shows that at high values of the transformation ratio, the second structure is more adapted for its practical use. Thus, the spiral design of the AECT makes it possible to reduce its linear dimensions at high values of the transformation \( n_3 \).

![Fig. 4. Construction of spiral AECT: 1 – a transforming element in the form of a plate with length \( a \), height \( b \) and width \( c \) coiled into a spiral with outer \( r_1 \) and inner \( r_2 \) radii; 2 – electrical insulating layers; 3 – layers; 4, 5 and 6, 7 – input and output electrical contacts, respectively; 8 – interturn electrical insulating layer.](image-url)
III. Anisotropic electrically conductive materials and their optimization

Currently, both natural and artificial anisotropic electrically conductive materials are known. The first class includes, for example, some elements of the fifth group of Mendeleev’s periodic table of elements such as bismuth (Bi) and antimony (Sb), for which \( k = 1.2 \div 1.5 \). The second class should include artificial materials that are obtained by sequential synthesis methods and directional crystallization, for example, \( CdSb \), \( Bi_2Te_3 \), as well as eutectic needle compositions \( CdSb-CoSb \) [8], \( ZnAs-As \) [9], the value of \( k \) for which is within \( 1.6 \div 3 \).

The use of such materials in transformers makes it possible to obtain the value of the conversion coefficient \( m = (9 \div 60.6)\% \), which is clearly insufficient for solving modern practical problems. A further increase in the value of the coefficient \( m \) is possible in the case of using artificially anisotropic materials, the calculation procedure for which is given in [10].

In the case of representation of an artificially anisotropic electrically conductive medium in the form of a rectangular parallelepiped of length \( l \), height \( h \) and width \( s \) (Fig. 5), made of vertically arranged alternating layers 1 and 2 with thicknesses \( d_1 \) and \( d_2 \), respectively, characterized by electrical conductivities \( \sigma_1 \) and \( \sigma_2 \) (\( \sigma_1 >> \sigma_2 \)).

The values of the longitudinal \( \sigma_{\|} \) and transverse \( \sigma_{\perp} \) components of the electrical conductivity tensor \( \vec{\sigma} \) of the medium are determined by the following expressions:

\[
\sigma_{\|} = \frac{\sigma_1 d_1 + \sigma_2 d_2}{d_1 + d_2}, \tag{20}
\]

\[
\sigma_{\perp} = \frac{\sigma_1 \sigma_2 (d_1 + d_2)}{\sigma_1 d_1 + \sigma_2 d_2}, \tag{21}
\]

and the value of the thickness \( d_1 \) and \( d_2 \) are related by the following relations:

\[
d_1 = d_2 \sqrt{\frac{\sigma_2}{\sigma_1}}. \tag{22}
\]

Thus, by selecting the appropriate materials for layers 1 and 2, as well as their thicknesses, an artificially anisotropic electrically conductive material with the desired conversion coefficient \( m \) is obtained.

For an AECT design with a transforming element based on an optimized artificially anisotropic material shown in Fig. 6, the transformation ratio \( n_4 \) is determined by the following expression:

\[
n_4 = \frac{(\sigma_1 d_1 + \sigma_2 d_2) (\sigma_1 d_2 + \sigma_2 d_1) - \sigma_1 \sigma_2 (d_1 + d_2)^2}{(\sigma_1 d_1 + \sigma_2 d_2) (\sigma_1 d_2 + \sigma_2 d_1) + \sigma_1 \sigma_2 (d_1 + d_2)^2}. \tag{23}
\]

Thus, the possibility of free choice of the appropriate materials leads to the real creation of AECT with the required functional characteristics.

Silicon (Si) is especially promising in this respect, which, depending on the degree of structural perfection, has both dielectric and metallic properties. The use of planar technology allows in this case to obtain anisotropic electrically conductive materials with \( k = 60 \div 80 \).

Devices based on the transformation phenomenon

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Fig. 5. Model of the structure of an artificially anisotropic conductive medium: 1 – a layer of material with electrical conductivity \( \sigma_1 \) and thickness \( d_1 \); 2 – a layer of material with electrical conductivity \( \sigma_2 \) and thickness \( d_2 \).

Fig. 6. Anisotropic transforming element with artificial anisotropic material.
considered above can be used as matching elements of various broadband systems, as well as nodes and blocks of electronics, instrumentation, metrology and computer technology. The use of the considered transformation principle will expand the scope of its use in metrology and measuring technology.

Conclusion

The possibility of transformation of alternating electric current by anisotropic electrically conductive media is shown. The above transformation principle will expand its use in various fields of science and technology.

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