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First Principles DFT Calculations of the Optical Properties of A₄BX₆ Group Crystals

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The results of investigating of the electron band energy structure and optical properties of A₄BX₆ (Tl₄HgI₆ and Tl₄CdI₆) group crystals are presented. The energy band structures of Tl₄HgI₆ and Tl₄CdI₆ crystals are calculated from the first principles within generalized gradient approximation (GGA). The band structure and reflection index were calculated using a pseudopotential method in the framework of density functional theory. Optical absorption edge in Tl₄HgI₆ and Tl₄CdI₆ is formed by direct optical transitions. The spectral dependence of the reflection index was calculated on the basis of the energy band results with using the Kramers–Kronig method. The spectra show pronounced anisotropy in $E||a(b)$ and $E||c$ polarizations. It was found the anomalous by large values of the birefringence ($\Delta n > 0.18$ for Tl₄HgI₆ and $\Delta n > 0.03$ for Tl₄CdI₆) in the visible and near infrared region.

Keywords: optical constants, birefringence, electron band-energy structure.

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Introduction

Searching for new functional materials and ways to control their properties belongs to the primary tasks of physics of semiconductors and insulators. Wide-gap semiconductor crystals have been used extensively in X- and γ -ray detectors, the chemical industry, and biomedicine [1-4]. Designing room temperature radiation detectors is an urgent problem of materials science. The A₄BX₆ crystals has a wide band gap ($E_g = 2.08$ eV [5] for Tl₄HgI₆ and $E_g = 2.83$ eV [6] for Tl₄CdI₆). The presence of thallium enhances the X- and γ -ray absorption coefficient of the material due to the high density ($\rho = 7.19$ g/cm³ [7] for Tl₄HgI₆ and $\rho = 6.87$ g/cm³ [7] for Tl₄CdI₆) and large atomic number Z ($Z_{\text{Ti}} = 81$, $Z_{\text{Hg}} = 80$, $Z_{\text{Cd}} = 48$, and $Z_{\text{I}} = 53$).

By now, the optical, mechanical, spectral, and energetic characteristics of the A₄BX₆ crystals have been well-investigated [2]-[14]. The energy band spectrum [6, 11, 13, 14] is calculated using the local density approximation (LDA) and exchange correlation functional based on the generalized gradient

approximation (GGA) in the Perdew, Burke, and Ernzerhof (PBE) parametrization.

For today we have not found in literature any work either theoretical or experimental on birefringence (BRF) of A₄BX₆ group crystals (except Tl₄HgI₆ [13]). In this paper, we presented the results of calculations of the optical constants of BRF of A₄BX₆ crystals.

I. Theoretical Calculation Technique

The theoretical calculations were made using the electron density functional theory (DFT). For ionic potentials, the ultrasoft Vanderbilt pseudopotentials were used [15]. To describe the exchange correlation energy of the electron subsystem, the GGA functional with the PBE parameterization [16] was used. To determine the difference between the experimental [7] and equilibrium theoretical lattice parameters (see Fig. 1 and Table 1), we used the structure-optimization procedure, which consists of finding the minimum full energy as a function of the crystal unit-cell volume [17].

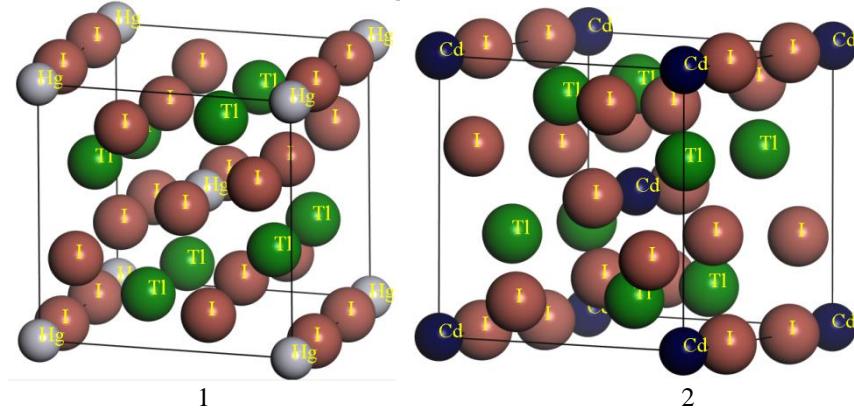


Fig. 1. Views of the initially optimized structure of the Tl_4HgI_6 (1) and Tl_4CdI_6 (2) crystals.

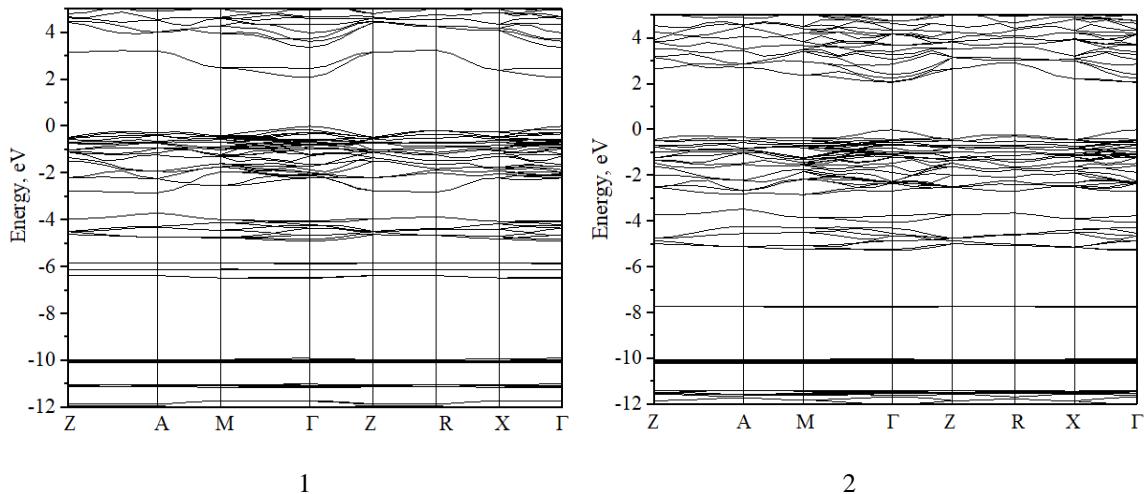


Fig. 2. Band energy diagram of the Tl_4HgI_6 (1) [13] and Tl_4CdI_6 (2) [6] crystals.

A value of $E_{\text{cut-off}} = 310$ eV for the energy of cutting-off the plane waves was used in calculations. The electron configurations of atoms are as follows: Tl — [Xe] 5d¹⁰6s²6p¹, I — [Kr] 5s²5p⁵, Hg — [Xe] 5d¹⁰6s², Cd — [Kr] 4d¹⁰5s² where [Kr] and [Xe] denote the inner core states. The integration over the Brillouin zone was performed on the $2 \times 2 \times 2$ grid of k points by the Monkhorsta-Pack scheme [18]. A self-consistent convergence of the total energy was taken as 5.0×10^{-7} eV/atom. Geometric optimizations of the lattice parameters and the atomic coordinates were performed using a Broyden–Fletcher–Goldfarb–Shanno minimization technique. The maximal ionic Hellmann–Feynman forces were set within 0.01 eV/Å, the maximum ionic displacement within 5.0×10^{-4} Å, and the

maximum stress within 0.02 GPa.

The simplest way to obtain data similar to the experimental ones is to use a so-called scissors operator, which changes the band gap by shifting the conduction bands toward higher energies [19]. This operator is based on the similarity of the dispersion dependences of the conduction band energies $E(\mathbf{k})$ determined by solving the Kohn–Sham equations [20]. The conduction bands of the calculated energy spectrum are usually shifted until they coincide with the experimental minimum energy gap E_g of the crystal.

II. Results and Discussion

Figure 2 shows the full energy band diagrams of the A_4BX_6 group crystals along the high-symmetry lines of the tetragonal Brillouin zone. Here, the energy is counted from the Fermi level.

The energy band diagram was constructed using the points of the Brillouin zone in the inverse space, which were as follows: Z(0, 0, 0.5), A(0.5, 0.5, 0.5), M(0.5, 0.5, 0), Γ (0, 0, 0), R(0, 0.5, 0.5), X(0, 0.5, 0).

An analysis of the results of theoretical calculations of the energy band spectrum shows that the smallest energy bandgap is localized in the center zone of the Brillouin zone (point Γ). This means that the crystal is

Table 1

Comparison of the experimental and equilibrium theoretical lattice parameters for A_4BX_6 group crystals

Crystal	a , nm	b , nm	c , nm
Tl_4HgI_6	0.9446 [7]	0.9446 [7]	0.92600 [7]
	0.9664	0.9664	0.9524
Tl_4CdI_6	0.9231 [7]	0.9231 [7]	0.9592 [7]
	0.9225	0.9225	0.9597

characterized by a direct energy bandgap.

The calculated value of the band gap E_g of Tl_4HgI_6 and Tl_4CdI_6 is estimated as 1.27 eV and 2.02 eV, respectively. In this relation, it is well known that DFT-based calculations of semiconductors in the LDA and GGA levels of theory usually underestimate the band gap E_g [19]. Using the experimental data of the band gap ($E_g = 2.08$ eV [5] for Tl_4HgI_6 and $E_g = 2.83$ eV [6] for Tl_4CdI_6) we have obtained the value of “scissor” factor being equal to 0.81 eV (the “scissor” factor may be used for comparison of theoretical and experimental optical spectra in the range of electron excitations).

When studying the optical properties, it is convenient to use complex dielectric function ϵ . Its imaginary part ϵ_2 can be calculated from the momentum matrix elements between the occupied and unoccupied wave functions [13]. Real part ϵ_1 of the dielectric function can be found from its imaginary part using the known Kramers–Kronig relationship [13]. Using the relationship:

$$n = \sqrt{\frac{(\epsilon_1^2 + \epsilon_2^2)^{1/2} + \epsilon_1}{2}}, \quad 1$$

as well as the calculated spectra of the real and imaginary

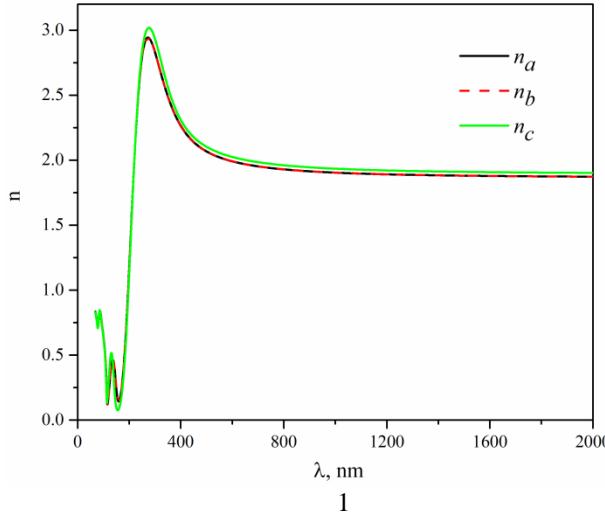


Fig. 3. Calculated spectra of the refractive index for three major crystallooptical directions of the Ti_4CdI_6 (1) and Ti_4HgI_6 (2) crystals.

parts of the dielectric function, the spectral dependences of the refractive index was found (Fig. 3). As can be seen from Fig. 3, the refractive indices satisfy the relation $n_c > n_{a,b}$ above 400 nm.

It can be seen that in the main spectral transmission region the refractive index increases towards shorter wavelength [21–29]. Such a behavior can be caused by increasing the exciton-phonon interaction and contribution of absorption coefficient in the process of formation of fundamental absorption edge.

Figure 4 presents the calculated anisotropic BRF dispersion for the A_4BX_6 crystals. The theoretical spectra show very good agreement with the experiment [13]. It is seen that the dispersion is normal ($d\Delta n/d\lambda < 0$) and the variation in the BRF itself with the wavelength is considerable ($d\Delta n/d\lambda \sim -3.03 \times 10^{-4} \text{ nm}^{-1}$ for Ti_4HgI_6 and $d\Delta n/d\lambda \sim -1.1 \times 10^{-5} \text{ nm}^{-1}$ for Ti_4CdI_6).

The anomalous by large values of difference ($\Delta n > 0.18$ for Ti_4HgI_6 and $\Delta n > 0.03$ for Ti_4CdI_6) of the BRF attract our attention for visible region of light. To our mind this behavior is caused by strong anisotropy of the optical functions ϵ_1, ϵ_2 [14]. For A_4BX_6 crystals, a comparison of BRF with other known crystals was given

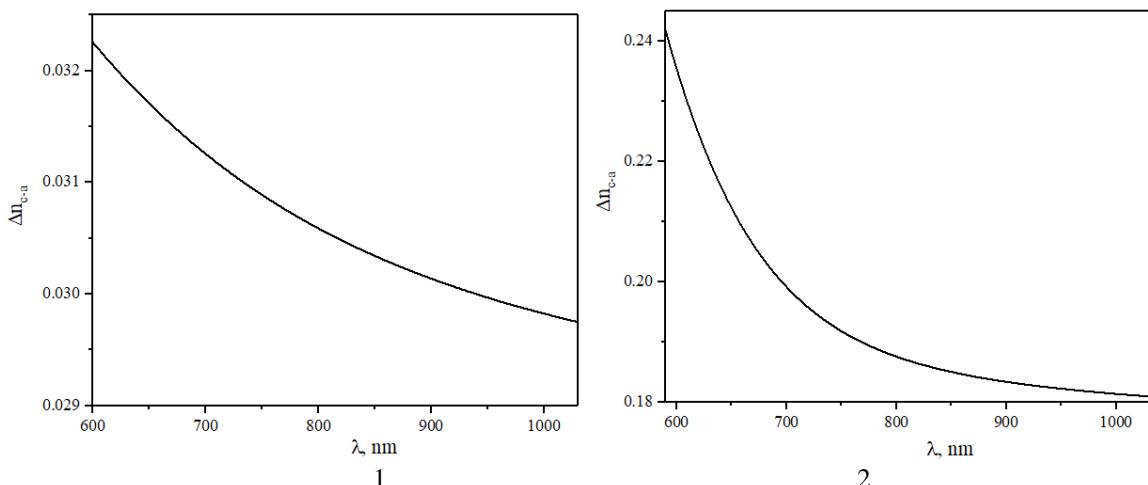
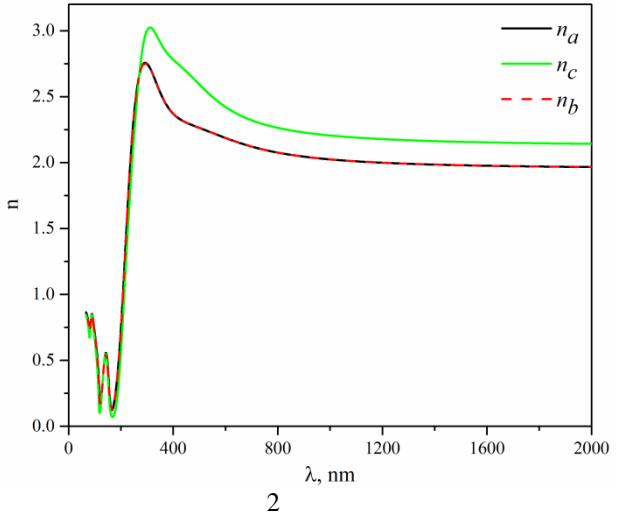
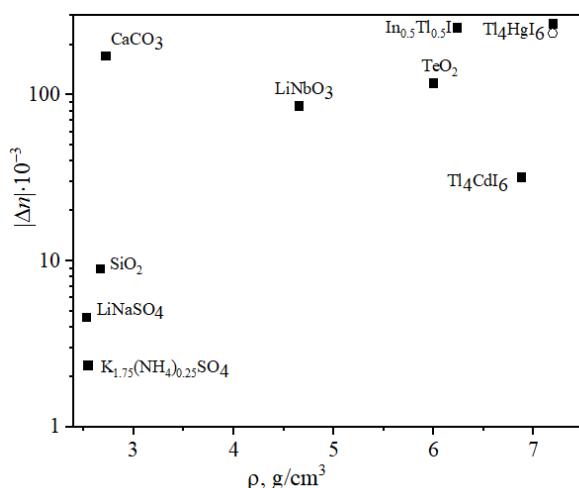


Fig. 4. Calculated spectra of BRF $\Delta n(\lambda)$ for Ti_4CdI_6 (1) and Ti_4HgI_6 (2) crystals.

Table 2Comparison of the BRF for A_4BX_6 crystals with other known crystals

Crystal	Crystal structure	BRF (Δn)	Density, g/cm ³
$CaCO_3^*$	Hexagonal	-0.172	2.715
TeO_2^*	Hexagonal	-0.118	5.99
$LiNbO_3^*$	Hexagonal	-0.086	4.65
SiO_2^*	Hexagonal	+0.009	2.66
$In_{0.5}Tl_{0.5}I$	Orthorhombic	+0.255 [27]	6.234(2) [30]
$K_{1.75}(NH_4)_{0.25}SO_4$ [31]	Orthorhombic	+0.00237	2.539
$LiNaSO_4$ [32]	Hexagonal	+0.0046	2.525
Tl_4HgI_6	Hexagonal	+0.235	7.19 [7]
Tl_4HgI_6	Hexagonal	+0.269 [13]	7.19 [13]
Tl_4CdI_6	Hexagonal	+0.032	6.87 [6]

*data taken from Ref. [33]

**Fig. 5.** Dependence of $|\Delta n| \sim f(\rho)$ for listed crystals from Table 2 (open circle at Tl_4HgI_6 correspond to our data).

in Table 2 (at room temperature for $\lambda \sim 600$ nm). Dependence of $|\Delta n| \sim f(\rho)$ for listed crystals from Table I present on Fig. 5.

Conclusion

The calculations of the energy band structure and

optical spectra of a A_4BX_6 crystals within the framework of density functional theory allow us to make the following conclusions. It was established from the energy band diagrams that the minimum band gap is localized at the center of the Brillouin zone (at the Γ point). This means that the crystals are direct-gap. The spectral dependence of the refractive index was calculated on the basis of the energy band results with using the Kramers–Kronig method. It was found the anomalous by large values of the BRF in the visible and near infrared region (600 - 1100 nm).

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Оптичні властивості кристалів групи A₄BX₆: розрахунок з перших принципів

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Представлено результати дослідження електронної енергетичної структури та оптичних властивостей кристалів групи A₄BX₆ (Tl₄HgI₆ та Tl₄CdI₆). Електронна енергетична структура кристалів Tl₄HgI₆ та Tl₄CdI₆ обчислюється за першими принципами в рамках узагальненого градієнта апроксимації (GGA). Зонну структуру та показник заломлення розраховували за допомогою псевдопотенційного методу в рамках теорії функціоналу густини. Край оптичного поглинання в Tl₄HgI₆ та Tl₄CdI₆ сформований прямими оптичними переходами. Спектральну залежність показника заломлення розраховували на основі результатів електронної енергетичної структури за допомогою методу Крамерса – Кроніга. Спектри демонструють виражену анізотропію в поляризаціях $E||a(b)$ та $E||c$. Виявлено аномально велике значення показника двопроменезаломлення ($\Delta n > 0.18$ for Tl₄HgI₆ and $\Delta n > 0.03$ for Tl₄CdI₆) у видимій та близькій інфрачервоній області.

Ключові слова: оптичні константи, двопроменезаломлення, електронна енергетична структура.