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Free Vibration Analysis of Sandwich Plate-Reinforced Foam Core Adopting Micro Aluminum Powder

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This paper presents an analytical study of the free vibration behavior of foam core sandwich plates strengthened with Aluminum micro spherical powder. Sandwich plate with a polyurethane foam core sandwiched between two Aluminum faces. To calculate the natural frequencies, use Kirchhoff's theory to drive the equation of sandwich plate vibration. Microparticle composite equations evaluated the stiffness characteristics of a foam-Aluminum core. The findings reveal that the impact of filling foam is effective; according to the free vibration analysis, the sandwich plate's free vibration and static behavior can be improved by using micro spherical powder foam in the vacant spherical gaps of the foam core. In comparison to other cores, the core foam-aluminum sandwich plate deflects less.

Keywords: sandwich plate; analytical solution; free vibration; polyurethane foam; aluminum powder.

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Introduction

The attractive sandwich composite was employed in aerospace, buildings, vehicles, etc. . It comes with lower weight, higher stiffness, high energy absorption, cost saving, and many other distinctions. The wide range of applications chooses the appropriate sandwich structure depending on the behavior and environment of the application [1]. At the same time, the sandwich structure is two parallel thin faces, upper and lower, separated by a thick core glued together. The face and core materials can be made of mantellic, non-mantellic, composite, or each alone, relying on the above [2]. Higher stiffness to lower weight ratio is today's corporate competition war. The structural components of the sandwich are shown in Fig. 1.

The modification for the sandwich plate structure included multi-steps, from reinforcement by fiber materials [3] to the additive of Nanomaterial [4], to the improvement of the static and dynamic characterizations for composite structure, which is used in multi-applications, [5]. Then, in the current year, the modification of the plate structure included an increase in the static and dynamic characterizations of the plate with a decrease in the weight of the plate and an increase in the strength (increasing the strength-to-weight ratio) by using a different technique for mechanical properties such as using of the reinforcement of polymer core materials and using functionally graded materials for solid and porous structure, [6-7]. In addition, using the Nanomaterials to reinforce the foam and functionally graded materials porous core structure modified the structure's mechanical properties with low weight increase [8]. In many applications, the core type used in sandwich panels is corrugated, honeycomb, lattice, solid, and foam. The diversity in core types comes from the demand for better performance with less weight [9]. Foams are extraordinary existence and can be found any ware around. Whenever bubbles congregate together, forming a foam, unique patterns that vary and continue to develop as they age, modify and lose liquid, resulting in such a lightweight substance. Often, they are disorganized combinations of various sizes of bubbles. However, orderly structures can be found [10].

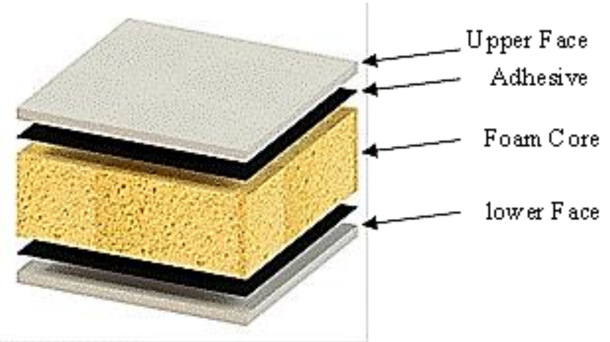

Fig. 1. Sandwich Structure Components with foam core.

Solid foams are a lightweight cellular engineering material commonly utilized in engineering design as the core material in composite sandwich constructions. They may be categorized based on the material type and can be foamed as metallic foams, ceramic foams, polymeric foams, and others. Polymeric foams are the most prevalent variety, and they are also known as plastic foams, cellular plastics, foamed plastics, expanded polymers, and so on. Chemical interactions between two or more substances generate a particular gaseous product, confined in a bubble phase and forming a foam [11].

Recently, many works have been proposed to improve the composite design. Methods using composite face sheets and metal foams cores [12], [13], [14], Aluminum matrix syntactic foam cores [15], Aluminium foam sandwich structures [16], and thermoplastic foams [17] have been proposed to modify the impact strength, dynamic characteristics, and enhancement strength and mechanical behavior of composite structures. Qusai H. Jebur et al. [18] conducted experimental work on a low-density polyethylene extruded foam. The average cell shape in the foamy direction is nearly 20% longer than in the transverse direction. Manmohan D. Goel et al. [19] achieved modeling and numerical simulation of foam sandwiches exposed to impulsive stress. Stiffened sandwich foam panels significantly enhance resistance to impulsive loads. According to [20], an analytical solution for vibration analysis of the functionally graded sandwich structures was proposed using polymer core and metallic skins based on various parameters. Murat Sen et al. [21] developed an analytical solution to describe the impacts of PU foam reinforcement thickness. The results show that with increasing PU foam reinforcement thickness, natural frequencies and damping ratios were seen to rise. The free vibration characteristics of composite sandwich plate PVC viscoelastic core were explored by Seyed Rezvani et al. [22].

Zhiqiang Fan et al. [23] developed a new constitutive relation for reinforced cenosphere polyurethane syntactic foam with a hierarchical cell structure using the stress-strain response under a wide strain rate range through dynamic compression tests. Many researchers have concentrated on the mechanical analyses of composite structures. Arunkumar et al. [24] analyzed the bending and free vibration of a Foam-filled truss core sandwich. The research was done theoretically and numerically. The impact of foam is effective in cellular and trapezoidal cores, according to the free vibration study. The failure characteristics of sandwich plates comprised of composite skins and foam cores subjected to impact loads are studied by Shuchang Long et al. [25] using experimental and finite element analysis (FEA) techniques. The model has the benefit of accurately mimicking failure aspects. Cigdem Caglayan et al. [26] used amorphous silica nanoparticles (SiO_2) as a dry powder and polyol to conduct experimental research to check the performance of the composite sandwich panel. The findings indicated that the shear thickening fluids filled PU foam core sandwich composites responded with a smaller damage breadth

Xudong Zhao et al. [27] investigated the flexural characteristics of hybrid polyurethane (PU) foam core theoretically and experimentally. The panel with the hybrid PU foam core shows apparent adequate strength, the failure ability can be increased substantially, and the hybrid PU foam core made the structure show a more ductility behavior. Moreover, Mylena L. Oliveira et al. [28] studied the effects of using Al powder to reinforce PU foam experimentally. Addition powder hydrogen-bonded to PU molecular chains and worked as a crosslinking agent, boosting the density. Thermodynamic stability and impact have been enhanced. Using numerical and experimental studies, Hosein Andami et al. [29] investigated the performance of stiff polyurethane foams as energy-absorbent cores. Increasing foam thickness significantly improved the mitigation capability. Pareta et al. [30] conducted an experimental study examining the mechanical properties of the sandwich plate with a PU foam core. Fly ash (FA) reinforcing particle in PU foam was used. The results of this investigation show that the mechanical properties were significantly improved. Beatrise Sture et al. [31] studied rigid polyurethane (PU) foam composites reinforced with sawdust, micro cellulose, and nano cellulose. According to an experimental investigation, inclusion did not substantially improve features.

P. Mohammadkhani et al. [32] explored the influence of steel wire on the energy absorption of a PU foam core. The study was done experimentally and numerically. The reinforcement enhances impact resistance, also potentially saving money. Furthermore, Atsushi Koyama et al. [33] presented an experimental study on the dynamic deformation properties of rigid polyurethane foam. It is concluded that the restriction circumstances and deformation direction influenced dynamic characteristics. Keleshteri and Jelovica [34] investigated the free vibration and buckling behavior of cylindrical sandwich panels with functionally graded metal foam core using third-order shear deformation theory.

The preceding investigations were focused on, as can be seen from the reviewed studies, the earlier research was primarily experimental and numerical, with a few theoretical studies. The majority of them are vibration-free, buckling, and flexural. They adopted a different type of reinforcing material, as can be seen above. This study investigates the vibration analysis for sandwich plate structures with Reinforced Foam Core. The solution for plate structure with Reinforced Foam Core includes calculating the natural frequency and dynamic response for a plate with different core parameters. Analytical technique to study the free vibration of the rectangular sandwich plate using two identical face sheets of aluminum and in between core of polyurethane (PU) foam, the (PU) foam to be strengthened with particle reinforcement adopting microparticles size of aluminum powder. The work was done theoretically after driving the differential governing equations with Kirchhoff theory under the assumptions that were made and using the equation that controls the (PU) foam/aluminum powder composite to conclude the modules of elasticity and the passion ratio, later to calculate the natural frequencies adopting varying densities of (PU) foam using MS Excel Sheet.

I. Microspherical reinforcement

Reinforcements associated with polymer matrices are micro spherical Reinforcements as Particulate composites. These Reinforcements are solid or hollow micro balls with dimensions ranging (from 10 to 150)μm.

Volume fraction ∀f can be as high as 50%.

Modulus of elasticity $E_{Reinforce}$"E_{Matrix}

Defining the constant that controls micro-spherical reinforcements (k) [35].

$$k = \frac{E_{fo}}{3(1-2v_{fo})}\left[1 + 3\left(\frac{1-v_{fo}}{1+v_{fo}}\right)\frac{v_{fo}}{1-v_{fo}}\right] \tag{1}$$

Where (E_{fo}, G_{fo} And v_{fo}) modulus of elasticity, modulus of rigidity, and the passion ratio, which are the elastic constants of (PU) Foam as matrix material, and (E_f, G_f And v_f) are the elastic constants of the composite (matrix + Reinforcements) are provided by the following relations:

$$G_f \approx \frac{E_{fo}}{2(1+v_{fo})}\left[1 + \frac{15}{2}\left(\frac{1-v_{fo}}{4-5v_{fo}}\right)\frac{v_{fo}}{1-v_{fo}}\right],$$
$$v_f \approx \frac{1}{2}\left(\frac{3k-2G}{3k+G}\right), \tag{2}$$
$$E_f \approx \frac{9KG}{3k+G}.$$

II. Analytical solution

To begin, the differential governing equations that define the sandwich structure's vibration behavior is derived. The equilibrium approach with Kirchhoff's theory, often known as classical plate theory, is a theory of thin plates. The following are the assumptions of the linear, elastic, small-deflection bending theory for thin plates [36].

1. The plate's thickness (*h*) is minuscule compared to its lateral dimensions.

2. Given that the plate's material is elastic, homogenous, and isotropic.

3. Initially, the plate is flat.

4. Midplane deflection is minimal. As a result, the slope of the deflected surface is very modest, square of the slope is insignificant compared to unity.

5. Straight lines normal to the middle plane stay straight and normal during deformation, and their length is unaffected. The vertical shear strains γ_{xz} & γ_{yz} are negligible, and the normal strain ε_{zz} may also be disregarded.

6. Because the stress is normal in the middle plane, σ_{zz}, is minor in comparison to the other stress components. It can be overlooked in stress-strain relationships.

7. Because the displacements of a plate are modest, the center surface is considered to remain unstrained after bending.

Using the prior assumptions and the equilibrium method, the differential equation of forced, undamped motion of plates (equilibrium in the z direction) has the form [37].

$$D\nabla^2\nabla^2 w(x,y,t) = p(x,y,t) - \rho h\frac{\partial^2 w}{\partial t^2}(x,y,t)$$
$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \rho h\frac{\partial^2 \omega}{\partial t^2} = p_z \tag{3}$$

As M_{xx}, M_{yy}, and M_{xy} Are the plate's bending and twisting moments per unit length and $W(x,y,t)$ is the z-direction deflection of the plate. The moments on a sandwich plate and their directions are shown in Fig. 2. For free vibration $p_z = 0$

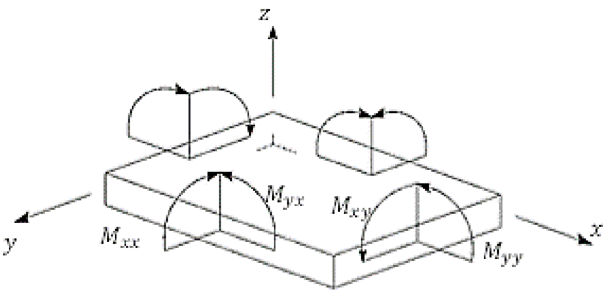

Fig. 2. Moments on flat sandwich layer.

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \rho h\frac{\partial^2 \omega}{\partial t^2} = 0$$

Displacement of strain According to the Kirchhoff hypothesis, the laminate may be stated in terms of the transverse displacement of the center surface of the plate, which results from the elastic body being strained due to the applied load.

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix} = \begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial u}{\partial y}\\ \frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x}\end{Bmatrix} == \begin{Bmatrix}\frac{\partial w_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial w_0}{\partial y} - z\frac{\partial^2 w}{\partial y^2}\\ \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} - 2z\frac{\partial^2 w}{\partial x\,\partial y}\end{Bmatrix} \tag{4}$$

Stresses can be described as transverse displacement thanks to stress-strain relationships [37].

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix} = \begin{bmatrix}\frac{E_{xx}}{1-v_{xy}v_{yx}} & \frac{E_{yy}v_{xy}}{1-v_{xy}v_{yx}} & 0\\ \frac{E_{yy}v_{xy}}{1-v_{xy}v_{yx}} & \frac{E_{yy}}{1-v_{xy}v_{yx}} & 0\\ 0 & 0 & G_{xy}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix} \tag{5}$$

The mechanical properties of plate materials sections in the x and y directions are E_{xx}, E_{yy}, G_{xy}, v_{xy}, v_{yx}.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{xx}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial x^2} + \frac{E_{yy}\nu_{xy}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial y^2} \\ \frac{E_{xx}\nu_{xy}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial x^2} + \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}} \frac{\partial^2 w}{\partial y^2} \\ 2G_{xy} \frac{\partial^2 w}{\partial x \, \partial y} \end{Bmatrix} \tag{6}$$

Stress Relationships for each layer of the sandwich plate can be expressed in terms of each property:

- Mechanical properties of the upper face part:

$E_{xx} = E_{yy} = E_u$ & $G_{xy} = G_u = \frac{E_u}{2(1+\nu_u)}$, also $\nu_{xy} = \nu_u$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_u}{1-\nu_u{}^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu_u \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_u}{1-\nu_u{}^2} \left(\nu_u \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_u}{(1+\nu_u)} \frac{\partial^2 w}{\partial x \, \partial y} \end{Bmatrix} \tag{7}$$

- Mechanical properties of the lower face part:

$E_{xx} = E_{yy} = E_l$ & $G_{xy} = G_l = \frac{E_l}{2(1+\nu_l)}$ Also $\nu_{xy} = \nu_l$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_l}{1-\nu_l{}^2} \left(\frac{\partial^2 w}{\partial x^2} + \nu_l \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_l}{1-\nu_l{}^2} \left(\nu_l \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ \frac{E_l}{(1+\nu_l)} \frac{\partial^2 w}{\partial x \, \partial y} \end{Bmatrix} \tag{8}$$

- Mechanical properties of foam core

$E_{xx} = E_{f1}$, $E_{yy} = E_{f2}$ & $G_{xy} = G_l$, also $\nu_{xy} = \nu_{12}$, $\nu_{yx} = \nu_{21}$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -z \begin{Bmatrix} \frac{E_{f1}}{1-\nu_{12}\nu_{21}} \frac{\partial^2 w}{\partial x^2} + \frac{E_{f2}\nu_{12}}{1-\nu_{12}\nu_{21}} \frac{\partial^2 w}{\partial y^2} \\ \frac{E_{f1}\nu_{12}}{1-\nu_{12}\nu_{21}} \frac{\partial^2 w}{\partial x^2} + \frac{E_{f2}}{1-\nu_{12}\nu_{21}} \frac{\partial^2 w}{\partial y^2} \\ 2G_l \frac{\partial^2 w}{\partial x \, \partial y} \end{Bmatrix} \tag{9}$$

Using harmonic form plate stresses in the mid-plane of the plate varies in the z-direction across the plate thickness, corresponding to the equation. In terms of stress components, express the bending and twisting moments, as well as the shear force as [38]:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-z/2}^{z/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z \, dz \tag{10}$$

As σ_x, σ_yAnd τ_{xy}Are the normal and share stresses on the plate. Also, the part of ρh can be expressed as:

$$\rho h = \int_{-z/2}^{z/2} \rho \, dz \tag{11}$$

For a sandwich plate made of two faces with a thickness upward h_u and downward h_l And in between thick core with a thickness h_f as shown in Fig. 3.

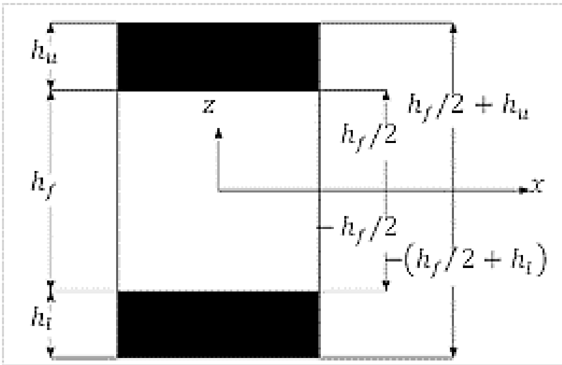

Fig. 3. Geometry of the sandwich plate layers.

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{Bmatrix} \int_{-\left(h_f/2+h_l\right)}^{-\left(h_f/2\right)} (\sigma_x)_l \, zdz + \int_{-\left(h_f/2\right)}^{\left(h_f/2\right)} (\sigma_x)_f \, zdz \\ + \int_{\left(h_f/2\right)}^{\left(h_f/2+h_u\right)} (\sigma_x)_l \, zdz \\ \int_{-\left(h_f/2+h_l\right)}^{-\left(h_f/2\right)} \left(\sigma_y\right)_l zdz + \int_{-\left(h_f/2\right)}^{\left(h_f/2\right)} \left(\sigma_y\right)_f zdz \\ + \int_{\left(h_f/2\right)}^{\left(h_f/2+h_u\right)} \left(\sigma_y\right)_u zdz \\ \int_{-\left(h_f/2+h_l\right)}^{-\left(h_f/2\right)} \left(\tau_{xy}\right)_l zdz + \int_{-\left(h_f/2\right)}^{\left(h_f/2\right)} \left(\tau_{xy}\right)_f zdz \\ + \int_{\left(h_f/2\right)}^{\left(h_f/2+h_u\right)} \left(\tau_{xy}\right)_u zdz \end{Bmatrix} \tag{12}$$

$$\rho h=\begin{bmatrix}\int_{-\left(h_f/_2+h_l\right)}^{-\left(h_f/_2\right)}\rho_l\,dz+\int_{-\left(h_f/_2\right)}^{\left(h_f/_2\right)}\rho_f\,dz\\+\int_{\left(-h_f/_2\right)}^{\left(h_f/_2+h_u\right)}\rho_u\,dz\end{bmatrix}\tag{13}$$

$$M_x=-\begin{bmatrix}\int_{-\left(h_f/_2+h_l\right)}^{-\left(h_f/_2\right)}\left[\frac{zE_l}{1-\nu_l^2}\left(\frac{\partial^2w}{\partial x^2}+\nu_l\frac{\partial^2w}{\partial y^2}\right)\right]zdz\\+\int_{-\left(h_f/_2\right)}^{\left(h_f/_2\right)}\begin{bmatrix}\frac{zE_{f1}}{1-\nu_{12}\nu_{21}}\frac{\partial^2w}{\partial x^2}\\+\frac{zE_{f2}\nu_{12}}{1-\nu_{12}\nu_{21}}\frac{\partial^2w}{\partial y^2}\end{bmatrix}zdz\\+\int_{\left(h_f/_2\right)}^{\left(h_f/_2+h_u\right)}\left[\frac{zE_u}{1-\nu_u^2}\left(\frac{\partial^2w}{\partial x^2}+\nu_u\frac{\partial^2w}{\partial y^2}\right)\right]zdz\end{bmatrix}$$

$$M_y=-\begin{bmatrix}\int_{-\left(h_f/_2+h_l\right)}^{-\left(h_f/_2\right)}\left[\frac{E_l}{1-\nu_l^2}\left(\nu_l\frac{\partial^2w}{\partial x^2}+\frac{\partial^2w}{\partial y^2}\right)\right]zdz\\+\int_{-\left(h_f/_2\right)}^{\left(h_f/_2\right)}\begin{bmatrix}\frac{zE_{f1}\nu_{12}}{1-\nu_{12}\nu_{21}}\frac{\partial^2w}{\partial x^2}\\+\frac{zE_{f2}}{1-\nu_{12}\nu_{21}}\frac{\partial^2w}{\partial y^2}\end{bmatrix}zdz\\+\int_{\left(h_f/_2\right)}^{\left(h_f/_2+h_u\right)}\left[\frac{zE_u}{1-\nu_u^2}\left(\nu_u\frac{\partial^2w}{\partial x^2}+\frac{\partial^2w}{\partial y^2}\right)\right]zdz\end{bmatrix}$$

$$M_{xy}=-\begin{bmatrix}\int_{-\left(h_f/_2+h_l\right)}^{-\left(h_f/_2\right)}\left[\frac{zE_l}{(1+\nu_l)}\frac{\partial^2w}{\partial x\,\partial y}\right]zdz\\+\int_{-\left(h_f/_2\right)}^{\left(h_f/_2\right)}\left[2zG_l\frac{\partial^2w}{\partial x\,\partial y}\right]zdz\\+\int_{\left(h_f/_2\right)}^{\left(h_f/_2+h_u\right)}\left[\frac{zE_u}{(1+\nu_u)}\frac{\partial^2w}{\partial x\,\partial y}\right]zdz\end{bmatrix}\tag{14}$$

After integrations, the equations of moments can be expressed as shown:

$$M_x=-\begin{bmatrix}\begin{bmatrix}\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)\\*\left(\frac{\partial^2w}{\partial x^2}+\nu_l\frac{\partial^2w}{\partial y^2}\right)\end{bmatrix}\\+\left[\left(\frac{h_f^3}{12}\right)\begin{bmatrix}\frac{E_{f1}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2w}{\partial x^2}\\+\frac{E_{f2}\nu_{12}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2w}{\partial y^2}\end{bmatrix}\right]\\+\begin{bmatrix}\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\\*\left(\frac{\partial^2w}{\partial x^2}+\nu_u\frac{\partial^2w}{\partial y^2}\right)\end{bmatrix}\end{bmatrix}$$

$$M_y=-\begin{bmatrix}\begin{bmatrix}\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)\\*\left(\nu_l\frac{\partial^2w}{\partial x^2}+\frac{\partial^2w}{\partial y^2}\right)\end{bmatrix}\\+\left(\frac{h_f^3}{12}\right)\begin{bmatrix}\frac{E_{f1}\nu_{12}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2w}{\partial x^2}\\+\frac{E_{f2}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2w}{\partial y^2}\end{bmatrix}\\+\begin{bmatrix}\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\\*\left(\nu_u\frac{\partial^2w}{\partial x^2}+\frac{\partial^2w}{\partial y^2}\right)\end{bmatrix}\end{bmatrix}$$

$$M_{xy}=-\begin{bmatrix}\left[\frac{E_l}{(1+\nu_l)}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)\frac{\partial^2w}{\partial x\,\partial y}\right]\\+\left[G_l\left(\frac{h_f^3}{6}\right)\frac{\partial^2w}{\partial x\,\partial y}\right]\\+\left[\frac{E_u}{(1+\nu_u)}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\frac{\partial^2w}{\partial x\,\partial y}\right]\end{bmatrix}\tag{15}$$

And also, for a term of the density of each layer multiplied by their height as written.

$$\rho h=\rho_lh_l+\rho_fh_f+\rho_uh_u\tag{16}$$

After integration where done, back substitution is resultant in the equilibrium equation.

$$\left(\begin{matrix}\begin{bmatrix}\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)\\+\frac{E_{f1}}{1-\nu_f^2}\left(\frac{h_f^3}{12}\right)\\+\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\end{bmatrix}\frac{\partial^4w}{\partial x^4}+\begin{bmatrix}\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)\\+\frac{E_{f2}}{1-\nu_f^2}\left(\frac{h_f^3}{12}\right)\\+\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\end{bmatrix}\frac{\partial^4w}{\partial y^4}+\\+\begin{bmatrix}\frac{2E_l}{(1+\nu_l)}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)\\+G_l\left(\frac{h_f^3}{3}\right)\\\frac{2E_u}{(1+\nu_u)}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\end{bmatrix}\frac{\partial^4w}{\partial x^2\,\partial y^2}\end{matrix}\right)+[\rho_lh_l+\rho_fh_f+\rho_uh_u]\frac{\partial^2w}{\partial t^2}=0\tag{17}$$

The above outcome is a 4th degree equation, with the solution of method of separation of variables assumption, as a function of displacement with variables x, y and function of time of t variable [39].

$$\begin{gathered}\omega(x,y,t)=W(x,y)T(t)\\T(t)=A\cos\omega t+B\sin\omega t\end{gathered}\tag{18}$$

And for a rectangular plate with length (a) and width (b) and simply supported on all the sides, as shown in fig. 4, the boundary conditions to be:

$$W(0,y)=0\text{ and }\left(\frac{\partial^2W_2}{\partial x^2}+\nu\frac{\partial^2W_2}{\partial y^2}\right)\Big|_{(0,y)}=0$$

$$W(a,y)=0\text{ and }\left(\frac{\partial^2W_2}{\partial x^2}+\nu\frac{\partial^2W_2}{\partial y^2}\right)\Big|_{(a,y)}=0$$

$$W(x,0)=0 \text{ and } \left(\frac{\partial^2 W_2}{\partial y^2}+\nu\frac{\partial^2 W_2}{\partial x^2}\right)\bigg|_{(x,0)}=0$$

$$W(x,b)=0 \text{ and } \left(\frac{\partial^2 W_2}{\partial y^2}+\nu\frac{\partial^2 W_2}{\partial x^2}\right)\bigg|_{(x,b)}=0 \qquad (19)$$

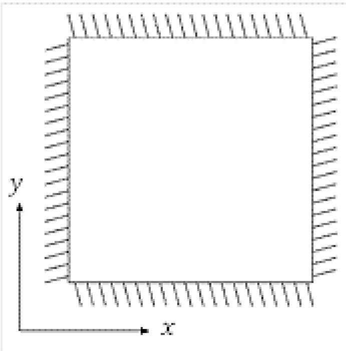

Fig. 4. Rectangular plate $(a * b)$ fixed in all directions.

With the applied condition, the displacement equation can be written as:

$$w=\mathrm{Sin}\frac{m\pi x}{a}\mathrm{Sin}\frac{n\pi y}{b} \qquad (20)$$

By driving the equation above the second derivative with x direction, y direction, and then xy direction, to solve the 4th degree equation.

$$\frac{\partial^4 w}{\partial x^4}=\frac{m^4\pi^4}{a^4}\mathrm{Sin}\frac{m\pi x}{a}\mathrm{Sin}\frac{n\pi y}{b}$$

$$\frac{\partial^4 w}{\partial y^4}=\frac{n^4\pi^4}{b^4}\mathrm{Sin}\frac{m\pi x}{a}\mathrm{Sin}\frac{n\pi y}{b} \qquad (21)$$

$$\frac{\partial^4 w}{\partial x^2\,\partial y^2}=\frac{m^2n^2\pi^4}{a^2b^2}\mathrm{Sin}\frac{m\pi x}{a}\mathrm{Sin}\frac{n\pi y}{b}$$

Back substitution the above in 4DOF equation as:

$$\left(\begin{aligned}&\left[\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)+\frac{E_{f1}}{1-\nu_{12}\nu_{21}}\left(\frac{h_f^3}{12}\right)+\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\right]\left(\frac{m^4\pi^4}{a^4}\right)\\&+\left[\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)+\frac{E_{f2}}{1-\nu_{12}\nu_{21}}\left(\frac{h_f^3}{12}\right)+\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\right]\frac{n^4\pi^4}{b^4}\\&+\left[\frac{2E_l}{(1+\nu_l)}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)+G_l\left(\frac{h_f^3}{3}\right)+\frac{2E_u}{(1+\nu_u)}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\right]\frac{m^2n^2\pi^4}{a^2b^2}\end{aligned}\right)W+[\rho_lh_l+\rho_fh_f+\rho_uh_u]\frac{\partial^2\omega}{\partial t^2}=0 \qquad (22)$$

By comparison, the resultant equation with the simple second-degree form, the result ω_{mn}^2 can be easily recognized:

$$\frac{\partial^2 w}{\partial t^2}+\omega_{mn}^2 wt=0 \qquad (23)$$

$$\omega_{mn}^2=\frac{\left(\begin{aligned}&\left[\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)+\frac{E_{f1}}{1-\nu_{12}\nu_{21}}\left(\frac{h_f^3}{12}\right)+\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\right]\left(\frac{m\pi}{a}\right)^4\\&+\left[\frac{E_l}{1-\nu_l^2}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)+\frac{E_{f2}}{1-\nu_{12}\nu_{21}}\left(\frac{h_f^3}{12}\right)+\frac{E_u}{1-\nu_u^2}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\right]\left(\frac{n\pi}{b}\right)^4\\&+\left[\frac{2E_l}{(1+\nu_l)}\left(\frac{h_l^3}{3}+\frac{h_l^2h_f}{2}+\frac{h_lh_f^2}{4}\right)+G_l\left(\frac{h_f^3}{3}\right)+\frac{2E_u}{(1+\nu_u)}\left(\frac{h_u^3}{3}+\frac{h_u^2h_f}{2}+\frac{h_uh_f^2}{4}\right)\right]\left(\frac{mn\pi^2}{ab}\right)^2\end{aligned}\right)}{\rho_lh_l+\rho_fh_f+\rho_uh_u} \qquad (24)$$

Since $a=b$ rectangular plate, the upper face and lower are the same $h_l=h_u=h_s$, also $v_{12}=v_{21}=v_f$, $E_{f1}=E_{f2}=E_f$. Taking $m=1$ & $n=1$, the equation can be expressed as:

$$\omega_{mn}^2=\frac{\left(\frac{\pi}{a}\right)^4\left(2\left[\frac{2E_s}{1-\nu_s^2}\left(\frac{h_s^3}{3}+\frac{h_s^2h_f}{2}+\frac{h_sh_f^2}{4}\right)+\frac{E_f}{1-\nu_f^2}\left(\frac{h_f^3}{12}\right)\right]+\left[\frac{4E_s}{(1+\nu_s)}\left(\frac{h_s^3}{3}+\frac{h_s^2h_f}{2}++\frac{h_sh_f^2}{4}\right)+G_s\left(\frac{h_f^3}{3}\right)\right]\right)}{2\rho_sh_s+\rho_fh_f} \qquad (25)$$

A rectangular sandwich plate has a natural frequency of ω_{mn}. All edges have simply supported boundary conditions.

III. Results

For rectangular sandwich plate $a = b = 30cm$. Using two Faces with a thickness of $h_s = 1mm$ of aluminum alloy (Al 1100-H12) has $E_s = 68.9$ GPa, $\rho_s = 2710\ kg/m^3$ & $\nu_s = 0.33$. Polyurethane foam (PU) adopted in the core part with a thickness of $h_f = 14mm$, PU properties, as inserted in the table below, has $\nu_f \sim 0.33$. The powder that has been used as a reinforcement particle to the PU of aluminum powder (Al 7429-90-5) with $E_{po.} = 68$ GPa, $\rho_{po.} = 2700\ kg/m^3$ & $\nu_{po.} = 0.36$.

To complete the result, applying various moduli of elasticity over varying foam densities according to the ASTM D1621, density varies between (60 to 450) kg/m^3 [40], and the result is listed below in table (1). The natural frequency and young's modulus are associated with adding micro powder, which can be seen in Fig. 5, also Fig. 6. The results are noticeable with the relation between the natural frequency and varying densities in Fig.7 and the young's modulus as shown in Fig.8.

Table 1.
The results of adopting varying density ranges with the variation in Volume Fraction.

$\forall f$ %	E_{fo} GPa	E_f GPa	ρ_{fo} kg/m^3	ρ_f kg/m^3	ω rad/sec
0.000	0.0045	0.00450	60	60	10087.0
0.005		0.00455		66	10020.0
0.010		0.00459		72	9954.4
0.015		0.00464		78	9890.0
0.020		0.00469		84	9826.8
0.025		0.00473		90	9764.8
0.000	0.0382	0.03820	100	100	9665.6
0.005		0.03859		110	9567.9
0.010		0.03899		120	9473.1
0.015		0.03938		130	9381.1
0.020		0.03979		140	9291.7
0.025		0.04019		150	9204.8
0.000	0.0595	0.05950	150	150	9205.7
0.005		0.06011		165	9079.8
0.010		0.06072		180	8958.9
0.015		0.06134		195	8842.8
0.020		0.06197		210	8731.1
0.025		0.06260		225	8623.4
0.000	0.0922	0.09220	200	200	8806.4
0.005		0.09314		220	8660.1
0.010		0.09410		240	8521.0
0.015		0.09506		260	8388.3
0.020		0.09603		280	8261.6
0.025		0.09701		300	8140.6
0.000	0.1276	0.12760	250	250	8455.2
0.005		0.12891		275	8294.1
0.010		0.13022		300	8141.8
0.015		0.13155		325	7997.7
0.020		0.13290		350	7861.0
0.025		0.13426		375	7731.0
0.000	0.1730	0.17300	300	300	8143.5
0.005		0.17477		330	7971.4
0.010		0.17656		360	7809.8
0.015		0.17836		390	7657.6
0.020		0.18018		420	7514.0
0.025		0.18203		450	7378.2
0.000	0.2229	0.22290	350	350	7864.3
0.005		0.22518		385	7684.1
0.010		0.22748		420	7515.7
0.015		0.22981		455	7358.0
0.020		0.23216		490	7209.8
0.025		0.23453		525	7070.2
0.000	0.2690	0.26900	400	400	7612.1
0.005		0.27175		440	7425.9
0.010		0.27453		480	7252.7
0.015		0.27734		520	7091.1
0.020		0.28017		560	6939.8
0.025		0.28303		600	6797.8
0.000	0.3229	0.32290	450	450	7383.2
0.005		0.32620		495	7192.5
0.010		0.32954		540	7015.9
0.015		0.33291		585	6851.7
0.020		0.33631		630	6698.5
0.025		0.33975		675	6555.2

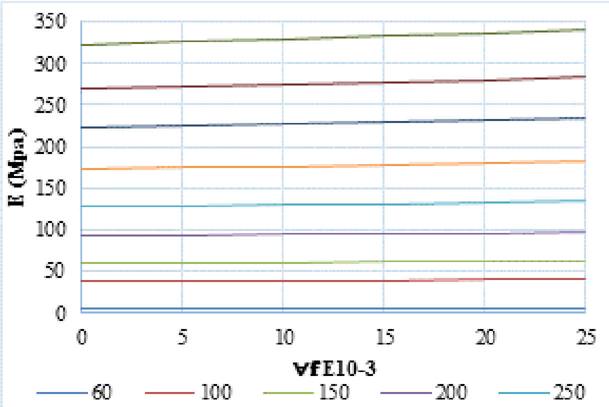

Fig. 5. Graph depicts elasticity Modulus and Volume fraction relationship at varying densities (60 to 450 kg/m^3).

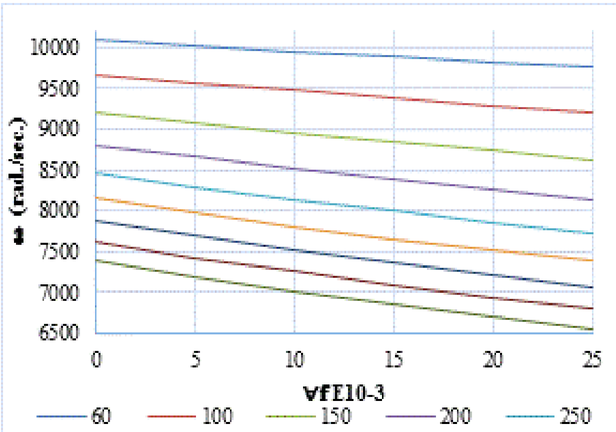

Fig. 6. Graph depicts natural frequency and Volume fraction relationship at varying densities (60 to 450 kg/m^3).

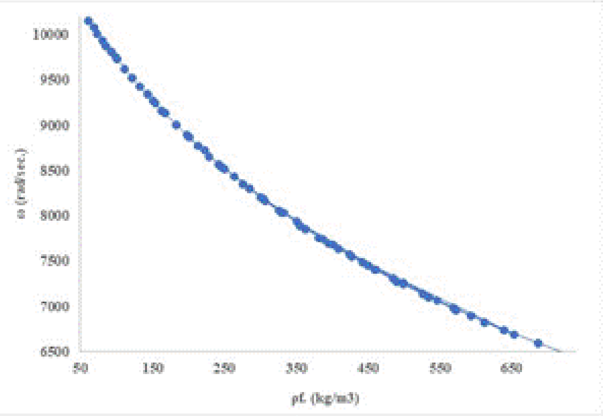

Fig. 7. Graph describes the natural frequency and density relationship.

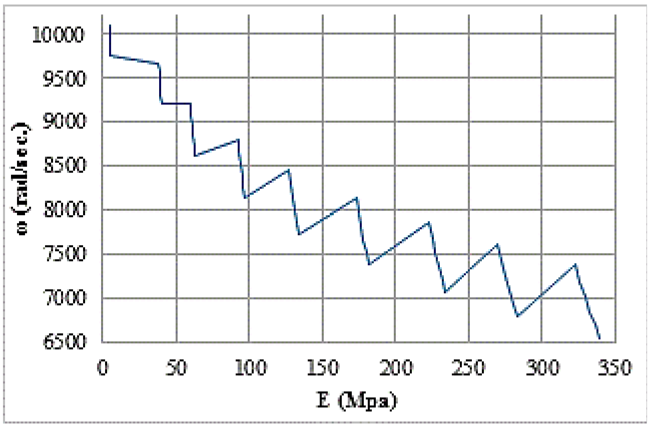

Fig. 8. Graph depicts natural frequency and elasticity Modulus relationship.

Conclusions

Several assumptions were made with the analytical solution of free vibration for a polyurethane foam core sandwich plate strengthened with aluminum micro spherical powder sandwiched between two aluminum faces. From the calculated natural frequencies, using Kirchhoff's theory in the vibration of the sandwich plate. According to the free vibration analysis, based on the findings above, we conclude that:

The impact of filling foam is effective.

The modules of elasticity increased with the addition, using micro spherical powder foam in the vacant spherical gaps of the foam core.

The sandwich plate's free vibration and static behavior can be improved by reducing the natural frequency.

Increasing proceed in two ways, first by using micro spherical powder with the befit of composite state and second by increasing the density of PU foam.

One assumption the solution is built on; the modules of elasticity are the same in the x & y direction; a hypothesis becomes a fact.

In comparison to other cores, the core foam-aluminum sandwich plate deflects less.

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Вільний вібраційний аналіз сендвіч-пластин з піни, зміцненої мікророзмірним алюмінієвим порошком

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У статті наведено аналітичне дослідження поведінки вільної вібрації сендвіч-пластин з піни, зміцненої алюмінієвим мікросферичним порошком. Сендвіч-пластина з пінополіуретану розміщена між двома алюмінієвими площинами. Для обчислення власної частоти використано теорію Кірхгофа для вібраційного рівняння сендвіч-пластини. Композитні рівняння мікрочастинок дозволили оцінити характеристики жорсткості піноалюмінієвої основи. Результати показали ефективний вплив наповнювальної піни; згідно із аналізом вільної вібрації, вільну вібрацію та статичну поведінку сендвіч-плит можна покращити за допомогою використання мікросферичної порошкоподібної піни у вільних сферичних порожнинах базового спіненого ядра. У порівнянні з іншими ядрами, сендвіч-пластина з піноалюмінію прогинається менше.

Ключові слова: сендвіч-пластини; аналітичний розв'язок; вільна вібрація; пінополіуретан; алюмінієвий порошок.