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Consideration of Boundary Conditions in the Scattering of Laser Radiation by Rough Fractal Surfaces

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The work within the scalar theory of Kirchhoff, solved the problem of scattering of laser light rough surface for the simulation which used two-dimensional modified Weierstrass function with a finite number of harmonics. Based on the analytical solution found numerically calculated scattering indicatrix laser fractal surfaces of several types for different angles of incidence and created a catalog of various specific types of scattering surfaces based on the Weierstrass function and the corresponding three-dimensional scattering indicatrix $\ln < \rho_S >$. It is shown that when scattering light by a surface of a fractal relief, the law of mirror reflection is violated, and the scattering image contains bursts of intensity in different directions.

Key words: fractal surfaces, Weierstrass function, scattering indicatrix, laser radiation, boundary conditions.

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Introduction

For modeling of rough surfaces in the last one or two decades, fractal functions are used. [1-6]. The application of fractals to the simulation of the terrain surface of rough surfaces is based on the fact that in many cases real surfaces are not either periodically pure or purely random. Proceeding from this, it can be expected that fractal functions, in which periodicity and randomness are in a certain way combined [1], will describe the real surfaces more adequately than previously used periodic and random functions. Despite some progress in the study of scattering of waves by fractal surfaces, there are not so many things done in this area. It should be noted that the overwhelming majority of published solutions to the scattering problem are obtained in the framework of the approximate Kirchhoff theory with the use of one-dimensional fractal functions for the simulation of the scattering surface. In this case, the model surface is fractal in only one direction, and in the other, depending on the modeling method, is either random or deterministic (for example, a folded surface). In addition, published analytical solutions to the problem of scattering electromagnetic waves by a fractal surface, use or too simplified surface models, or contain errors and therefore need to be clarified.

I. Weierstrass-Mandelbrot function

In theoretical studies of the scattering of electromagnetic waves by rough surfaces, there is a need to use one or another mathematical model of rough surface. By this time, different authors used deterministic, random and fractal functions to describe the rough surface relief. Often, for mathematical modeling of a roughness profile, used the Weierstrass function or its various modifications, which contain an infinite number of harmonics and are self-similar on an arbitrarily small scale. The presence of an infinite number of harmonics causes the inconvenience of these functions in terms of physical applications, and the one-dimensionality of a fractal model makes it impossible to construct fractal surfaces in all directions. Therefore we take for surface modeling one of the modifications that it looked at [1, 2] - two-dimensional band limited Weierstrass function:

$$W(x,y) = c_W \sum_{n=0}^{N-1} \sum_{m=1}^{M} q^{(D-3)n} \sin\left\{ Kq^n \left[x\cos\frac{2\pi m}{M} + y\sin\frac{2\pi m}{M} \right] + \varphi_{nm} \right\}, \quad (1)$$

where c_w – normalization constant; $q > 1$ – the main spatial frequency of the surface; K – fundamental wave number; D ($2 < D < 3$) – fractal dimension of the surface; N, M – the number of overtones; φ_{nm} – phase terms that have a uniform distribution over the interval [-π, π].

Function (1) is a combination of random structure and deterministic period. It is anisotropic in two directions, if the numbers of harmonics are not very large. It has derivatives and at the same time - self-similar. Since natural surfaces are not purely random or purely periodic, function (1) can serve as a good approximation for describing natural surfaces. The surface, modeled by the Weierstrass function, has many scales and its roughness may vary depending on the scale being considered.

II. Scattering of electromagnetic waves on fractal surface

When the electromagnetic wave falls to the surface of the rough surface, its scattering occurs - the reflected wave propagates not only in the direction of mirror reflection, but also in different directions. The intensity of the radiation scattered in one direction or another is defined as the parameters of the actual surface (reflection coefficient, height, shape and character of the location of the inequalities), and the parameters of the incident wave (frequency, polarization) and the parameters of the experiment (angle of incidence). The initial wave falls to a rough surface at an angle θ_1 and dissipates in all directions. The scattered wave is observed with the help of a detector in a direction characterized by polar θ_2 and azimuthal θ_3 angles. The measured magnitude is the intensity of light I_S scattered in the direction (θ_2, θ_3). Our task is to establish a connection between the intensity of light scattered by the fractal surface in one direction or another and the parameters of the surface and construct the indicatrix of scattering $I_s = I_s(\theta_2, \theta_3)$ of the electromagnetic wave by a rough surface (1).

The base formula of Kirchhoff method allows to find a scattered field $E_s(\vec{r}, t)$ under the following conditions [7,8]:

– the falling wave is monochromatic and plane;

– the scattered surface is rough inside rectangle ($-X<x_0<X$, $-Y<y_0<Y$) and smooth outside the border;

– the size of the rough area is much larger than the wavelength;

– all points of the surface have a complete gradient;

– the reflection coefficient is the same at all points of the surface;

– the scattered field is observed in the wave zone, *i.e.*, far enough away from the scattering surface.

Under these conditions, the scattered field can be expressed by the following formula [7]:

$$E_S(\vec{r}) = -ikrF(\theta_1, \theta_2, \theta_3)\frac{\exp(ikr)}{2\pi r}\int_{S_0}\exp[ik\varphi(x_0, y_0)]dx_0 dy_0 + E_e(\vec{r}), \tag{2}$$

where k is a wave number of the falling wave, R is a scattering coefficient, $F(\theta_1,\theta_2,\theta_3) = -\frac{R}{2C}(A^2 + B^2 + C^2)$ is an angle factor, $h(x_0, y_0) = c_w W(x_0, y_0)$, $A = \sin\theta_1 - \sin\theta_2\cos\theta_3$, $B = -\sin\theta_2\sin\theta_3$, $C = -\cos\theta_1 - \cos\theta_2$, a phase function is $\phi(x_0, y_0) = Ax_0 + By_0 + Ch(x_0, y_0)$, $E_e(\vec{r}) = -\frac{R}{C}\cdot\frac{\exp(ikr)}{4\pi r}(AI_1 + BI_2)$ – boundary term,

$$I_1 = \int_{-Y}^{Y}\left[e^{ik\varphi(X, y_0)} - e^{ik\varphi(-X, y_0)}\right]dy_0, \quad I_2 = \int_{-X}^{X}\left[e^{ik\varphi(x_0, Y)} - e^{ik\varphi(x_0, -Y)}\right]dx_0. \tag{3}$$

If set the parameters of the scattering surface (c_w (or σ), $D, q, K, N, M, X, Y, \varphi_{nm}$), the parameters of the falling wave k (or $\lambda = \frac{2\pi}{k}$) and the parameters of the experiment ($\theta_1, \theta_2, \theta_3$), then the formula $I_S = \vec{E}_s \cdot \vec{E}_s^*$ (where $\vec{E}_s$ is an electric field of the scattering wave) can be calculate the intensity of the scattered wave. Therefore, the problem of finding the intensity of the scattered wave I_S is reduced to finding a scattered field $\vec{E}_s$.

For conducting calculations it is necessary to operate the average for the ensemble of surfaces with intensity $\langle I_S \rangle = \langle \vec{E}_s \cdot \vec{E}_s^* \rangle$. This intensity is proportional to the intensity of the wave $I_0 = \left(\frac{2kXY\cos\theta_1}{\pi r}\right)^2$ reflected from the corresponding smooth surface, so it is more convenient to use the average scatter coefficient for theoretical analysis of the calculation results $\langle \rho_S \rangle \equiv \frac{\langle I_S \rangle}{I_0} = \frac{\langle \vec{E}_S \vec{E}_S^* \rangle}{I_0}$ (I_0 – the intensity of the wave reflected from the corresponding smooth surface). After calculating $\langle I_s \rangle$ we get the exact expression:

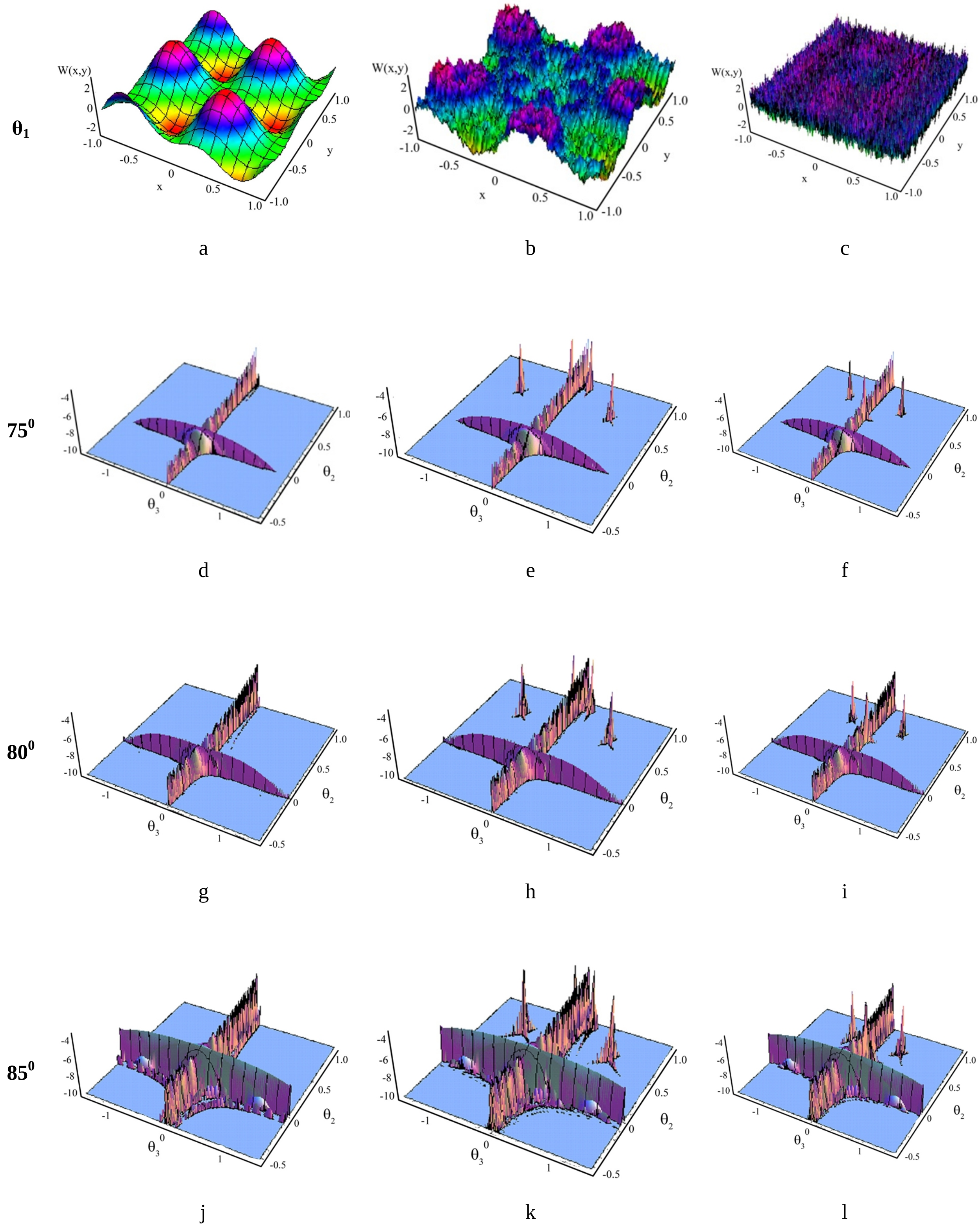

Fig. 1. Dependences of the scattering indicatrix $\ln < \rho_s >$ from the scattering angles θ_2 and θ_3 for the three angles of fall $\theta_1 = 75, 80, 85^\circ$, taking into account the boundary conditions for three different types of fractal surfaces, characterized by the following set of parameters: $M = 3$, $N = 2$, $D = 2.01$, $q = 1.01$; $N = 5$, $M = 5$, $D = 2.5$, $q = 3$; $N = 10$, $M = 10$, $D = 2.99$, $q = 7$, respectively.

$$\langle\rho_S\rangle \approx \left[\frac{F(\theta_1,\theta_2,\theta_3)}{\cos\theta_1}\right]^2 \begin{Bmatrix} \left[1-(k\sigma C)^2\right]\operatorname{sinc}^2(kAX)\operatorname{sinc}^2(kBY)+ \\ +\frac{1}{2}c_f^2\sum_{nm} q^{2(D-3)n}\operatorname{sinc}^2\left[\left(kA+Kq^n\cos\frac{2\pi m}{M}\right)X\right]\times \\ \operatorname{sinc}^2\left[\left(kB+Kq^n\sin\frac{2\pi m}{M}\right)Y\right] \end{Bmatrix} + \left[\frac{R}{2C\cos\theta_1}\left(A^2+B^2\right)\right]^2\operatorname{sinc}^2(kAX)\operatorname{sinc}^2(kBY), \quad (4)$$

where $c_f = kc_w C = k\sigma C\left[\frac{2}{M}\cdot\frac{1-q^{2(D-3)}}{1-q^{2N(D-3)}}\right]^{\frac{1}{2}}$.

Note that a similar solution to the scattering problem was found in [1,2,9], but our results have certain differences from the results of these works. In particular, the expression (4) for the averaged scattering coefficient contains an additional term:

$$\left[\frac{R}{2C\cos\theta_1}\left(A^2+B^2\right)\right]^2\operatorname{sinc}^2(kAX)\operatorname{sinc}^2(kBY), \quad (5)$$

It is connected by its origin with the contribution of the boundaries of a fragment of the scattering surface. This additional term gives a significant contribution to the resulting scattering indicatrix for certain geometry of the experiment. In addition, the expression we obtained (4) differs from the corresponding expression from [2] the numerical values of the coefficients before $(k\sigma C)^2$ and c_f^2.

III. Results of numerical calculations of the scattering coefficient

On the basis of expression (4) we carried out numerical calculations of the scattering coefficient and constructed graphs of dependence $\langle\rho_s\rangle$ on the polar θ_2 and azimuthal angles θ_3 (normalized scattering indicatrix) for different fractal surfaces (for different values of the fractal dimension *D*) and the angles of falling θ_1. The mean square altitude, the fundamental wave number of the surface *K* and the dimensions (*X, Y*) of the surface fragment are given in units k, since the wave number of the falling wave k is included in the calculation formulas in the form of dimensionless combinations $k\sigma$, kX and kY. We adopted $R = 1$, that is, did not take into account the real dependence of the reflection coefficient *R* on the length of the falling wave λ and the angle of falling θ_1.

Numerical calculations of three-dimensional indications of scattering were carried out using the original program developed by us in Mathematics 5.1 environment (Fig. 1). In this figure, only the boundary conditions of the scattering indicatrix for the angles 75^0, 80^0 and 85^0 are presented, since at at lower angles θ_1 there is no observed contribution of boundary conditions in the general spectrum of the scattering indicatrix.

The influence of each of the parameters *q, K, D, N, M* on the character of the surface profile is quite complex and is determined by the values of all other parameters. For example, with the value of $D = 2.1$, which is close to the minimum ($D = 2$), an increase in the value of *q* almost does not change the appearance of the surface. As the magnitude of *D* increases, the surface profile becomes more sensitive to the value. Note that with the increase of *N, M, D* and *q*, the spatial homogeneity of the surface increases on a large scale: large-scale "hills" disappear, and small-scale heterogeneities increasingly resemble ripples on a flat surface.

From the analysis of fig. 1, it follows that the boundary conditions substantially change the picture of scattering.

New peaks arise, the magnitude and shape of which depend on the angle of incidence. As the angle of incidence θ from 75 to 85° increases, the intensity of the peaks increases approximately three times.

The shape of the peaks and their location does not depend on the type (fractal structure) of the scattering surface. That is, the shape and intensity of the peaks are the same for each of the three types of scattering surface considered.

Additional peaks arising in the scattering picture, due to the contribution from the boundaries of the fragment of the scattering surface, are symmetrical in relation to the main peak.

Conclusions

It is shown that when scattering light by a surface of a fractal relief, the law of mirror reflection is violated, and the scattering image contains bursts of intensity in different directions. For *D* values that little differ from the integers, the largest intensity of the scattered wave is observed in the direction of the reflected wave and, in addition, there are other directions (lateral petals) formed as a result of Bragg's scattering, in which there are bursts of intensity. Taking into account the contribution of the boundaries of the fragment of the scattering surface somewhat changes the scattering pattern, especially for the angles of the fall $\theta_1 > 75^\circ$. New peaks arise, the size and shape of which do not depend on the angle of incidence.

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Врахування крайових умов при розсіюванні лазерного випромінювання шорсткими фрактальними поверхнями

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В роботі в рамках скалярної теорії Кірхгофа, розв’язано задачу розсіяння лазерного випромінювання шорсткою поверхнею для моделювання якої використовувалась двовимірна модифікація функції Вейєрштрасса зі скінченим числом гармонік. На основі знайденого аналітичного розв’язку чисельно розраховано індикатриси розсіювання лазерного випромінювання фрактальними поверхнями декількох типів для різних кутів падіння і створено каталог різноманітних характерних типів розсіюючих поверхонь на основі функції Вейерштраса а також відповідних тривимірних індикатрис розсіювання $\ln < \rho_s >$.

Ключові слова: фрактальні поверхні, функція Вейєрштрасса, індикатриси розсіювання, лазерне опромінення, крайові умови.