PHYSICS AND CHEMISTRY OF SOLIDSTATE
V. 18, № 2 (2017) P. 243-248
DOI: 10.15330/pcss.18.2.243-248

ФІЗИКА І ХІМІЯ ТВЕРДОГО ТІЛА
Т. 18, № 2 (2017) С. 243-248

UDK 539.3

ISSN 1729-4428

A.V. Pavliuk

Dynamics of Three-layer Cylindrical Shells Elliptical Cross-Section With a Longitudinal-Transverse Discrete Ribbed Filler

S.P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine, Nesterova str., 3, Kyiv, Ukraine, aniutapavliuk@gmail.com

In this paper, we consider the equations of non-axisymmetric oscillations of discretely reinforced multilayer cylindrical shells of elliptical section. When analyzing the elements of the elastic structure, a refinement model of the theory of shells and rods of the Timoshenko type is used. The numerical method of solving the dynamic equations is based on the integro- interpolation method of constructing the finite-difference schemes for equations with discontinuous coefficients. The problem of dynamic behavior of a three-layer longitudinal-transversal reinforced cylindrical shell of an elliptical section under a distributed nonstationary load is investigated. A solution of the problem on dynamic behaviour of the three-layered cylindrical shell with some discrete longitudinal-transverse ribbed filler is considered for distributed non-stationary loading.

Keywords: three-layer cylindrical shell, elliptic cross-section, Timoshenko-type theory, forced vibrations, numerical solution.

Article acted received 07.05.2017; accepted for publication 05.06.2017.

Introduction

The evaluation of the stress-strain state of a three-layer cylindrical shell with discrete ribbed filler is a rather complex task, the solution of which requires the development of certain theoretical models of layered plates and membranes. The implementation of these models causes the need to improve and develop effective numerical methods for calculating these structural elements. The complexity of constructing mechanical models of multilayer shells and the application of fundamentally different kinematic and static hypotheses leads to a large variety of calculation schemes and equations [1 - 22]. It is known that when constructing multilayer shell variants, there are two main approaches to constructing mathematical models that are based on the use of unified hypotheses for the entire package [3, 5, 7-11, 14-22] and hypotheses that take into account the kinematic and static characteristics of each layer [1, 6]. In accordance with the terminology proposed in [3, 6], the model and the theory of the second approach are called "discrete-structural" when considering three-layer shells with ribbed filler, the following approaches are also valid: the constructive-orthotropic model of three-layer membranes and the model taking into account the discrete location of the filler elements . Within the framework of the second approach axisymmetric and non-axisymmetric oscillations of three-layer shells with ribbed filler under non-stationary loads [11, 12] are considered.

I. Formulation of the problem

A three-layer cylindrical shell of an elliptic section with discrete longitudinal and transverse ribbed filler under the action of internal distributed non-stationary load is considered. The inhomogeneous three-layer elastic structure is two cylindrical shells of an elliptical cross-section (internal and external sheathing), which are rigidly interconnected by a system of longitudinal and transverse discrete ribs. Schematic representation of the original design is presented in Fig. 1

The coefficients of the first quadratic form and the curvature of the coordinate surface of the initial shells are taken as follows

$$A_1 = 1,\ k_2 = 0, \quad (1)$$

$$A_2 = (a_k^2 \cos^2 a_2 + b_k^2 \sin^2 a_2)^{1/2},$$

$$k_2 = a_k b_k (a_k^2 \cos^2 a_2 + b_k^2 \sin^2 a_2)^{-3/2},\ k=1,2;$$

where a_k and b_k – the axis of the ellipse, which characterizes the cross-section of the corresponding cylindrical shell.

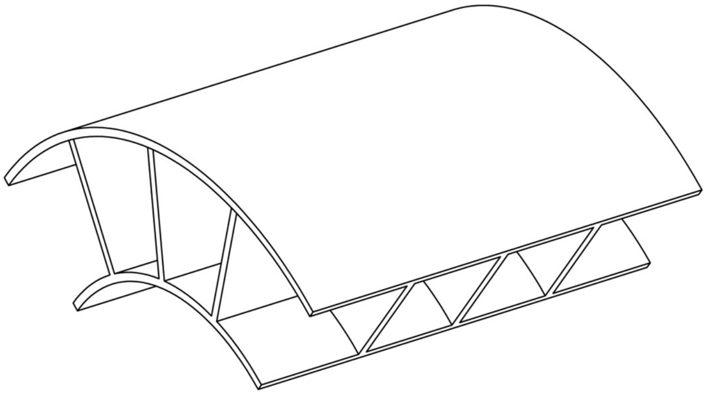

Fig. 1. Schematic representation of a three-layer cylindrical shell of an elliptic cross-section with a discrete longitudinal and transverse ribbed filler.

It is accepted that the deformed state of the inner and outer sheathing (respectively, indices 1 and 2) can be determined by the generalized vectors of displacements of the corresponding medial surfaces $\overline{U}_1 = (u_1^1, u_2^1, u_3^1, j_1^1, j_2^1)^T$ and $\overline{U}_2 = (u_1^2, u_2^2, u_3^2, j_1^2, j_2^2)^T$. When considering the elements of the discrete filler it is assumed that the deformed state of the rib, directed along the axis a_1, is determined by the vector of the center line displacement of the cross section of the *i*-th rib $\overline{U}_i = (u_{1i}, u_{2i}, u_{3i}, j_{1i}, j_{2i})$, and the deformed state of the transverse *j*-th rib, directed along the axis a_2, can be determined by a generalized vector of displacements $\overline{U}_j = (u_{1j}, u_{2j}, u_{3j}, j_{1j}, j_{2j})^T$ [2, 11].

To derive equation of oscillation of a three-layer elastic structure with a discrete filler, the variational principle of the stationary position of Hamilton-Ostrogradsky [2] is used. After standard transformations in the variational equation, taking into account the expressions for the potential and kinetic energies for the sheaths and edges in [2,11], we obtain two groups of equations. The equation of oscillations three-layered cylindrical shell elliptical cross section with discrete longitudinal-transverse filler written as:

- for internal and external linings

$$\frac{\partial T_{11}^k}{\partial s_1} + \frac{\partial S^k}{\partial s_2} = r_k h_k \frac{\partial^2 u_1^k}{\partial t^2};$$

$$\frac{\partial S^k}{\partial s_1} + \frac{\partial T_{22}^k}{\partial s_2} + k_2 T_{23}^k = r_k h_k \frac{\partial^2 u_2^k}{\partial t^2};$$

$$\frac{\partial T_{13}^k}{\partial s_1} + \frac{\partial T_{23}^k}{\partial s_2} - k_2 T_{22}^k + P_3^k(s_1, s_2, t) = r_k h_k \frac{\partial^2 u_3^k}{\partial t^2}; \qquad (2)$$

$$\frac{\partial M_{11}^k}{\partial s_1} + \frac{\partial H^k}{\partial s_2} - T_{13}^k = r_k \frac{h_k^3}{12} \frac{\partial^2 j_1^k}{\partial t^2};$$

$$\frac{\partial H^k}{\partial s_1} + \frac{\partial M_{22}^k}{\partial s_2} - T_{23}^k = r_k \frac{h_k^3}{12} \frac{\partial^2 j_2^k}{\partial t^2}; \; k = 1,2;$$

- for the *i*-th longitudinal rib

$$[S]_i + \frac{\partial T_{11i}}{\partial s_1} = r_i F_i \frac{\partial^2 u_{1i}}{\partial t^2};$$

$$[T_{22}]_i + \frac{\partial T_{12i}}{\partial s_1} = r_i F_i \frac{\partial^2 u_{2i}}{\partial t^2}; \qquad (3)$$

$$[T_{23}]_i + \frac{\partial T_{13i}}{\partial s_1} = r_i F_i \frac{\partial^2 u_{3i}}{\partial t^2};$$

$$[H]_i + \frac{\partial M_{11i}}{\partial s_1} - T_{13i} = r_i I_{1i} \frac{\partial^2 j_{1i}}{\partial t^2};$$

$$[M_{22}]_i + \frac{\partial M_{12i}}{\partial s_1} = r_i I_{kri} \frac{\partial^2 j_{2i}}{\partial t^2}; \; i = \overline{1, I}.$$

In ratios (3), the magnitudes of the type $[S]_i, [T_{22}]_i, [T_{23}]_i$, $[H]_i, [M_{22}]_i$ correspond to the total value of the forces-moments of the outer and inner sheaths acting on the *i*-th discrete element of the filler.

for the *j*-th transverse rib

$$\frac{\partial T_{21j}}{\partial s_2} + [T_{11}]_j = r_j F_j \left(\frac{\partial^2 u_1}{\partial t^2} \pm h_{cj} \frac{\partial^2 j_1}{\partial t^2} \right), \qquad (4)$$

$$\frac{\partial T_{22j}}{\partial s_2} + k_{2j} T_{23j} + [S]_j = r_j F_j \left(\frac{\partial^2 u_2}{\partial t^2} \pm h_{cj} \frac{\partial^2 j_2}{\partial t^2} \right),$$

$$\frac{\partial T_{23j}}{\partial s_2} - k_{2j} T_{22j} + [T_{13}]_j = r_j F_j \frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{\partial M_{21j}}{\partial s_2} \pm h_{cj} \frac{\partial T_{21j}}{\partial s_2} + [M_{11}]_j = r_j F_j \left(\pm h_{cj} \frac{\partial^2 u_1}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{crj}}{F_j} \right) \frac{\partial^2 j_1}{\partial t^2} \right)$$

$$\frac{\partial M_{22j}}{\partial s_2} - T_{23j} \pm h_{cj} \left(\frac{\partial T_{22j}}{\partial s_2} + k_{2j} T_{23j} \right) + [H]_j =$$

$$= r_j F_j \left(\pm h_{cj} \frac{\partial^2 u_2}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{2j}}{F_j} \right) \frac{\partial^2 j_2}{\partial t^2} \right)$$

$$\frac{\partial T_{21j}}{\partial s_2} + [T_{11}]_j = r_j F_j \left(\frac{\partial^2 u_1}{\partial t^2} \pm h_{cj} \frac{\partial^2 j_1}{\partial t^2} \right), \qquad (4)$$

$$\frac{\partial T_{22j}}{\partial s_2} + k_{2j} T_{23j} + [S]_j = r_j F_j \left(\frac{\partial^2 u_2}{\partial t^2} \pm h_{cj} \frac{\partial^2 j_2}{\partial t^2} \right),$$

$$\frac{\partial T_{23j}}{\partial s_2} - k_{2j}T_{22j} + [T_{13}]_j = r_j F_j \frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{\partial M_{21j}}{\partial s_2} \pm h_{cj}\frac{\partial T_{21j}}{\partial s_2} + [M_{11}]_j = r_j F_j\left(\pm h_{cj}\frac{\partial^2 u_1}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{crj}}{F_j}\right)\frac{\partial^2 j_1}{\partial t^2}\right)$$

$$\frac{\partial M_{22j}}{\partial s_2} - T_{23j} \pm h_{cj}\left(\frac{\partial T_{22j}}{\partial s_2} + k_{2j}T_{23j}\right) + [H]_j =$$

$$= r_j F_j\left(\pm h_{cj}\frac{\partial^2 u_2}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{2j}}{F_j}\right)\frac{\partial^2 j_2}{\partial t^2}\right)$$

In the equations of oscillation of the discrete reinforcing ribs (4), the designations of the type $[S]_j$ correspond to the total action of the values of the forces - moments of the smooth cylindrical shell of the elliptic section on the *j*-th reinforcement rib.

In the equations (2) – (4) u_1, u_2, u_3, j_1, j_2 – components of a generalized motion vector of the median surface of the shell; r, r_i, r_j – densitys material shell *i*-th, *j*-th ribs respectively; *h* – the thickness of the shell; $h_{ci} = 0{,}5(h + h_i)$; h_i – height of the cross-section of the *i*-th rib; $h_{cj} = 0{,}5(h + h_j)$; h_j – Height of the cross-section of the *j*- th rib. Values $[f]_i = f^+ - f^-$, де $f^\pm$ – the value of the functions on the right and left on the rupture line (line designing the center of gravity of the *i*-th rib on the median surface of the cylindrical shell). Accordingly, quantities are determined $[f]_j = f^+ - f^-$.

The values of effort-moments in the equations of oscillation for the shell (2) are related to the corresponding values of deformation by the following relations

$$T_{11} = B_{11}(e_{11} + n_2 e_{22}), \quad T_{22} = B_{22}(e_{22} + n_1 e_{11}), \qquad (5)$$

$$T_{13} = B_{13}e_{13}, \ T_{23} = B_{23}e_{23}, \ S = B_{12}e_{12},$$

$$M_{11} = D_{11}(k_{11} + n_2 k_{22}),$$

$$M_{22} = D_{22}(k_{22} + n_1 k_{11}), \ H = D_{12}k_{12},$$

$$e_{11} = \frac{\partial u_1}{\partial s_1}, \qquad e_{22} = \frac{\partial u_2}{\partial s_2} + k_2 u_3,$$

$$e_{12} = \frac{\partial u_1}{\partial s_2} + \frac{\partial u_2}{\partial s_1}, \qquad e_{13} = f_1 + \frac{\partial u_3}{\partial s_1},$$

$$e_{23} = f_2 + \frac{\partial u_3}{\partial s_2} - k_2 u_2,$$

$$k_{11} = \frac{\partial j_1}{\partial s_1}, \quad k_{22} = \frac{\partial j_2}{\partial s_2}, \qquad k_{12} = \frac{\partial j_1}{\partial s_2} + \frac{\partial j_2}{\partial s_1}$$

In ratios (5) the following notation is introduced:

$$B_{11} = \frac{E_1 h}{1 - n_1 n_2}, \quad B_{22} = \frac{E_2 h}{1 - n_1 n_2},$$

$$B_{12} = G_{12}h, \ B_{13} = G_{13}h, \quad B_{23} = G_{23}h,$$

$$D_{11} = \frac{E_1 h^3}{12(1 - n_1 n_2)}, D_{22} = \frac{E_2 h^3}{12(1 - n_1 n_2)}, \ D_{12} = G_{12}\frac{h^3}{12},$$

where E_1, E_2, G_{12}, G_{13}, G_{23}, n_1, n_2 – physical and mechanical parameters of orthotropic shell material.

The values of effort-moments in the oscillation equations for the i-th rib (3) are related to the corresponding values of the deformations according to the relations

$$T_{11i} = E_i F_i e_{11i}, \ T_{12i} = G_i F_i e_{12i}, \ T_{13i} = G_i F_i e_{13i}, \quad (6)$$

$$M_{11i} = E_i I_{1i} k_{11i}, \ M_{12i} = G_i I_{cri} k_{12i},$$

$$e_{11i} = \frac{\partial u_1}{\partial s_1} \pm h_{ci}\frac{\partial j_1}{\partial s_1}, \qquad e_{22i} = \frac{\partial u_2}{\partial s_2} \pm h_{ci}\frac{\partial j_2}{\partial s_1},$$

$$e_{13} = j_1 + \frac{\partial u_3}{\partial s_1}, \qquad k_{11i} = \frac{\partial j_1}{\partial s_1}, \qquad k_{11i} = \frac{\partial j_2}{\partial s_1}.$$

In ratios (6) E_i, G_i – physical and mechanical parameters of ribs material; F_i, I_{1i}, I_{cri} – geometric parameters of the cross-section of i -th rib.

The values of effort-moments in the oscillation equations for the j -th rib (4) are related to the corresponding values of the deformations according to the relations

$$T_{11j} = E_i F_i e_{11j}, \ T_{12j} = G_j F_j e_{12j}, \ T_{13j} = G_j F_j e_{13j}, \quad (7)$$

$$M_{11j} = E_j I_{1j} k_{11j}, \ M_{12j} = G_j I_{torj} k_{12j},$$

$$e_{11j} = \frac{\partial u_1}{\partial s_1} \pm h_{cj}\frac{\partial j_1}{\partial s_1}, \qquad e_{22j} = \frac{\partial u_2}{\partial s_2} \pm h_{cj}\frac{\partial j_2}{\partial s_1},$$

$$\varepsilon_{13} = \varphi_1 + \frac{\partial u_3}{\partial s_1}, \qquad \kappa_{11j} = \frac{\partial \varphi_1}{\partial s_1}, \qquad \kappa_{11j} = \frac{\partial \varphi_2}{\partial s_1}.$$

In ratios (7) E_j, G_j – physical and mechanical parameters of ribs material; F_j, I_{1j}, I_{torj} – geometric parameters of the cross-section of j -th rib.

The equations of oscillations (2)-(7) are supplemented by the corresponding boundary and initial conditions.

II. Research results

2.1. Numerical algorithm.

The numerical algorithm for the solution of the initial boundary value problem (2) – (7) is based on the application of the integro-interpolation method of constructing difference relations on the spatial coordinates s_1, s_2 and the explicit approximation of the time coordinate t [2, 11].

According to the initial formulation of the problem, the solution is sought in a smooth domain (equation (2), (5)) and is glued on lines of discontinuities (equations (3), (4)). Let us build solutions in a smooth region $D = \{s_{10} \le s_1 \le s_{1N};\ s_{20} \le s_2 \le s_{2N}\}$. Choose a subregion $D_{kl}^1 \subset D$,

$D_{kl}^1 = \{s_{1k-1/2} \le s_1 \le s_{1k+1/2};\ s_{2l-1/2} \le s_2 \le s_{2l+1/2}\}$

and integrate the equation of oscillation (2) in this subregion. As a result, we obtain such a difference in the ratio of finding solutions in the $(n+1)$-th time layer.

$$\frac{T_{11k+1/2,l}^n - T_{11k-1/2,l}^n}{\Delta s_1} + \frac{S_{k,l+1/2}^n - S_{k,l-1/2}^n}{\Delta s_2} = rh(u_{1k,l}^n)_{\bar{t}t}, \quad (8)$$

$$\frac{S_{k+1/2,l}^n - S_{k-1/2,l}^n}{\Delta s_1} + \frac{T_{22k,l+1/2}^n - T_{22k,l-1/2}^n}{\Delta s_2} +$$

$$+ \frac{k_{2l}}{2}(T_{23k,l+1/2}^n - T_{23k,l-1/2}^n) = rh(u_{2k,l}^n)_{\bar{t}t},$$

$$\frac{T_{13k+1/2,l}^n - T_{13k-1/2,l}^n}{\Delta s_1} + \frac{T_{23k,l+1/2}^n - T_{23k,l-1/2}^n}{\Delta s_2} -$$

$$- \frac{k_{2l}}{2}(T_{22k,l+1/2}^n - T_{22k,l-1/2}^n) + P_{3k,l}^n = rh(u_{3k,l}^n)_{\bar{t}t},$$

$$\frac{M_{11k+1/2,l}^n - M_{11k-1/2,l}^n}{\Delta s_1} + \frac{H_{k,l+1/2}^n - H_{k,l-1/2}^n}{\Delta s_2} -$$

$$- \frac{1}{2}(T_{13k+1/2,l}^n + T_{13k-1/2,l}^n) = r\frac{h^3}{12}(j_{1k,l}^n)_{\bar{t}t},$$

$$\frac{H_{k+1/2,l}^n - H_{k-1/2,l}^n}{\Delta s_1} + \frac{M_{22k,l+1/2}^n - M_{22k,l-1/2}^n}{\Delta s_2} -$$

$$- \frac{1}{2}(T_{23k,l+1/2}^n + T_{23k,l-1/2}^n) = r\frac{h^3}{12}(j_{2k,l}^n)_{\bar{t}t}.$$

So in difference ratios the values of generalized displacements u_1, u_2, u_3, j_1, j_2 are correlated to the whole nodes of the spatial difference grid, and the values of effort-moments (correspondingly deformations) are correlated to half-knots $(k \pm 1/2, l), (k, l \pm 1/2)$. To obtain the agreed difference relations for effort-moments of the equation (5), they are integrated by region

$$D_{kl}^2 = \{s_{1k-1} \le s_1 \le s_{1k};\ s_{2l-1/2} \le s_2 \le s_{2l+1/2}\},$$

$$D_{kl}^3 = \{s_{1k} \le s_1 \le s_{1k+1};\ s_{2l-1/2} \le s_2 \le s_{2l+1/2}\},$$

etc. In ratios (8) the notation of difference derivatives is introduced according to [11]. Similarly constructed value difference equations fluctuations reinforcing *i*-th and *j*-th ribs. The above mentioned approach to the construction of difference schemes allows us to fulfill the law of conservation of the total mechanical energy of the output elastic system at a difference level.

2.2. Numerical results.

As a partial case of a three-layer cylindrical shell of an elliptic section, the problem of forced fluctuations of three-layer cylindrical shells of circular cross section with a longitudinal-transverse discrete ribbed filler with internally distributed pulsed loading is considered.

The problem of forced fluctuations of a three-layer cylindrical shell with a discrete longitudinal-transverse ribbed filler with internal distributed pulse loading is considered. It is assumed that the edges of the shell and the elements of the longitudinal filler are rigidly fixed. The boundary conditions for this case $x = 0,\ x = L$ have the following form

$$u_1^k = u_2^k = u_3^k = j_1^k = j_2^k = 0,\ k = \overline{1,2};$$

$$u_{1i} = u_{2i} = u_{3i} = j_{1i} = j_{2i} = 0,\ i = \overline{1,I}.$$

The initial conditions for the given system of equations are zero. The problem was considered in the following geometric and physical-mechanical parameters:

$L/h_1 = 80,\quad h_1 = h_2,$ $h_j = h_i,\ j = \overline{1,J};$

$R_1/h_1 = 20,\quad h_i = 2h_1,\ i = \overline{1,I};$

$$E_1^1 = E_1^2 = E_i = E_j = 7 \cdot 10^{10}\ \text{Па};$$

$n_1^1 = n_1^2 = 0.3;\quad r_1 = r_2 = r_i = r_j = 2.7 \cdot 10^3$, kg/m^3,

where R_k, h_k – the radius of the middle surface and thickness of the inner lining; L – length of construction. Considered the case of a longitudinal-transverse discrete filler at $I = 4$ and $J = 3$, When the discrete elements are evenly spaced along the spatial coordinates between the inner and outer sheaths. Centers of cross-sectional gravity of discrete elements of the aggregate are projected onto the corresponding middle surfaces of the panels along the lines $y_i = (i-1)pR/2,\quad i = \overline{1,4}$ and $x_j = jL/4,\quad j = \overline{1,3}$. Normal impulse load was given in the form $P_3^1 = A \cdot [h(t) - h(t-T)]$, where $h(t)$ – Heaviside's function, A – load amplitude, T – load time. In the calculations it was laid $A = 10^6$ Pa; $T = 50 \cdot 10^{-6}$ s.

The results of calculations of this task are shown in Figure 2. The curve with index 1 corresponds to the value of the inner shell, and the curve with the index 2 corresponds to the value of the outer shell. In particular, the dependence of the value u_3 between the longitudinal edges on the axis of symmetry on the spatial coordinate x at time $t = 7.5T$. Based on the presented material, the location of the transverse discrete reinforcing ribs ($x_j = jL/4,\quad j = \overline{1,3}$) – these are points of connection of curves with indexes 1 and 2. In fig. 3 shows the dependence of the magnitude u_3 along the center of the weight of the cross section of the longitudinal discrete edge from the spatial coordinate x at the instant of time $t = 7.5T$. In this case, there is one curve, depicted in the figure. The comparison of the values of u_3 along the symmetry line between the ribs (Fig. 2) and the longitudinal ribs find it possible to characterize the effect of the longitudinal-transverse discrete aggregate on the distribution of the kinematic parameters of the output elastic structure. Calculations of the stress-strain state of the dynamic behavior of the three-layer membranes, taking into account the ribbed filler, were carried out on a time interval $t = 40T \div 60T$. In this paper, the characteristic dependences of the stress-strain state of the three-layer shell are presented. In particular, moments of time when the corresponding kinematic and static values reached the maximum values were considered. It was observed that the maximum of the values of u_3 along the symmetry line between the edges and along the edge is reached at the time $t = 7.5T$, where T – duration of non-stationary load. Comparative analysis values u_3 Along the edges (Fig. 3) and value u_3 the line of symmetry between the ribs shows (Fig.2), that the difference in maximum values of u_3 reaches an order of 1.8 times.

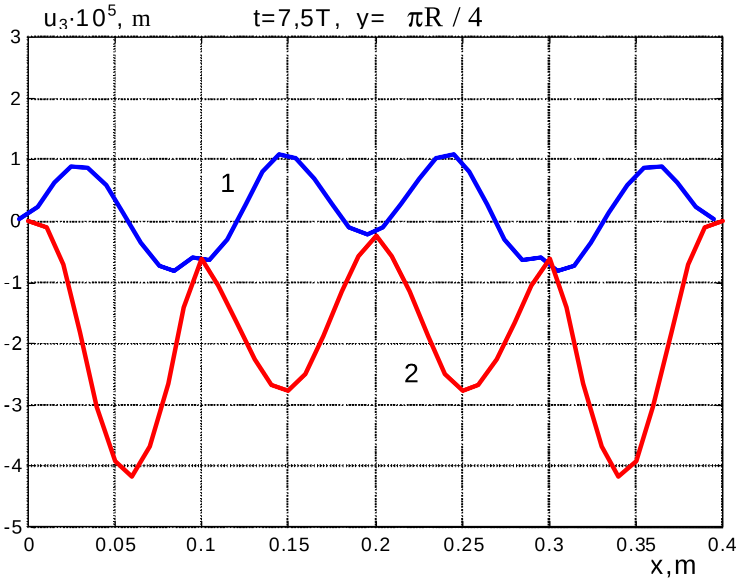

Fig. 2. Dependence of the value u_3 between the edges on the axis of symmetry on the spatial coordinate x at the instant of time $t = 7.5T$.

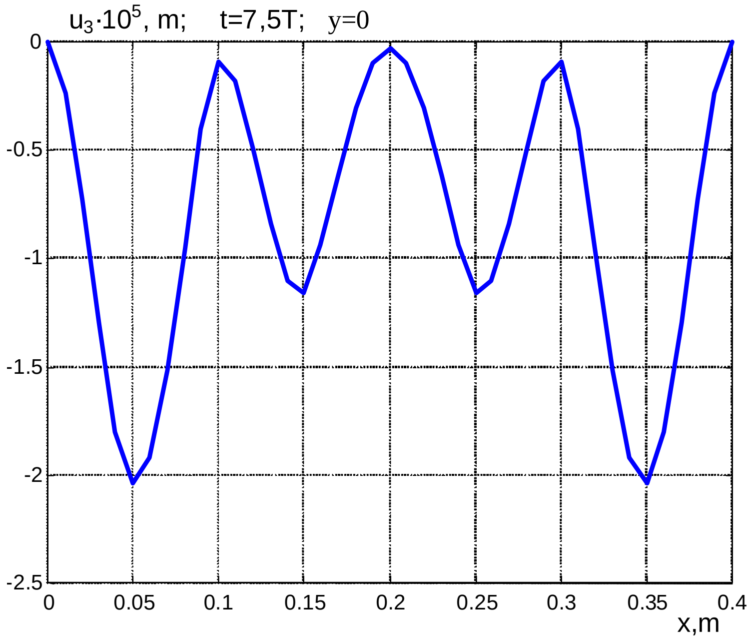

Fig. 3. The dependence of the value u_3 along the center line of the cross-sectional dimension of the longitudinal discrete edge from the spatial coordinate x at the instant of time $t = 7.5T$.

Conclusions

In this paper, based on the Hamilton-Ostrogradsky variational principle, we obtain the equations of oscillation of three-layer cylindrical shells of an elliptic section with discrete ribbed filler. When considering the elements of an elastic structure, models of shells and ribs are used in accordance with the hypothesis of Tymoshenko. An effective numerical method is developed for solving the obtained equations, which is based on the application of integro-interpolation relations by spatial coordinates and an explicit finite-difference scheme in time coordinate. The solution of the problem of the dynamic behavior of a three-layer cylindrical shell of an elliptic section with discrete ribbed filler under the action of a pulsed load is obtained. The analysis of the received results is given.

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А.В. Павлюк

Динаміка тришарової циліндричної оболонки еліптичного перерізу з поздовжньо-поперечним ребристим дискретним наповнювачем

Інститут механіки Національної академії наук України ім. С.П. Тимошенка, вул. Нестерова 3, 03057, Київ, Україна, aniutapavliuk@gmail.com

В роботі розглядаються рівняння неосесиметричних коливань дискретно підкріплених тришарових циліндричних оболонок еліптичного перерізу. При аналізі елементів пружної структури використовується уточнююча модель теорії оболонок і стержнів типу Тимошенка. Досліджено задачу динамічної поведінки тришарової поздовжньо-поперечної підкріпленої циліндричної оболонки еліптичного перерізу при розподіленому нестаціонарному навантаженні.

Ключові слова: тришарова циліндрична оболонка, еліптичний переріз, теорія типу Тимошенка, вимушені коливання, чисельний розв’язок.